

# Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.5-  
 $a+b-x+c-x^2-p-d+e-x+f-x^2-q$

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# Chapter 1

## Introduction

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This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 118 ]. This is test number [ 23 ].

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. IntegrateAlgebraic under Mathematica 12.3.1 on windows 10. [https://github.com/stblake/algebraic\\_integration](https://github.com/stblake/algebraic_integration). September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 ( 118 )	0.00 ( 0 )
Mathematica	100.00 ( 118 )	0.00 ( 0 )
Maple	100.00 ( 118 )	0.00 ( 0 )
Fricas	94.07 ( 111 )	5.93 ( 7 )
Giac	76.27 ( 90 )	23.73 ( 28 )
IntegrateAlgebraic	61.02 ( 72 )	38.98 ( 46 )
Maxima	56.78 ( 67 )	43.22 ( 51 )
Mupad	44.92 ( 53 )	55.08 ( 65 )
Sympy	36.44 ( 43 )	% 63.56 ( 75 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

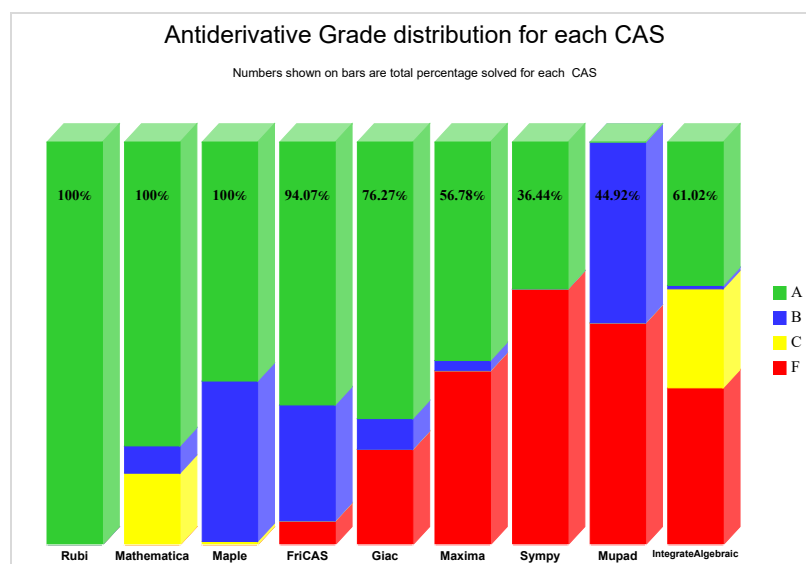
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

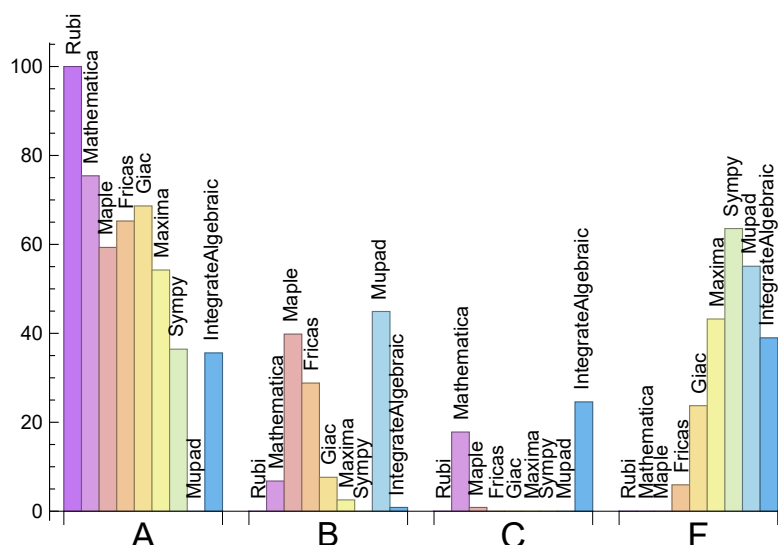
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	75.42	6.78	17.80	0.00
Giac	68.64	7.63	0.00	23.73
Fricas	65.25	28.81	0.00	5.93
Maple	59.32	39.83	0.85	0.00
Maxima	54.24	2.54	0.00	43.22
Sympy	36.44	0.00	0.00	63.56
IntegrateAlgebraic	35.59	0.85	24.58	38.98
Mupad	N/A	44.92	0.00	55.08

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	7	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	46	93.48 %	6.52 %	0.00 %
Giac	28	3.57 %	17.86 %	78.57 %
Maxima	51	60.78 %	0.00 %	39.22 %
Sympy	75	84.00 %	16.00 %	0.00 %
Mupad	65	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

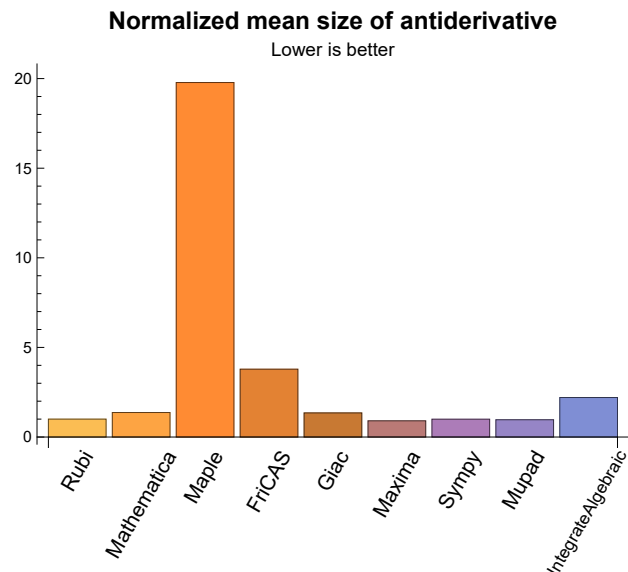
### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

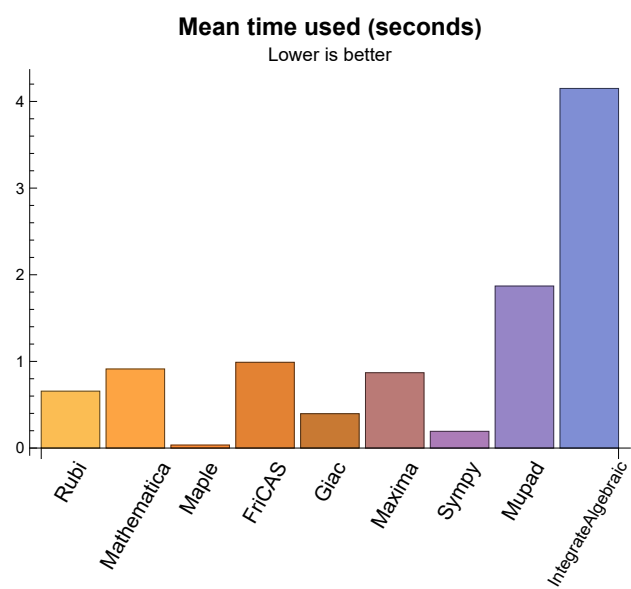
System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.66	184.69	1.00	124.00	1.00
Mathematica	0.91	319.82	1.37	94.00	1.00
Maple	0.03	6393.43	19.78	134.00	0.83
Maxima	0.87	90.12	0.90	72.00	0.83
Fricas	0.99	827.68	3.79	112.00	1.24
Sympy	0.19	76.86	0.99	73.00	0.98
Giac	0.40	203.17	1.35	72.50	0.79
Mupad	1.87	105.40	0.96	64.00	0.83
IntegrateAlgebraic	4.15	1095.07	2.20	187.50	1.10

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.







## 1.4 list of integrals that has no closed form antiderivative

{

## 1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {6, 103, 104, 105, 110}

**IntegrateAlgebraic** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

### 1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

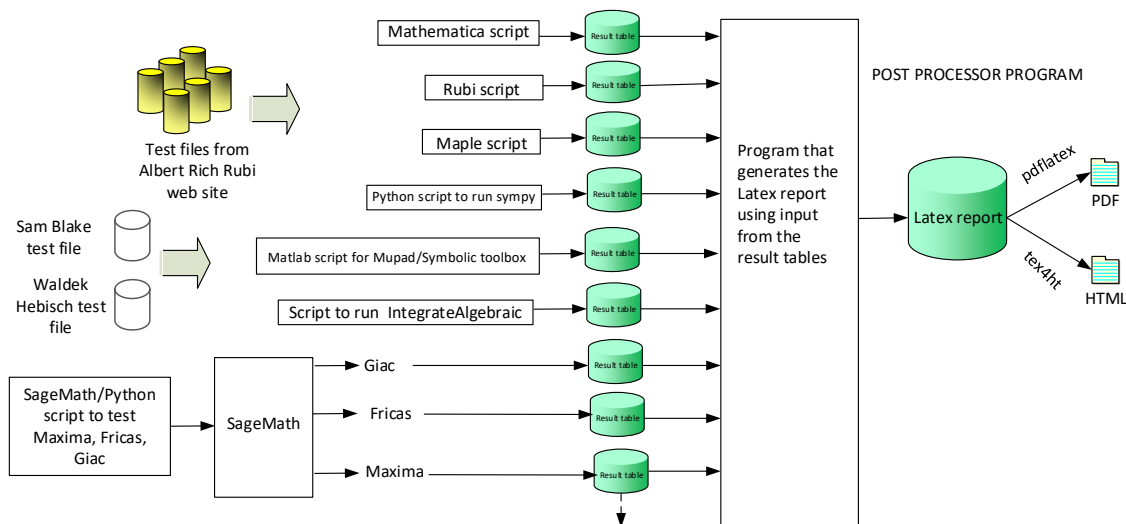
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x) \sim 2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"  
*The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.  
*The following fields present only in Rubi Tables*
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**





# Chapter 2

## detailed summary tables of results

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## 2.1 List of integrals sorted by grade for each CAS

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### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 69, 70, 71, 72, 76, 77, 78, 79, 83, 84, 85, 86, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117 }

B grade: { 2, 3, 4, 5, 6, 7, 104, 105 }

C grade: { 8, 11, 59, 60, 61, 66, 67, 68, 73, 74, 75, 80, 81, 82, 87, 88, 89, 94, 95, 96, 118 }

F grade: { }

### 2.1.3 Maple

A grade: { 1, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 69, 70, 71, 72, 76, 77, 78, 79, 83, 84, 85, 86, 90, 93, 108, 117 }

B grade: { 2, 3, 4, 5, 6, 7, 11, 59, 60, 61, 66, 67, 68, 73, 74, 75, 80, 81, 82, 87, 88, 89, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 118 }

C grade: { 9 }

F grade: { }

### 2.1.4 Maxima

A grade: { 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 69, 70, 71, 72, 76, 77, 78, 79, 83, 84, 85, 86, 93 }

B grade: { 90, 91, 92 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 59, 60, 61, 66, 67, 68, 73, 74, 75, 80, 81, 82, 87, 88, 89, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

### 2.1.5 FriCAS

A grade: { 1, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 69, 70, 71, 72, 76, 77, 78, 79, 83, 84, 85, 90, 91, 93, 97, 98, 102, 106, 107, 108, 116 }

B grade: { 2, 3, 4, 5, 6, 7, 59, 60, 61, 66, 67, 68, 73, 74, 75, 80, 81, 82, 86, 87, 88, 89, 92, 94, 95, 96, 101, 109, 111, 112, 113, 115, 117, 118 }

C grade: { }

F grade: { 99, 100, 103, 104, 105, 110, 114 }

### 2.1.6 Sympy

A grade: { 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

### 2.1.7 Giac

A grade: { 1, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 69, 70, 71, 72, 76, 77, 78, 79, 83, 84, 85, 86, 90, 91, 92, 93, 97, 98, 102, 106, 107, 108, 111, 112, 113, 115, 116, 117 }

B grade: { 2, 3, 4, 5, 8, 10, 11, 101, 118 }

C grade: { }

F grade: { 6, 7, 59, 60, 61, 66, 67, 68, 73, 74, 75, 80, 81, 82, 87, 88, 89, 94, 95, 96, 99, 100, 103, 104, 105, 109, 110, 114 }

### 2.1.8 Mupad

A grade: { }

B grade: { 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 86, 93, 97, 98, 113, 117 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 118 }

### 2.1.9 Integrate Algebraic

A grade: { 1, 8, 9, 10, 11, 55, 56, 57, 58, 62, 63, 64, 65, 69, 70, 71, 72, 76, 77, 78, 79, 83, 84, 85, 86, 90, 91, 92, 93, 97, 98, 102, 106, 107, 108, 111, 112, 113, 115, 116, 117, 118 }

B grade: { 101 }

C grade: { 2, 3, 4, 7, 59, 60, 61, 66, 67, 68, 73, 74, 75, 80, 81, 82, 87, 88, 89, 94, 95, 96, 99, 100, 103, 104, 109, 110, 114 }

F grade: { 5, 6, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 105 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$ . To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	87	136	0	205	0	84	-1	96
N.S.	1	1.00	0.85	1.33	0.00	2.01	0.00	0.82	-0.01	0.94
time (sec)	N/A	0.094	0.195	0.028	0.000	0.430	0.000	0.256	0.000	0.439
Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F(-2)	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	178	491	0	1079	0	847	-1	162
N.S.	1	1.00	2.17	5.99	0.00	13.16	0.00	10.33	-0.01	1.98
time (sec)	N/A	0.111	0.394	0.060	0.000	0.662	0.000	3.037	0.000	0.505
Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F(-2)	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	161	307	0	813	0	703	-1	146
N.S.	1	1.00	2.44	4.65	0.00	12.32	0.00	10.65	-0.02	2.21
time (sec)	N/A	0.079	0.228	0.029	0.000	0.516	0.000	1.898	0.000	0.444
Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	296	829	0	1544	0	1166	-1	239
N.S.	1	1.00	2.29	6.43	0.00	11.97	0.00	9.04	-0.01	1.85
time (sec)	N/A	0.169	0.923	0.033	0.000	0.843	0.000	2.265	0.000	0.863
Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	1746	1884	0	3818	0	2986	-1	0
N.S.	1	1.00	7.79	8.41	0.00	17.04	0.00	13.33	-0.00	0.00
time (sec)	N/A	0.427	6.382	0.032	0.000	4.820	0.000	4.870	0.000	180.014

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	328	328	3382	3695	0	8134	0	0	-1	0
N.S.	1	1.00	10.31	11.27	0.00	24.80	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.970	6.607	0.038	0.000	20.449	0.000	0.000	0.000	180.018
Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F(-1)	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	490	1377	0	2005	0	0	-1	266
N.S.	1	1.00	3.02	8.50	0.00	12.38	0.00	0.00	-0.01	1.64
time (sec)	N/A	0.306	1.965	0.033	0.000	1.573	0.000	0.000	0.000	1.337
Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	84	27	0	38	0	52	-1	55
N.S.	1	1.00	3.00	0.96	0.00	1.36	0.00	1.86	-0.04	1.96
time (sec)	N/A	0.017	0.074	0.022	0.000	0.398	0.000	0.219	0.000	0.276
Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	27	16	0	22	0	49	-1	41
N.S.	1	1.00	0.56	0.33	0.00	0.46	0.00	1.02	-0.02	0.85
time (sec)	N/A	0.015	0.020	0.046	0.000	0.404	0.000	0.229	0.000	0.141
Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	66	84	0	82	0	143	-1	62
N.S.	1	1.00	0.94	1.20	0.00	1.17	0.00	2.04	-0.01	0.89
time (sec)	N/A	0.051	0.088	0.021	0.000	0.413	0.000	0.260	0.000	0.276
Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	159	341	0	161	0	171	-1	80
N.S.	1	1.00	1.62	3.48	0.00	1.64	0.00	1.74	-0.01	0.82
time (sec)	N/A	0.210	0.395	0.030	0.000	0.424	0.000	0.215	0.000	0.288

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	55	54	54	65	54	54	0
N.S.	1	1.00	1.00	0.81	0.79	0.79	0.96	0.79	0.79	0.00
time (sec)	N/A	0.049	0.003	0.001	0.460	0.337	0.086	0.206	0.062	0.000
Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	45	44	44	53	44	44	0
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.95	0.79	0.79	0.00
time (sec)	N/A	0.038	0.002	0.001	0.447	0.342	0.081	0.181	0.034	0.000
Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	35	34	34	41	34	34	0
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.93	0.77	0.77	0.00
time (sec)	N/A	0.029	0.001	0.001	0.434	0.341	0.075	0.209	0.025	0.000
Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.016	0.001	0.001	0.432	0.341	0.062	0.182	0.023	0.000
Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	34	33	33	49	33	35	0
N.S.	1	1.00	1.00	0.81	0.79	0.79	1.17	0.79	0.83	0.00
time (sec)	N/A	0.040	0.016	0.005	0.975	0.393	0.141	0.176	3.393	0.000
Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	34	36	45	42	36	35	0
N.S.	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	0.81	0.00
time (sec)	N/A	0.027	0.015	0.005	0.969	0.388	0.157	0.209	0.038	0.000



Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	53	47	56	75	63	46	55	0
N.S.	1	1.00	0.83	0.73	0.88	1.17	0.98	0.72	0.86	0.00
time (sec)	N/A	0.037	0.026	0.005	0.958	0.390	0.182	0.175	0.049	0.001
Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	80	65	64	64	76	64	64	0
N.S.	1	1.00	1.00	0.81	0.80	0.80	0.95	0.80	0.80	0.00
time (sec)	N/A	0.060	0.003	0.002	0.452	0.345	0.102	0.201	0.085	0.000
Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	66	55	54	54	63	54	54	0
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.95	0.82	0.82	0.00
time (sec)	N/A	0.048	0.002	0.001	0.438	0.338	0.086	0.215	0.054	0.000
Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	45	44	44	51	44	44	0
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.94	0.81	0.81	0.00
time (sec)	N/A	0.042	0.002	0.001	0.437	0.332	0.082	0.202	0.034	0.000
Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	35	34	34	41	34	34	0
N.S.	1	1.00	1.00	0.76	0.74	0.74	0.89	0.74	0.74	0.00
time (sec)	N/A	0.030	0.001	0.001	0.435	0.337	0.072	0.192	0.025	0.000
Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	53	44	43	43	63	43	45	0
N.S.	1	1.00	0.95	0.79	0.77	0.77	1.12	0.77	0.80	0.00
time (sec)	N/A	0.051	0.020	0.004	0.968	0.392	0.150	0.210	3.449	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	59	51	52	78	65	52	51	0
N.S.	1	1.00	0.94	0.81	0.83	1.24	1.03	0.83	0.81	0.00
time (sec)	N/A	0.061	0.032	0.006	0.961	0.396	0.198	0.172	0.050	0.000
Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	53	47	56	75	63	46	55	0
N.S.	1	1.00	0.83	0.73	0.88	1.17	0.98	0.72	0.86	0.00
time (sec)	N/A	0.052	0.024	0.006	0.968	0.383	0.199	0.185	3.442	0.000
Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	63	57	76	105	83	56	75	0
N.S.	1	1.00	0.74	0.67	0.89	1.24	0.98	0.66	0.88	0.00
time (sec)	N/A	0.063	0.044	0.007	0.962	0.377	0.229	0.183	3.468	0.000
Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	96	75	74	74	92	74	74	0
N.S.	1	1.00	1.00	0.78	0.77	0.77	0.96	0.77	0.77	0.00
time (sec)	N/A	0.071	0.004	0.000	0.432	0.339	0.097	0.176	0.121	0.000
Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	82	65	64	64	78	64	64	0
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.95	0.78	0.78	0.00
time (sec)	N/A	0.056	0.002	0.001	0.426	0.329	0.090	0.205	0.081	0.000
Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	55	54	54	65	54	54	0
N.S.	1	1.00	1.00	0.81	0.79	0.79	0.96	0.79	0.79	0.00
time (sec)	N/A	0.051	0.002	0.002	0.429	0.337	0.084	0.202	0.054	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	45	44	44	53	44	44	0
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.95	0.79	0.79	0.00
time (sec)	N/A	0.032	0.001	0.001	0.434	0.332	0.077	0.201	0.033	0.000
Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	63	54	53	53	76	53	55	0
N.S.	1	1.00	0.90	0.77	0.76	0.76	1.09	0.76	0.79	0.00
time (sec)	N/A	0.053	0.023	0.004	0.963	0.408	0.159	0.211	0.043	0.000
Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	77	61	62	88	78	62	61	0
N.S.	1	1.00	1.00	0.79	0.81	1.14	1.01	0.81	0.79	0.00
time (sec)	N/A	0.072	0.027	0.008	0.961	0.408	0.188	0.206	3.431	0.001
Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	78	63	72	118	85	62	71	0
N.S.	1	1.00	0.93	0.75	0.86	1.40	1.01	0.74	0.85	0.00
time (sec)	N/A	0.087	0.037	0.009	0.970	0.393	0.235	0.189	3.426	0.000
Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	72	64	63	63	87	63	65	0
N.S.	1	1.00	0.86	0.76	0.75	0.75	1.04	0.75	0.77	0.00
time (sec)	N/A	0.057	0.028	0.006	0.976	0.397	0.167	0.211	3.441	0.001
Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	63	54	53	53	73	53	55	0
N.S.	1	1.00	0.90	0.77	0.76	0.76	1.04	0.76	0.79	0.00
time (sec)	N/A	0.056	0.021	0.004	0.968	0.414	0.152	0.213	0.042	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	52	44	43	43	60	43	45	0
N.S.	1	1.00	0.93	0.79	0.77	0.77	1.07	0.77	0.80	0.00
time (sec)	N/A	0.050	0.017	0.004	0.958	0.434	0.155	0.182	3.445	0.000
Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	34	33	33	46	33	35	0
N.S.	1	1.00	1.00	0.81	0.79	0.79	1.10	0.79	0.83	0.00
time (sec)	N/A	0.035	0.010	0.004	0.950	0.395	0.141	0.207	0.039	0.000
Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	60	59	59	83	59	79	0
N.S.	1	1.00	1.00	0.82	0.81	0.81	1.14	0.81	1.08	0.00
time (sec)	N/A	0.053	0.032	0.006	0.963	0.407	0.244	0.191	0.187	0.000
Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	94	77	78	117	102	78	95	0
N.S.	1	1.00	1.00	0.82	0.83	1.24	1.09	0.83	1.01	0.00
time (sec)	N/A	0.089	0.083	0.007	0.960	0.412	0.322	0.198	3.567	0.000
Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	104	89	98	177	119	88	115	0
N.S.	1	1.00	0.90	0.77	0.85	1.54	1.03	0.77	1.00	0.00
time (sec)	N/A	0.124	0.155	0.008	0.959	0.402	0.363	0.180	0.179	0.000
Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	91	71	72	98	90	72	72	0
N.S.	1	1.00	1.00	0.78	0.79	1.08	0.99	0.79	0.79	0.00
time (sec)	N/A	0.086	0.052	0.010	0.964	0.393	0.198	0.183	3.462	0.001

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	77	61	62	88	75	62	61	0
N.S.	1	1.00	1.00	0.79	0.81	1.14	0.97	0.81	0.79	0.00
time (sec)	N/A	0.073	0.028	0.008	0.960	0.432	0.191	0.195	3.424	0.000
Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	51	52	78	61	52	52	0
N.S.	1	1.00	1.00	0.81	0.83	1.24	0.97	0.83	0.83	0.00
time (sec)	N/A	0.063	0.031	0.007	0.966	0.407	0.189	0.173	3.404	0.000
Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	34	36	45	42	36	36	0
N.S.	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	0.84	0.00
time (sec)	N/A	0.026	0.015	0.004	0.964	0.401	0.154	0.189	0.040	0.000
Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	94	77	78	117	102	78	96	0
N.S.	1	1.00	1.00	0.82	0.83	1.24	1.09	0.83	1.02	0.00
time (sec)	N/A	0.088	0.061	0.007	0.963	0.414	0.319	0.192	3.578	0.000
Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	106	94	96	167	122	96	115	0
N.S.	1	1.00	0.83	0.74	0.76	1.31	0.96	0.76	0.91	0.00
time (sec)	N/A	0.123	0.057	0.010	0.961	0.434	0.359	0.193	0.177	0.000
Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	136	106	118	237	143	110	135	0
N.S.	1	1.00	0.92	0.72	0.80	1.60	0.97	0.74	0.91	0.00
time (sec)	N/A	0.161	0.076	0.011	0.968	0.409	0.400	0.202	3.586	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	98	73	82	128	95	72	81	0
N.S.	1	1.00	1.00	0.74	0.84	1.31	0.97	0.73	0.83	0.00
time (sec)	N/A	0.113	0.038	0.008	0.951	0.411	0.241	0.207	0.045	0.000
Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	84	63	72	118	82	62	72	0
N.S.	1	1.00	1.00	0.75	0.86	1.40	0.98	0.74	0.86	0.00
time (sec)	N/A	0.087	0.037	0.007	0.958	0.394	0.229	0.183	0.049	0.001
Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	51	47	56	75	63	46	56	0
N.S.	1	1.00	0.80	0.73	0.88	1.17	0.98	0.72	0.88	0.00
time (sec)	N/A	0.053	0.029	0.006	0.961	0.387	0.201	0.186	3.474	0.000
Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	51	47	56	75	61	46	55	0
N.S.	1	1.00	0.80	0.73	0.88	1.17	0.95	0.72	0.86	0.00
time (sec)	N/A	0.033	0.028	0.007	0.960	0.418	0.183	0.205	0.044	0.000
Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	99	89	98	177	122	88	116	0
N.S.	1	1.00	0.86	0.77	0.85	1.54	1.06	0.77	1.01	0.00
time (sec)	N/A	0.124	0.161	0.008	0.973	0.431	0.357	0.211	3.580	0.000
Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	136	106	116	227	143	110	136	0
N.S.	1	1.00	0.85	0.66	0.72	1.42	0.89	0.69	0.85	0.00
time (sec)	N/A	0.160	0.124	0.011	0.974	0.426	0.427	0.217	3.597	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	181	181	151	118	138	297	163	116	155	0
N.S.	1	1.00	0.83	0.65	0.76	1.64	0.90	0.64	0.86	0.00
time (sec)	N/A	0.204	0.095	0.013	0.972	0.428	0.470	0.219	3.588	0.001
Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	85	166	177	98	0	93	221	100
N.S.	1	1.00	0.41	0.80	0.85	0.47	0.00	0.45	1.06	0.48
time (sec)	N/A	0.309	0.296	0.031	1.006	0.437	0.000	0.249	5.028	1.083
Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	75	132	143	88	0	83	187	90
N.S.	1	1.00	0.45	0.80	0.86	0.53	0.00	0.50	1.13	0.54
time (sec)	N/A	0.181	0.171	0.008	0.982	0.434	0.000	0.254	4.692	0.869
Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	65	98	109	78	0	73	153	80
N.S.	1	1.00	0.52	0.79	0.88	0.63	0.00	0.59	1.23	0.65
time (sec)	N/A	0.099	0.105	0.007	0.976	0.419	0.000	0.217	4.185	0.562
Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	55	64	75	68	0	63	119	70
N.S.	1	1.00	0.67	0.78	0.91	0.83	0.00	0.77	1.45	0.85
time (sec)	N/A	0.040	0.052	0.006	0.961	0.411	0.000	0.238	3.835	0.366
Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	185	2065	0	2016	0	0	-1	211
N.S.	1	1.00	1.06	11.87	0.00	11.59	0.00	0.00	-0.01	1.21
time (sec)	N/A	0.441	0.457	0.149	0.000	0.994	0.000	0.000	0.000	0.346

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	214	16357	0	2102	0	0	-1	423
N.S.	1	1.00	1.14	87.01	0.00	11.18	0.00	0.00	-0.01	2.25
time (sec)	N/A	0.393	1.036	0.232	0.000	1.194	0.000	0.000	0.000	0.705
Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	299	44343	0	2182	0	0	-1	396
N.S.	1	1.00	1.34	198.85	0.00	9.78	0.00	0.00	-0.00	1.78
time (sec)	N/A	0.459	2.075	0.372	0.000	1.437	0.000	0.000	0.000	0.824
Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	231	231	95	185	206	108	0	103	-1	110
N.S.	1	1.00	0.41	0.80	0.89	0.47	0.00	0.45	-0.00	0.48
time (sec)	N/A	0.342	0.367	0.035	1.022	0.479	0.000	0.255	0.000	1.372
Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	85	151	172	98	0	93	-1	100
N.S.	1	1.00	0.45	0.80	0.91	0.52	0.00	0.49	-0.01	0.53
time (sec)	N/A	0.190	0.231	0.007	0.991	0.460	0.000	0.256	0.000	1.056
Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	75	117	138	88	0	83	-1	90
N.S.	1	1.00	0.51	0.80	0.94	0.60	0.00	0.56	-0.01	0.61
time (sec)	N/A	0.122	0.141	0.009	0.983	0.455	0.000	0.257	0.000	0.824
Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	65	83	104	78	0	73	-1	80
N.S.	1	1.00	0.62	0.79	0.99	0.74	0.00	0.70	-0.01	0.76
time (sec)	N/A	0.050	0.080	0.006	0.963	0.410	0.000	0.230	0.000	0.581



Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	197	197	310	3460	0	2027	0	0	-1	235
N.S.	1	1.00	1.57	17.56	0.00	10.29	0.00	0.00	-0.01	1.19
time (sec)	N/A	0.488	0.693	0.053	0.000	0.994	0.000	0.000	0.000	0.530
Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	232	232	530	28185	0	2150	0	0	-1	422
N.S.	1	1.00	2.28	121.49	0.00	9.27	0.00	0.00	-0.00	1.82
time (sec)	N/A	0.575	2.562	0.158	0.000	1.291	0.000	0.000	0.000	0.680
Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	1262	81552	0	2183	0	0	-1	578
N.S.	1	1.00	5.66	365.70	0.00	9.79	0.00	0.00	-0.00	2.59
time (sec)	N/A	0.433	5.342	0.350	0.000	1.270	0.000	0.000	0.000	1.059
Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	254	105	204	235	118	0	113	-1	120
N.S.	1	1.00	0.41	0.80	0.93	0.46	0.00	0.44	-0.00	0.47
time (sec)	N/A	0.373	0.452	0.040	1.021	0.419	0.000	0.536	0.000	1.834
Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	95	170	201	108	0	103	-1	110
N.S.	1	1.00	0.45	0.80	0.95	0.51	0.00	0.49	-0.00	0.52
time (sec)	N/A	0.220	0.296	0.009	0.996	0.417	0.000	0.549	0.000	1.336
Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	85	136	167	98	0	93	-1	100
N.S.	1	1.00	0.50	0.80	0.98	0.58	0.00	0.55	-0.01	0.59
time (sec)	N/A	0.130	0.183	0.007	1.028	0.415	0.000	0.516	0.000	1.048

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	75	102	133	88	0	83	-1	90
N.S.	1	1.00	0.59	0.80	1.04	0.69	0.00	0.65	-0.01	0.70
time (sec)	N/A	0.062	0.112	0.006	0.974	0.411	0.000	0.391	0.000	0.843
Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	222	222	229	4860	0	2010	0	0	-1	245
N.S.	1	1.00	1.03	21.89	0.00	9.05	0.00	0.00	-0.00	1.10
time (sec)	N/A	0.539	1.047	0.053	0.000	1.040	0.000	0.000	0.000	0.762
Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	685	40028	0	2161	0	0	-1	442
N.S.	1	1.00	2.69	156.97	0.00	8.47	0.00	0.00	-0.00	1.73
time (sec)	N/A	0.660	1.655	0.157	0.000	1.317	0.000	0.000	0.000	0.970
Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	281	1009	119458	0	2240	0	0	-1	627
N.S.	1	1.00	3.59	425.12	0.00	7.97	0.00	0.00	-0.00	2.23
time (sec)	N/A	0.655	1.996	0.384	0.000	1.479	0.000	0.000	0.000	1.085
Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	75	147	148	88	0	83	-1	90
N.S.	1	1.00	0.41	0.79	0.80	0.48	0.00	0.45	-0.01	0.49
time (sec)	N/A	0.312	0.241	0.019	0.999	0.433	0.000	0.503	0.000	0.802
Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	65	113	114	78	0	73	-1	80
N.S.	1	1.00	0.45	0.79	0.80	0.55	0.00	0.51	-0.01	0.56
time (sec)	N/A	0.168	0.134	0.008	0.977	0.407	0.000	0.524	0.000	0.598

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	55	79	80	68	0	63	-1	70
N.S.	1	1.00	0.54	0.78	0.79	0.67	0.00	0.62	-0.01	0.69
time (sec)	N/A	0.088	0.075	0.008	0.965	0.408	0.000	0.487	0.000	0.490
Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	45	45	46	58	0	53	-1	60
N.S.	1	1.00	0.76	0.76	0.78	0.98	0.00	0.90	-0.02	1.02
time (sec)	N/A	0.033	0.039	0.006	0.956	0.415	0.000	0.530	0.000	0.276
Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	176	684	0	2002	0	0	-1	135
N.S.	1	1.00	1.19	4.62	0.00	13.53	0.00	0.00	-0.01	0.91
time (sec)	N/A	0.314	0.300	0.004	0.000	1.132	0.000	0.000	0.000	0.359
Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	287	5225	0	2102	0	0	-1	230
N.S.	1	1.00	1.53	27.79	0.00	11.18	0.00	0.00	-0.01	1.22
time (sec)	N/A	0.429	0.996	0.006	0.000	1.250	0.000	0.000	0.000	0.543
Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	1277	13040	0	2183	0	0	-1	396
N.S.	1	1.00	5.73	58.48	0.00	9.79	0.00	0.00	-0.00	1.78
time (sec)	N/A	0.469	6.230	0.007	0.000	1.284	0.000	0.000	0.000	0.843
Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	95	166	148	112	0	82	-1	90
N.S.	1	1.00	0.57	1.00	0.89	0.67	0.00	0.49	-0.01	0.54
time (sec)	N/A	0.204	0.373	0.029	0.985	0.411	0.000	0.251	0.000	1.321

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	65	132	114	102	0	72	-1	80
N.S.	1	1.00	0.52	1.06	0.92	0.82	0.00	0.58	-0.01	0.65
time (sec)	N/A	0.127	0.232	0.008	0.973	0.419	0.000	0.281	0.000	0.844
Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	55	98	80	92	0	62	-1	70
N.S.	1	1.00	0.67	1.20	0.98	1.12	0.00	0.76	-0.01	0.85
time (sec)	N/A	0.071	0.143	0.008	0.959	0.415	0.000	0.237	0.000	0.728
Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	64	46	82	0	53	87	60
N.S.	1	1.00	1.00	1.42	1.02	1.82	0.00	1.18	1.93	1.33
time (sec)	N/A	0.029	0.076	0.006	0.955	0.415	0.000	0.234	0.227	0.424
Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	202	718	0	2083	0	0	-1	199
N.S.	1	1.00	1.15	4.08	0.00	11.84	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.408	1.287	0.034	0.000	1.168	0.000	0.000	0.000	0.493
Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	740	5942	0	2173	0	0	-1	406
N.S.	1	1.00	3.51	28.16	0.00	10.30	0.00	0.00	-0.00	1.92
time (sec)	N/A	0.473	1.525	0.099	0.000	1.259	0.000	0.000	0.000	0.945
Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	231	18981	0	2263	0	0	-1	611
N.S.	1	1.00	0.94	77.16	0.00	9.20	0.00	0.00	-0.00	2.48
time (sec)	N/A	0.525	2.206	0.208	0.000	1.386	0.000	0.000	0.000	1.513

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	75	214	253	132	0	81	-1	90
N.S.	1	1.00	0.51	1.46	1.72	0.90	0.00	0.55	-0.01	0.61
time (sec)	N/A	0.167	0.513	0.031	1.002	0.423	0.000	0.252	0.000	1.449
Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	65	180	219	122	0	72	-1	80
N.S.	1	1.00	0.62	1.71	2.09	1.16	0.00	0.69	-0.01	0.76
time (sec)	N/A	0.105	0.338	0.008	1.020	0.426	0.000	0.268	0.000	1.028
Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	55	146	185	112	0	61	-1	70
N.S.	1	1.00	0.81	2.15	2.72	1.65	0.00	0.90	-0.01	1.03
time (sec)	N/A	0.061	0.236	0.007	0.978	0.408	0.000	0.267	0.000	0.737
Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	33	30	59	51	0	29	29	33
N.S.	1	1.00	0.70	0.64	1.26	1.09	0.00	0.62	0.62	0.70
time (sec)	N/A	0.022	0.103	0.005	0.432	0.404	0.000	0.232	0.090	0.515
Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	218	751	0	2133	0	0	-1	209
N.S.	1	1.00	1.10	3.77	0.00	10.72	0.00	0.00	-0.01	1.05
time (sec)	N/A	0.456	0.868	0.039	0.000	1.239	0.000	0.000	0.000	0.826
Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	296	5975	0	2253	0	0	-1	416
N.S.	1	1.00	1.26	25.53	0.00	9.63	0.00	0.00	-0.00	1.78
time (sec)	N/A	0.543	1.153	0.104	0.000	1.194	0.000	0.000	0.000	1.418

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	242	19014	0	2343	0	0	-1	601
N.S.	1	1.00	0.90	70.68	0.00	8.71	0.00	0.00	-0.00	2.23
time (sec)	N/A	0.589	1.958	0.209	0.000	1.327	0.000	0.000	0.000	1.611
Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	436	436	657	1429	0	1269	0	638	1299	655
N.S.	1	1.00	1.51	3.28	0.00	2.91	0.00	1.46	2.98	1.50
time (sec)	N/A	0.789	0.935	0.023	0.000	0.739	0.000	0.365	5.312	2.823
Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	175	173	453	0	465	0	212	320	207
N.S.	1	1.00	0.99	2.59	0.00	2.66	0.00	1.21	1.83	1.18
time (sec)	N/A	0.164	0.273	0.007	0.000	0.472	0.000	0.299	3.906	0.770
Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	431	431	417	6019	0	0	0	0	-1	402
N.S.	1	1.00	0.97	13.97	0.00	0.00	0.00	0.00	-0.00	0.93
time (sec)	N/A	1.051	1.219	0.052	0.000	0.000	0.000	0.000	0.000	0.601
Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	488	488	555	22287	0	0	0	0	-1	946
N.S.	1	1.00	1.14	45.67	0.00	0.00	0.00	0.00	-0.00	1.94
time (sec)	N/A	2.929	5.093	0.037	0.000	0.000	0.000	0.000	0.000	2.554
Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	564	564	829	2458	0	2179	0	1150	-1	1210
N.S.	1	1.00	1.47	4.36	0.00	3.86	0.00	2.04	-0.00	2.15
time (sec)	N/A	0.935	1.762	0.025	0.000	1.939	0.000	0.598	0.000	7.792

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	392	862	0	839	0	417	-1	408
N.S.	1	1.00	1.66	3.65	0.00	3.56	0.00	1.77	-0.00	1.73
time (sec)	N/A	0.230	0.606	0.009	0.000	0.842	0.000	0.612	0.000	1.701

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	679	678	1232	22523	0	0	0	0	-1	1160
N.S.	1	1.00	1.81	33.17	0.00	0.00	0.00	0.00	-0.00	1.71
time (sec)	N/A	11.033	4.576	0.029	0.000	0.000	0.000	0.000	0.000	1.670

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	F(-1)	F(-1)	F(-2)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	704	704	2843	72576	0	0	0	0	-1	2427
N.S.	1	1.00	4.04	103.09	0.00	0.00	0.00	0.00	-0.00	3.45
time (sec)	N/A	11.950	6.831	0.039	0.000	0.000	0.000	0.000	0.000	5.181

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	671	669	4727	178044	0	0	0	0	-1	0
N.S.	1	1.00	7.04	265.34	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	11.597	7.229	0.066	0.000	0.000	0.000	0.000	0.000	180.012

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	717	717	615	1930	0	1583	0	824	-1	847
N.S.	1	1.00	0.86	2.69	0.00	2.21	0.00	1.15	-0.00	1.18
time (sec)	N/A	2.710	1.346	0.028	0.000	1.407	0.000	0.528	0.000	4.221

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	251	706	0	637	0	304	-1	296
N.S.	1	1.00	0.79	2.23	0.00	2.02	0.00	0.96	-0.00	0.94
time (sec)	N/A	0.626	0.503	0.014	0.000	0.800	0.000	0.632	0.000	1.089

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	96	185	0	227	0	98	-1	102
N.S.	1	1.00	0.83	1.59	0.00	1.96	0.00	0.84	-0.01	0.88
time (sec)	N/A	0.111	0.146	0.008	0.000	0.634	0.000	0.499	0.000	0.480
Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	374	374	376	761	0	11287	0	0	-1	211
N.S.	1	1.00	1.01	2.03	0.00	30.18	0.00	0.00	-0.00	0.56
time (sec)	N/A	0.579	1.454	0.025	0.000	7.139	0.000	0.000	0.000	0.442
Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	789	787	1377	3858	0	0	0	0	-1	55521
N.S.	1	1.00	1.75	4.89	0.00	0.00	0.00	0.00	-0.00	70.37
time (sec)	N/A	8.210	6.689	0.031	0.000	0.000	0.000	0.000	0.000	175.847
Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	649	649	745	2827	0	3143	0	1099	-1	1100
N.S.	1	1.00	1.15	4.36	0.00	4.84	0.00	1.69	-0.00	1.69
time (sec)	N/A	2.106	1.664	0.030	0.000	1.952	0.000	0.423	0.000	5.978
Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	309	309	288	1011	0	1305	0	407	-1	375
N.S.	1	1.00	0.93	3.27	0.00	4.22	0.00	1.32	-0.00	1.21
time (sec)	N/A	0.447	0.733	0.015	0.000	1.238	0.000	0.352	0.000	2.163
Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	113	249	0	429	0	122	143	111
N.S.	1	1.00	1.02	2.24	0.00	3.86	0.00	1.10	1.29	1.00
time (sec)	N/A	0.080	0.311	0.006	0.000	0.784	0.000	0.312	3.734	0.551



Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	666	666	700	4099	0	0	0	0	-1	730
N.S.	1	1.00	1.05	6.15	0.00	0.00	0.00	0.00	-0.00	1.10
time (sec)	N/A	1.829	5.343	0.028	0.000	0.000	0.000	0.000	0.000	1.609
Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	891	891	872	4635	0	3995	0	1401	-1	1411
N.S.	1	1.00	0.98	5.20	0.00	4.48	0.00	1.57	-0.00	1.58
time (sec)	N/A	1.768	2.191	0.031	0.000	5.385	0.000	0.455	0.000	37.604
Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	444	444	387	1786	0	1581	0	587	-1	555
N.S.	1	1.00	0.87	4.02	0.00	3.56	0.00	1.32	-0.00	1.25
time (sec)	N/A	0.451	1.257	0.014	0.000	3.503	0.000	0.338	0.000	3.156
Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	147	185	0	286	0	240	175	176
N.S.	1	1.00	1.12	1.41	0.00	2.18	0.00	1.83	1.34	1.34
time (sec)	N/A	0.085	0.378	0.007	0.000	2.597	0.000	0.322	3.708	1.165
Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	81	144	0	154	0	205	-1	53
N.S.	1	1.00	1.59	2.82	0.00	3.02	0.00	4.02	-0.02	1.04
time (sec)	N/A	0.066	0.041	0.024	0.000	0.445	0.000	0.276	0.000	0.336

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. The column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [11] had the largest ratio of [.4444]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.00	29	0.138
2	A	2	2	1.00	31	0.065
3	A	2	2	1.00	27	0.074
4	A	4	4	1.00	27	0.148
5	A	5	5	1.00	27	0.185
6	A	6	5	1.00	27	0.185
7	A	4	4	1.00	31	0.129
8	A	2	2	1.00	23	0.087
9	A	3	3	1.00	22	0.136
10	A	6	5	1.00	18	0.278
11	A	16	12	1.00	27	0.444
12	A	2	1	1.00	23	0.043
13	A	2	1	1.00	23	0.043
14	A	2	1	1.00	23	0.043
15	A	2	1	1.00	21	0.048
16	A	6	5	1.00	23	0.217
17	A	4	4	1.00	23	0.174
18	A	5	5	1.00	23	0.217
19	A	2	1	1.00	25	0.040
20	A	2	1	1.00	25	0.040
21	A	2	1	1.00	25	0.040
22	A	2	1	1.00	23	0.043
23	A	6	5	1.00	25	0.200
24	A	7	6	1.00	25	0.240
25	A	5	4	1.00	25	0.160
26	A	6	5	1.00	25	0.200
27	A	2	1	1.00	25	0.040
28	A	2	1	1.00	25	0.040
29	A	2	1	1.00	25	0.040
30	A	2	1	1.00	23	0.043
31	A	6	5	1.00	25	0.200
32	A	7	6	1.00	25	0.240
33	A	8	6	1.00	25	0.240
34	A	6	5	1.00	25	0.200
35	A	6	5	1.00	25	0.200
36	A	6	5	1.00	25	0.200
37	A	6	5	1.00	23	0.217
38	A	9	5	1.00	25	0.200
39	A	10	6	1.00	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	11	7	1.00	25	0.280
41	A	7	6	1.00	25	0.240
42	A	7	6	1.00	25	0.240
43	A	7	6	1.00	25	0.240
44	A	4	4	1.00	23	0.174
45	A	10	6	1.00	25	0.240
46	A	11	7	1.00	25	0.280
47	A	12	7	1.00	25	0.280
48	A	8	6	1.00	25	0.240
49	A	8	6	1.00	25	0.240
50	A	5	4	1.00	25	0.160
51	A	5	5	1.00	23	0.217
52	A	11	7	1.00	25	0.280
53	A	12	7	1.00	25	0.280
54	A	13	7	1.00	25	0.280
55	A	11	5	1.00	27	0.185
56	A	9	5	1.00	27	0.185
57	A	7	5	1.00	27	0.185
58	A	5	5	1.00	25	0.200
59	A	8	7	1.00	27	0.259
60	A	6	5	1.00	27	0.185
61	A	7	6	1.00	27	0.222
62	A	12	5	1.00	27	0.185
63	A	10	5	1.00	27	0.185
64	A	8	5	1.00	27	0.185
65	A	6	5	1.00	25	0.200
66	A	9	8	1.00	27	0.296
67	A	10	9	1.00	27	0.333
68	A	7	6	1.00	27	0.222
69	A	13	5	1.00	27	0.185
70	A	11	5	1.00	27	0.185
71	A	9	5	1.00	27	0.185
72	A	7	5	1.00	25	0.200
73	A	10	9	1.00	27	0.333
74	A	11	9	1.00	27	0.333
75	A	11	10	1.00	27	0.370
76	A	10	4	1.00	27	0.148
77	A	8	4	1.00	27	0.148
78	A	6	4	1.00	27	0.148
79	A	4	4	1.00	25	0.160
80	A	5	4	1.00	27	0.148
81	A	6	5	1.00	27	0.185
82	A	7	6	1.00	27	0.222
83	A	9	5	1.00	27	0.185
84	A	7	5	1.00	27	0.185
85	A	5	5	1.00	27	0.185
86	A	4	4	1.00	25	0.160
87	A	6	5	1.00	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	7	6	1.00	27	0.222
89	A	8	6	1.00	27	0.222
90	A	8	5	1.00	27	0.185
91	A	6	5	1.00	27	0.185
92	A	5	4	1.00	27	0.148
93	A	3	3	1.00	25	0.120
94	A	7	6	1.00	27	0.222
95	A	8	6	1.00	27	0.222
96	A	9	6	1.00	27	0.222
97	A	7	5	1.00	27	0.185
98	A	5	5	1.00	25	0.200
99	A	8	5	1.00	27	0.185
100	A	6	4	1.00	27	0.148
101	A	8	5	1.00	27	0.185
102	A	6	5	1.00	25	0.200
103	A	9	6	1.00	27	0.222
104	A	10	7	1.00	27	0.259
105	A	7	5	1.00	27	0.185
106	A	8	4	1.00	27	0.148
107	A	6	4	1.00	27	0.148
108	A	4	4	1.00	25	0.160
109	A	5	3	1.00	27	0.111
110	A	6	4	1.00	27	0.148
111	A	7	5	1.00	27	0.185
112	A	5	5	1.00	27	0.185
113	A	4	4	1.00	25	0.160
114	A	6	4	1.00	27	0.148
115	A	6	5	1.00	27	0.185
116	A	5	4	1.00	27	0.148
117	A	3	3	1.00	25	0.120
118	A	5	4	1.00	27	0.148

# Chapter 3

## Listing of integrals

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3.13	$\int (3-x+2x^2)(2+3x+5x^2)^3 dx$	98
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3.29	$\int (3-x+2x^2)^3 (2+3x+5x^2)^2 dx$	140
3.30	$\int (3-x+2x^2)^3 (2+3x+5x^2) dx$	142
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3.34	$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2} dx$	155
3.35	$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2} dx$	158
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3.37	$\int \frac{2+3x+5x^2}{3-x+2x^2} dx$	164
3.38	$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)} dx$	167
3.39	$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2} dx$	170
3.40	$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3} dx$	174
3.41	$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2} dx$	179
3.42	$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2} dx$	183
3.43	$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2} dx$	187
3.44	$\int \frac{2+3x+5x^2}{(3-x+2x^2)^2} dx$	191
3.45	$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)} dx$	194
3.46	$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx$	198
3.47	$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3} dx$	203
3.48	$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3} dx$	208
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3.51	$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3} dx$	219

3.52	$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)} dx$	222
3.53	$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2} dx$	227
3.54	$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3} dx$	232
3.55	$\int \sqrt{3-x+2x^2} (2+3x+5x^2)^4 dx$	237
3.56	$\int \sqrt{3-x+2x^2} (2+3x+5x^2)^3 dx$	241
3.57	$\int \sqrt{3-x+2x^2} (2+3x+5x^2)^2 dx$	245
3.58	$\int \sqrt{3-x+2x^2} (2+3x+5x^2) dx$	248
3.59	$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2} dx$	251
3.60	$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2} dx$	257
3.61	$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3} dx$	262
3.62	$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^4 dx$	268
3.63	$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^3 dx$	272
3.64	$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^2 dx$	276
3.65	$\int (3-x+2x^2)^{3/2} (2+3x+5x^2) dx$	279
3.66	$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2} dx$	282
3.67	$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} dx$	289
3.68	$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3} dx$	295
3.69	$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^4 dx$	301
3.70	$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^3 dx$	305
3.71	$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^2 dx$	309
3.72	$\int (3-x+2x^2)^{5/2} (2+3x+5x^2) dx$	313
3.73	$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2} dx$	316
3.74	$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2} dx$	324
3.75	$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3} dx$	330
3.76	$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}} dx$	337
3.77	$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}} dx$	341
3.78	$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}} dx$	344
3.79	$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}} dx$	347
3.80	$\int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx$	350
3.81	$\int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)^2} dx$	355
3.82	$\int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)^3} dx$	360

3.83	$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}} dx$	367
3.84	$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}} dx$	371
3.85	$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}} dx$	375
3.86	$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}} dx$	378
3.87	$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)} dx$	381
3.88	$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2} dx$	386
3.89	$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3} dx$	392
3.90	$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}} dx$	399
3.91	$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}} dx$	403
3.92	$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}} dx$	407
3.93	$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}} dx$	410
3.94	$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)} dx$	413
3.95	$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2} dx$	419
3.96	$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3} dx$	425
3.97	$\int \sqrt{a+bx+cx^2} (d+ex+fx^2)^2 dx$	432
3.98	$\int \sqrt{a+bx+cx^2} (d+ex+fx^2) dx$	437
3.99	$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$	441
3.100	$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2} dx$	445
3.101	$\int (a+bx+cx^2)^{3/2} (d+ex+fx^2)^2 dx$	449
3.102	$\int (a+bx+cx^2)^{3/2} (d+ex+fx^2) dx$	455
3.103	$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2} dx$	459
3.104	$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2} dx$	463
3.105	$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3} dx$	472
3.106	$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}} dx$	477
3.107	$\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}} dx$	483
3.108	$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}} dx$	487
3.109	$\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	490
3.110	$\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)^2} dx$	497



3.111	$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}} dx$	502
3.112	$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}} dx$	509
3.113	$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}} dx$	513
3.114	$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$	517
3.115	$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}} dx$	522
3.116	$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}} dx$	530
3.117	$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}} dx$	535
3.118	$\int \frac{1}{\sqrt{-7+2x+5x^2}(8+12x+5x^2)} dx$	538

$$3.1 \quad \int \frac{a+bx+\frac{bf^2x^2}{e}}{\sqrt{d+ex+fx^2}} dx$$

**Optimal.** Leaf size=102

$$\frac{\left(8af - b\left(\frac{4df}{e} + e\right)\right) \tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}} + \frac{bx\sqrt{d+ex+fx^2}}{2e} + \frac{b\sqrt{d+ex+fx^2}}{4f}$$

**Rubi [A]** time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1661, 640, 621, 206}

$$\frac{\left(8af - b\left(\frac{4df}{e} + e\right)\right) \tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}} + \frac{bx\sqrt{d+ex+fx^2}}{2e} + \frac{b\sqrt{d+ex+fx^2}}{4f}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + (b\*f\*x^2)/e)/Sqrt[d + e\*x + f\*x^2], x]

[Out] (b\*Sqrt[d + e\*x + f\*x^2])/(4\*f) + (b\*x\*Sqrt[d + e\*x + f\*x^2])/(2\*e) + ((8\*a\*f - b\*(e + (4\*d\*f)/e))\*ArcTanh[(e + 2\*f\*x)/(2\*Sqrt[f]\*Sqrt[d + e\*x + f\*x^2])])/(8\*f^(3/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{a + bx + \frac{bfx^2}{e}}{\sqrt{d + ex + fx^2}} dx &= \frac{bx\sqrt{d + ex + fx^2}}{2e} + \frac{\int \frac{(2a - \frac{bd}{e})f + \frac{bfx}{2}}{\sqrt{d + ex + fx^2}} dx}{2f} \\
&= \frac{b\sqrt{d + ex + fx^2}}{4f} + \frac{bx\sqrt{d + ex + fx^2}}{2e} + \frac{\left(-be + 8af - \frac{4bdf}{e}\right) \int \frac{1}{\sqrt{d + ex + fx^2}} dx}{8f} \\
&= \frac{b\sqrt{d + ex + fx^2}}{4f} + \frac{bx\sqrt{d + ex + fx^2}}{2e} + \frac{\left(-be + 8af - \frac{4bdf}{e}\right) \text{Subst}\left(\int \frac{1}{4f - x^2} dx, x, \frac{e + \sqrt{d + ex + fx^2}}{2f}\right)}{4f} \\
&= \frac{b\sqrt{d + ex + fx^2}}{4f} + \frac{bx\sqrt{d + ex + fx^2}}{2e} - \frac{\left(be - 8af + \frac{4bdf}{e}\right) \tanh^{-1}\left(\frac{e + 2fx}{2\sqrt{f}\sqrt{d + ex + fx^2}}\right)}{8f^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.19, size = 87, normalized size = 0.85

$$\frac{2b\sqrt{f}(e + 2fx)\sqrt{d + x(e + fx)} - (b(4df + e^2) - 8aef) \tanh^{-1}\left(\frac{e + 2fx}{2\sqrt{f}\sqrt{d + x(e + fx)}}\right)}{8ef^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + (b\*f\*x^2)/e)/Sqrt[d + e\*x + f\*x^2], x]

[Out] (2\*b\*Sqrt[f]\*(e + 2\*f\*x)\*Sqrt[d + x\*(e + f\*x)] - (-8\*a\*e\*f + b\*(e^2 + 4\*d\*f)) \* ArcTanh[(e + 2\*f\*x)/(2\*Sqrt[f]\*Sqrt[d + x\*(e + f\*x)])]) / (8\*e\*f^(3/2))

**IntegrateAlgebraic [A]** time = 0.44, size = 96, normalized size = 0.94

$$\frac{(-8aef + 4bdf + be^2) \log\left(-2ef^{3/2}\sqrt{d + ex + fx^2} + e^2f + 2ef^2x\right)}{8ef^{3/2}} + \frac{b(e + 2fx)\sqrt{d + ex + fx^2}}{4ef}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b\*x + (b\*f\*x^2)/e)/Sqrt[d + e\*x + f\*x^2], x]

[Out] (b\*(e + 2\*f\*x)\*Sqrt[d + e\*x + f\*x^2]) / (4\*e\*f) + ((b\*e^2 + 4\*b\*d\*f - 8\*a\*e\*f) \* Log[e^2\*f + 2\*e\*f^2\*x - 2\*e\*f^(3/2)\*Sqrt[d + e\*x + f\*x^2]]) / (8\*e\*f^(3/2))

**fricas [A]** time = 0.43, size = 205, normalized size = 2.01

$$\left[ \frac{(be^2 + 4(bd - 2ae)f)\sqrt{f} \log\left(-8f^2x^2 - 8efx - e^2 - 4\sqrt{fx^2 + ex + d}(2fx + e)\sqrt{f} - 4df\right) - 4(2bf^2x + bef)\sqrt{fx^2 + ex + d}}{16ef^2}, \frac{(be^2 + 4(bd - 2ae)f)\sqrt{-f} \arctan\left(\frac{\sqrt{fx^2 + ex + d}(2fx + e)\sqrt{-f}}{2(f^2 + efx + df)}\right) + 2(2bf^2x + bef)\sqrt{fx^2 + ex + d}}{8ef^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x+b\*f\*x^2/e)/(f\*x^2+e\*x+d)^(1/2), x, algorithm="fricas")

[Out] [-1/16\*((b\*e^2 + 4\*(b\*d - 2\*a\*e)\*f)\*sqrt(f)\*log(-8\*f^2\*x^2 - 8\*e\*f\*x - e^2 - 4\*sqrt(f\*x^2 + e\*x + d)\*(2\*f\*x + e)\*sqrt(f) - 4\*d\*f) - 4\*(2\*b\*f^2\*x + b\*e\*f)\*sqrt(f\*x^2 + e\*x + d))/(e\*f^2), 1/8\*((b\*e^2 + 4\*(b\*d - 2\*a\*e)\*f)\*sqrt(-f)\*arctan(1/2\*sqrt(f\*x^2 + e\*x + d)\*(2\*f\*x + e)\*sqrt(-f)/(f^2\*x^2 + e\*f\*x + d\*f)) + 2\*(2\*b\*f^2\*x + b\*e\*f)\*sqrt(f\*x^2 + e\*x + d))/(e\*f^2)]

**giac [A]** time = 0.26, size = 84, normalized size = 0.82

$$\frac{1}{4}\sqrt{fx^2 + xe + d}\left(2bx e^{(-1)} + \frac{b}{f}\right) + \frac{(4bdf - 8afe + be^2)e^{(-1)} \log\left(\left|-2\left(\sqrt{f}x - \sqrt{fx^2 + xe + d}\right)\sqrt{f} - e\right|\right)}{8f^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x+b\*f\*x^2/e)/(f\*x^2+e\*x+d)^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{4}\sqrt{f x^2 + x e + d} (2 b x e^{-1} + b/f) + \frac{1}{8} (4 b d f - 8 a f e + b e^2) e^{-1} \log(\text{abs}(-2(\sqrt{f} x - \sqrt{f x^2 + x e + d}) \sqrt{f} - e)) / f^{3/2}$

**maple** [A] time = 0.03, size = 136, normalized size = 1.33

$$\frac{a \ln\left(\frac{f x + \frac{e}{2}}{\sqrt{f}} + \sqrt{f x^2 + e x + d}\right)}{\sqrt{f}} - \frac{b d \ln\left(\frac{f x + \frac{e}{2}}{\sqrt{f}} + \sqrt{f x^2 + e x + d}\right)}{2 e \sqrt{f}} - \frac{b e \ln\left(\frac{f x + \frac{e}{2}}{\sqrt{f}} + \sqrt{f x^2 + e x + d}\right)}{8 f^{\frac{3}{2}}} + \frac{\sqrt{f x^2 + e x + d} b x}{2 e} + \frac{\sqrt{f x^2 + e x + d} b}{4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x+b\*f\*x^2/e)/(f\*x^2+e\*x+d)^(1/2),x)

[Out]  $\frac{1}{2} b x (f x^2 + e x + d)^{1/2} / e + \frac{1}{4} b (f x^2 + e x + d)^{1/2} / f - \frac{1}{8} e b / f^{3/2} \ln\left(\frac{(1/2 e + f x) / f^{1/2} + (f x^2 + e x + d)^{1/2}}{(1/2 e + f x) / f^{1/2} + (f x^2 + e x + d)^{1/2}}\right) + a \ln\left(\frac{(1/2 e + f x) / f^{1/2} + (f x^2 + e x + d)^{1/2}}{(1/2 e + f x) / f^{1/2} + (f x^2 + e x + d)^{1/2}}\right)$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x+b\*f\*x^2/e)/(f\*x^2+e\*x+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*d\*f-e^2>0)', see `assume?` for more details) Is 4\*d\*f-e^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b x + \frac{b f x^2}{e}}{\sqrt{f x^2 + e x + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x + (b\*f\*x^2)/e)/(d + e\*x + f\*x^2)^(1/2),x)

[Out] int((a + b\*x + (b\*f\*x^2)/e)/(d + e\*x + f\*x^2)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a e}{\sqrt{d + e x + f x^2}} dx + \int \frac{b e x}{\sqrt{d + e x + f x^2}} dx + \int \frac{b f x^2}{\sqrt{d + e x + f x^2}} dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x+b\*f\*x\*\*2/e)/(f\*x\*\*2+e\*x+d)\*\*(1/2),x)

[Out] (Integral(a\*e/sqrt(d + e\*x + f\*x\*\*2), x) + Integral(b\*e\*x/sqrt(d + e\*x + f\*x\*\*2), x) + Integral(b\*f\*x\*\*2/sqrt(d + e\*x + f\*x\*\*2), x))/e

$$3.2 \quad \int \frac{1}{\sqrt{d+ex+fx^2} \left( a+bx+\frac{bf x^2}{e} \right)} dx$$

**Optimal.** Leaf size=82

$$-\frac{2\sqrt{e} \tanh^{-1} \left( \frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}} \right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

**Rubi [A]** time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$ , Rules used = {982, 208}

$$-\frac{2\sqrt{e} \tanh^{-1} \left( \frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}} \right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d + e\*x + f\*x^2]\*(a + b\*x + (b\*f\*x^2)/e)),x]

[Out] (-2\*Sqrt[e]\*ArcTanh[(Sqrt[b\*d - a\*e]\*(e + 2\*f\*x))/(Sqrt[e]\*Sqrt[b\*e - 4\*a\*f]\*Sqrt[d + e\*x + f\*x^2])]/(Sqrt[b\*d - a\*e]\*Sqrt[b\*e - 4\*a\*f]))

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 982**

Int[1/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] :> Dist[-2\*e, Subst[Int[1/(e\*(b\*e - 4\*a\*f) - (b\*d - a\*e)\*x^2), x], x, (e + 2\*f\*x)/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && EqQ[c\*e - b\*f, 0]

**Rubi steps**

$$\int \frac{1}{\sqrt{d+ex+fx^2} \left( a+bx+\frac{bf x^2}{e} \right)} dx = - \left( (2e) \text{Subst} \left( \int \frac{1}{e(be-4af) - (bd-ae)x^2} dx, x, \frac{e+2fx}{\sqrt{d+ex+fx^2}} \right) \right) \\ = - \frac{2\sqrt{e} \tanh^{-1} \left( \frac{\sqrt{bd-ae}(e+2fx)}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}} \right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

**Mathematica [B]** time = 0.39, size = 178, normalized size = 2.17

$$\frac{\sqrt{e} \left( \tanh^{-1} \left( \frac{-\sqrt{e}(e+2fx)\sqrt{be-4af} - \sqrt{b}(e^2-4df)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}} \right) + \tanh^{-1} \left( \frac{\sqrt{b}(e^2-4df) - \sqrt{e}(e+2fx)\sqrt{be-4af}}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}} \right) \right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d + e\*x + f\*x^2]\*(a + b\*x + (b\*f\*x^2)/e)),x]

[Out]  $(\sqrt{e} * (\operatorname{ArcTanh}[-(\sqrt{b} * (e^2 - 4 * d * f)) - \sqrt{e} * \sqrt{b * e - 4 * a * f}] * (e + 2 * f * x)) / (4 * \sqrt{b * d - a * e} * f * \sqrt{d + x * (e + f * x)})) + \operatorname{ArcTanh}[(\sqrt{b} * (e^2 - 4 * d * f) - \sqrt{e} * \sqrt{b * e - 4 * a * f}] * (e + 2 * f * x)) / (4 * \sqrt{b * d - a * e} * f * \sqrt{d + x * (e + f * x)})) / (\sqrt{b * d - a * e} * \sqrt{b * e - 4 * a * f})$

IntegrateAlgebraic [C] time = 0.50, size = 162, normalized size = 1.98

$$e \operatorname{RootSum} \left[ \#1^4 b f - 2 \#1^3 b e \sqrt{f} + 4 \#1^2 a e f - 2 \#1^2 b d f + \#1^2 b e^2 - 4 \#1 a e^2 \sqrt{f} + 2 \#1 b d e \sqrt{f} + a e^3 + b d^2 f - b d e^2, \frac{\log(-\#1 + \sqrt{d + e x + f x^2} - \sqrt{f} x)}{\#1^2 (-b) \sqrt{f} + \#1 b e - 2 a e \sqrt{f} + b d \sqrt{f}} \& \right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[d + e\*x + f\*x^2]\*(a + b\*x + (b\*f\*x^2)/e)),x]

[Out]  $e \operatorname{RootSum}[-(b * d * e^2) + a * e^3 + b * d^2 * f + 2 * b * d * e * \operatorname{Sqrt}[f] * \#1 - 4 * a * e^2 * \operatorname{Sqrt}[f] * \#1 + b * e^2 * \#1^2 - 2 * b * d * f * \#1^2 + 4 * a * e * f * \#1^2 - 2 * b * e * \operatorname{Sqrt}[f] * \#1^3 + b * f * \#1^4 \& , \operatorname{Log}[-(\operatorname{Sqrt}[f] * x) + \operatorname{Sqrt}[d + e * x + f * x^2] - \#1] / (b * d * \operatorname{Sqrt}[f] - 2 * a * e * \operatorname{Sqrt}[f] + b * e * \#1 - b * \operatorname{Sqrt}[f] * \#1^2) \& ]$

fricas [B] time = 0.66, size = 1079, normalized size = 13.16

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x+b\*f\*x^2/e)/(f\*x^2+e\*x+d)^(1/2),x, algorithm="fricas")

[Out]  $[1/2 * \operatorname{sqrt}(e / (b^2 * d * e - a * b * e^2 - 4 * (a * b * d - a^2 * e) * f)) * \log((8 * b^2 * d^2 * e^4 - 8 * a * b * d * e^5 + a^2 * e^6 + 16 * a^2 * d^2 * e^2 * f^2 + (b^2 * e^4 * f^2 + 16 * (b^2 * d^2 - 8 * a * b * d * e + 8 * a^2 * e^2) * f^4 + 8 * (3 * b^2 * d * e^2 - 4 * a * b * e^3) * f^3) * x^4 + 2 * (b^2 * e^5 * f + 16 * (b^2 * d^2 * e - 8 * a * b * d * e^2 + 8 * a^2 * e^3) * f^3 + 8 * (3 * b^2 * d * e^3 - 4 * a * b * e^4) * f^2) * x^3 + (b^2 * e^6 - 32 * (3 * a * b * d^2 * e - 4 * a^2 * d * e^2) * f^3 + 16 * (3 * b^2 * d^2 * e^2 - 13 * a * b * d * e^3 + 10 * a^2 * e^4) * f^2 + 2 * (16 * b^2 * d * e^4 - 19 * a * b * e^5) * f) * x^2 - 8 * (4 * a * b * d^2 * e^3 - 3 * a^2 * d * e^4) * f + 2 * (4 * b^2 * d * e^5 - 3 * a * b * e^6 - 16 * (3 * a * b * d^2 * e^2 - 4 * a^2 * d * e^3) * f^2 + 8 * (2 * b^2 * d^2 * e^3 - 5 * a * b * d * e^4 + 2 * a^2 * e^5) * f) * x - 4 * (2 * b^3 * d^2 * e^4 - 3 * a * b^2 * d * e^5 + a^2 * b * e^6 - 2 * (16 * (a * b^2 * d^2 - 3 * a^2 * b * d * e + 2 * a^3 * e^2) * f^4 - 4 * (b^3 * d^2 * e - 4 * a * b^2 * d * e^2 + 3 * a^2 * b * e^3) * f^3 - (b^3 * d * e^3 - a * b^2 * e^4) * f^2) * x^3 + 16 * (a^2 * b * d^2 * e^2 - a^3 * d * e^3) * f^2 - 3 * (16 * (a * b^2 * d^2 * e - 3 * a^2 * b * d * e^2 + 2 * a^3 * e^3) * f^3 - 4 * (b^3 * d^2 * e^2 - 4 * a * b^2 * d * e^3 + 3 * a^2 * b * e^4) * f^2 - (b^3 * d * e^4 - a * b^2 * e^5) * f) * x^2 - 4 * (3 * a * b^2 * d^2 * e^3 - 4 * a^2 * b * d * e^4 + a^3 * e^5) * f + (b^3 * d * e^5 - a * b^2 * e^6 + 32 * (a^2 * b * d^2 * e - a^3 * d * e^2) * f^3 - 40 * (a * b^2 * d^2 * e^2 - 2 * a^2 * b * d * e^3 + a^3 * e^4) * f^2 + 2 * (4 * b^3 * d^2 * e^3 - 11 * a * b^2 * d * e^4 + 7 * a^2 * b * e^5) * f) * x) * \operatorname{sqrt}(f * x^2 + e * x + d) * \operatorname{sqrt}(e / (b^2 * d * e - a * b * e^2 - 4 * (a * b * d - a^2 * e) * f)) / (b^2 * f^2 * x^4 + 2 * b^2 * e * f * x^3 + 2 * a * b * e^2 * x + a^2 * e^2 + (b^2 * e^2 + 2 * a * b * e * f) * x^2)), -\operatorname{sqrt}(-e / (b^2 * d * e - a * b * e^2 - 4 * (a * b * d - a^2 * e) * f)) * \operatorname{arctan}(-1/2 * (2 * b * d * e^2 - a * e^3 - 4 * a * d * e * f + (b * e^2 * f + 4 * (b * d - 2 * a * e) * f^2) * x^2 + (b * e^3 + 4 * (b * d * e - 2 * a * e^2) * f) * x) * \operatorname{sqrt}(f * x^2 + e * x + d) * \operatorname{sqrt}(-e / (b^2 * d * e - a * b * e^2 - 4 * (a * b * d - a^2 * e) * f)) / (2 * e * f^2 * x^3 + 3 * e^2 * f * x^2 + d * e^2 + (e^3 + 2 * d * e * f) * x))]$

giac [B] time = 3.04, size = 847, normalized size = 10.33

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x+b\*f\*x^2/e)/(f\*x^2+e\*x+d)^(1/2),x, algorithm="giac")

[Out]  $\operatorname{sqrt}(-4 * a * b * d * f * e + b^2 * d * e^2 + 4 * a^2 * f * e^2 - a * b * e^3) * \log(\operatorname{abs}(-4 * (\operatorname{sqrt}(f) * x - \operatorname{sqrt}(f * x^2 + x * e + d))^2 * b * d * f^2 + 8 * (\operatorname{sqrt}(f) * x - \operatorname{sqrt}(f * x^2 + x * e + d))^2 * a * f^2 * e - 4 * (\operatorname{sqrt}(f) * x - \operatorname{sqrt}(f * x^2 + x * e + d)) * b * d * f^{(3/2)} * e + 4 * b * d^2 * f^2 - (\operatorname{sqrt}(f) * x - \operatorname{sqrt}(f * x^2 + x * e + d))^2 * b * f * e^2 + 8 * (\operatorname{sqrt}(f) * x - \operatorname{sqrt}(f * x^2 + x * e + d)) * b * d * e^2 - 4 * (a * b * d - a^2 * e) * f) * x^2 + (b * e^2 * f + 4 * (b * d - 2 * a * e) * f^2) * x^2 + (b * e^3 + 4 * (b * d * e - 2 * a * e^2) * f) * x) * \operatorname{sqrt}(f * x^2 + x * e + d) * \operatorname{sqrt}(-e / (b^2 * d * e - a * b * e^2 - 4 * (a * b * d - a^2 * e) * f)) / (2 * e * f^2 * x^3 + 3 * e^2 * f * x^2 + d * e^2 + (e^3 + 2 * d * e * f) * x))$

$$f*x^2 + x*e + d)) * a*f^{(3/2)}*e^2 + 4*\sqrt{-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3}*(\sqrt{f}*x - \sqrt{f*x^2 + x*e + d})^2*f^{(3/2)} - 3*b*d*f*e^2 - (\sqrt{f}*x - \sqrt{f*x^2 + x*e + d})*b*\sqrt{f}*e^3 + 4*\sqrt{-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3}*(\sqrt{f}*x - \sqrt{f*x^2 + x*e + d})*f*e + 2*a*f*e^3 + \sqrt{-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3}*\sqrt{f}*e^2)/(4*a*b*d*f - b^2*d*e - 4*a^2*f*e + a*b*e^2) - \sqrt{-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3}*\log(\text{abs}(-4*(\sqrt{f}*x - \sqrt{f*x^2 + x*e + d})^2*b*d*f^2 + 8*(\sqrt{f}*x - \sqrt{f*x^2 + x*e + d})^2*a*f^2*e - 4*(\sqrt{f}*x - \sqrt{f*x^2 + x*e + d})*b*d*f^{(3/2)}*e + 4*b*d^2*f^2 - (\sqrt{f}*x - \sqrt{f*x^2 + x*e + d})^2*b*f*e^2 + 8*(\sqrt{f}*x - \sqrt{f*x^2 + x*e + d})*a*f^{(3/2)}*e^2 - 4*\sqrt{-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3}*(\sqrt{f}*x - \sqrt{f*x^2 + x*e + d})^2*f^{(3/2)} - 3*b*d*f*e^2 - (\sqrt{f}*x - \sqrt{f*x^2 + x*e + d})*b*\sqrt{f}*e^3 - 4*\sqrt{-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3}*(\sqrt{f}*x - \sqrt{f*x^2 + x*e + d})*f*e + 2*a*f*e^3 - \sqrt{-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3}*\sqrt{f}*e^2))/(4*a*b*d*f - b^2*d*e - 4*a^2*f*e + a*b*e^2)$$

**maple [B]** time = 0.06, size = 491, normalized size = 5.99

$$e \ln \left( \frac{-\frac{2(ac-bd)}{b} \sqrt{-(4af-be)bc} \left( x + \frac{-be + \sqrt{-(4af-be)bc}}{b} \right) + 2 \sqrt{\frac{ac-bd}{b}} \sqrt{\left( x + \frac{-be + \sqrt{-(4af-be)bc}}{b} \right)^2 f + \frac{\sqrt{-(4af-be)bc} \left( x + \frac{-be + \sqrt{-(4af-be)bc}}{b} \right)}{b}}}{x + \frac{-be + \sqrt{-(4af-be)bc}}{b}} \right) + e \ln \left( \frac{-\frac{2(ac-bd)}{b} \sqrt{-(4af-be)bc} \left( x + \frac{be + \sqrt{-(4af-be)bc}}{b} \right) + 2 \sqrt{\frac{ac-bd}{b}} \sqrt{\left( x + \frac{be + \sqrt{-(4af-be)bc}}{b} \right)^2 f - \frac{\sqrt{-(4af-be)bc} \left( x + \frac{be + \sqrt{-(4af-be)bc}}{b} \right)}{b}}}{x + \frac{be + \sqrt{-(4af-be)bc}}{b}} \right)}{\sqrt{-(4af-be)bc} \sqrt{\frac{ac-bd}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*x+b\*f\*x^2/e)/(f\*x^2+e\*x+d)^(1/2),x)

[Out] 
$$-e/(-b*e*(4*a*f-b*e))^{(1/2)}/(-1/b*(a*e-b*d))^{(1/2)}*\ln\left(\frac{-2/b*(a*e-b*d)+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)}*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)}}{(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+e/(-b*e*(4*a*f-b*e))^{(1/2)}/(-1/b*(a*e-b*d))^{(1/2)}*\ln\left(\frac{-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)}*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)}}{(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)\right)}\right)$$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x+b\*f\*x^2/e)/(f\*x^2+e\*x+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*f-b\*e>0)', see `assume?` for more details)Is 4\*a\*f-b\*e positive, negative or zero?

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{f x^2 + e x + d} \left( a + b x + \frac{b f x^2}{e} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d + e\*x + f\*x^2)^(1/2)\*(a + b\*x + (b\*f\*x^2)/e)),x)

[Out] int(1/((d + e\*x + f\*x^2)^(1/2)\*(a + b\*x + (b\*f\*x^2)/e)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e \int \frac{1}{ae\sqrt{d+ex+fx^2} + bex\sqrt{d+ex+fx^2} + bfx^2\sqrt{d+ex+fx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x+b\*f\*x\*\*2/e)/(f\*x\*\*2+e\*x+d)\*\*(1/2), x)

[Out] e\*Integral(1/(a\*e\*sqrt(d + e\*x + f\*x\*\*2) + b\*e\*x\*sqrt(d + e\*x + f\*x\*\*2) + b\*f\*x\*\*2\*sqrt(d + e\*x + f\*x\*\*2)), x)



$$3.3 \quad \int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)} dx$$

**Optimal.** Leaf size=66

$$\frac{2 \tanh^{-1} \left( \frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd} \sqrt{a+bx+cx^2}} \right)}{\sqrt{a-d} \sqrt{b^2-4cd}}$$

**Rubi [A]** time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$ , Rules used = {982, 208}

$$\frac{2 \tanh^{-1} \left( \frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd} \sqrt{a+bx+cx^2}} \right)}{\sqrt{a-d} \sqrt{b^2-4cd}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)),x]

[Out] (-2\*ArcTanh[(Sqrt[a - d]\*(b + 2\*c\*x))/(Sqrt[b^2 - 4\*c\*d]\*Sqrt[a + b\*x + c\*x^2])])/(Sqrt[a - d]\*Sqrt[b^2 - 4\*c\*d])

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 982**

Int[1/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] :> Dist[-2\*e, Subst[Int[1/(e\*(b\*e - 4\*a\*f) - (b\*d - a\*e)\*x^2), x], x, (e + 2\*f\*x)/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && EqQ[c\*e - b\*f, 0]

**Rubi steps**

$$\begin{aligned} \int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)} dx &= - \left( (2b) \text{Subst} \left( \int \frac{1}{b(b^2-4cd) - (ab-bd)x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}} \right) \right) \\ &= - \frac{2 \tanh^{-1} \left( \frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd} \sqrt{a+bx+cx^2}} \right)}{\sqrt{a-d} \sqrt{b^2-4cd}} \end{aligned}$$

**Mathematica [B]** time = 0.23, size = 161, normalized size = 2.44

$$\frac{\tanh^{-1} \left( \frac{4ac-2cx\sqrt{b^2-4cd}-b(\sqrt{b^2-4cd}+b)}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}} \right) + \tanh^{-1} \left( \frac{-2c(2a+x\sqrt{b^2-4cd})-b\sqrt{b^2-4cd}+b^2}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}} \right)}{\sqrt{a-d}\sqrt{b^2-4cd}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)),x]

[Out] (ArcTanh[(4\*a\*c - b\*(b + Sqrt[b^2 - 4\*c\*d]) - 2\*c\*Sqrt[b^2 - 4\*c\*d]\*x)/(4\*c\*Sqrt[a - d]\*Sqrt[a + x\*(b + c\*x)])] + ArcTanh[(b^2 - b\*Sqrt[b^2 - 4\*c\*d] -

$$\frac{2*c*(2*a + \text{Sqrt}[b^2 - 4*c*d]*x))/(4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + x*(b + c*x)])}{(\text{Sqrt}[a - d]*\text{Sqrt}[b^2 - 4*c*d])}$$

**IntegrateAlgebraic [C]** time = 0.44, size = 146, normalized size = 2.21

$$\text{RootSum} \left[ \#1^4 c - 2\#1^3 b \sqrt{c} - 2\#1^2 a c + \#1^2 b^2 + 4\#1^2 c d + 2\#1 a b \sqrt{c} - 4\#1 b \sqrt{c} d + a^2 c - a b^2 + b^2 d \&, \frac{\log(-\#1 + \sqrt{a + b x + c x^2} - \sqrt{c} x)}{\#1^2 (-\sqrt{c}) + \#1 b + a \sqrt{c} - 2 \sqrt{c} d} \& \right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)),x]

[Out] RootSum[-(a\*b^2) + a^2\*c + b^2\*d + 2\*a\*b\*Sqrt[c]\*#1 - 4\*b\*Sqrt[c]\*d\*#1 + b^2\*#1^2 - 2\*a\*c\*#1^2 + 4\*c\*d\*#1^2 - 2\*b\*Sqrt[c]\*#1^3 + c\*#1^4 & , Log[-(Sqrt[c]\*x) + Sqrt[a + b\*x + c\*x^2] - #1]/(a\*Sqrt[c] - 2\*Sqrt[c]\*d + b\*#1 - Sqrt[c]\*#1^2) & ]

**fricas [B]** time = 0.52, size = 813, normalized size = 12.32

$$\frac{\log\left(\frac{\sqrt{a+b x+c x^2}-\sqrt{c} x}{\sqrt{a+b x+c x^2}+\sqrt{c} x}\right)}{\sqrt{a+b x+c x^2}+\sqrt{c} x} + \frac{\log\left(\frac{\sqrt{a+b x+c x^2}-\sqrt{c} x}{\sqrt{a+b x+c x^2}+\sqrt{c} x}\right)}{\sqrt{a+b x+c x^2}+\sqrt{c} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] [1/2\*log((8\*a^2\*b^4 + (b^4\*c^2 + 24\*a\*b^2\*c^3 + 16\*a^2\*c^4 + 128\*c^4\*d^2 - 32\*(b^2\*c^3 + 4\*a\*c^4)\*d)\*x^4 + 2\*(b^5\*c + 24\*a\*b^3\*c^2 + 16\*a^2\*b\*c^3 + 12\*8\*b\*c^3\*d^2 - 32\*(b^3\*c^2 + 4\*a\*b\*c^3)\*d)\*x^3 + (b^4 + 24\*a\*b^2\*c + 16\*a^2\*c^2)\*d^2 + (b^6 + 32\*a\*b^4\*c + 48\*a^2\*b^2\*c^2 + 32\*(5\*b^2\*c^2 + 4\*a\*c^3)\*d^2 - 2\*(19\*b^4\*c + 104\*a\*b^2\*c^2 + 48\*a^2\*c^3)\*d)\*x^2 - 4\*(2\*a\*b^3 + 2\*(b^2\*c^2 + 4\*a\*c^3 - 8\*c^3\*d)\*x^3 + 3\*(b^3\*c + 4\*a\*b\*c^2 - 8\*b\*c^2\*d)\*x^2 - (b^3 + 4\*a\*b\*c)\*d + (b^4 + 8\*a\*b^2\*c - 2\*(5\*b^2\*c + 4\*a\*c^2)\*d)\*x)\*sqrt(a\*b^2 + 4\*c\*d^2 - (b^2 + 4\*a\*c)\*d)\*sqrt(c\*x^2 + b\*x + a) - 8\*(a\*b^4 + 4\*a^2\*b^2\*c)\*d + 2\*(4\*a\*b^5 + 16\*a^2\*b^3\*c + 16\*(b^3\*c + 4\*a\*b\*c^2)\*d^2 - (3\*b^5 + 40\*a\*b^3\*c + 48\*a^2\*b\*c^2)\*d)\*x)/(c^2\*x^4 + 2\*b\*c\*x^3 + 2\*b\*d\*x + (b^2 + 2\*c\*d)\*x^2 + d^2))/sqrt(a\*b^2 + 4\*c\*d^2 - (b^2 + 4\*a\*c)\*d), -sqrt(-a\*b^2 - 4\*c\*d^2 + (b^2 + 4\*a\*c)\*d)\*arctan(-1/2\*(2\*a\*b^2 + (b^2\*c + 4\*a\*c^2 - 8\*c^2\*d)\*x^2 - (b^2 + 4\*a\*c)\*d + (b^3 + 4\*a\*b\*c - 8\*b\*c\*d)\*x)\*sqrt(-a\*b^2 - 4\*c\*d^2 + (b^2 + 4\*a\*c)\*d)\*sqrt(c\*x^2 + b\*x + a)/(a^2\*b^3 + 4\*a\*b\*c\*d^2 + 2\*(a\*b^2\*c^2 + 4\*c^3\*d^2 - (b^2\*c^2 + 4\*a\*c^3)\*d)\*x^3 + 3\*(a\*b^3\*c + 4\*b\*c^2\*d^2 - (b^3\*c + 4\*a\*b\*c^2)\*d)\*x^2 - (a\*b^3 + 4\*a^2\*b\*c)\*d + (a\*b^4 + 2\*a^2\*b^2\*c + 4\*(b^2\*c + 2\*a\*c^2)\*d^2 - (b^4 + 6\*a\*b^2\*c + 8\*a^2\*c^2)\*d)\*x)/(a\*b^2 + 4\*c\*d^2 - (b^2 + 4\*a\*c)\*d)]

**giac [B]** time = 1.90, size = 703, normalized size = 10.65

$$\frac{\log\left(\frac{\sqrt{a+b x+c x^2}-\sqrt{c} x}{\sqrt{a+b x+c x^2}+\sqrt{c} x}\right)}{\sqrt{a+b x+c x^2}+\sqrt{c} x} + \frac{\log\left(\frac{\sqrt{a+b x+c x^2}-\sqrt{c} x}{\sqrt{a+b x+c x^2}+\sqrt{c} x}\right)}{\sqrt{a+b x+c x^2}+\sqrt{c} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out] -log(abs(-(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))^2\*b^2\*c - 4\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))^2\*a\*c^2 + 8\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))^2\*c^2\*d - (sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*b^3\*sqrt(c) - 4\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*a\*b\*c^(3/2) + 8\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*b\*c^(3/2)\*d - 3\*a\*b^2\*c + 4\*sqrt(a\*b^2 - b^2\*d - 4\*a\*c\*d + 4\*c\*d^2)\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))^2\*c^(3/2) + 4\*a^2\*c^2 + 2\*b^2\*c\*d + 4\*sqrt(a\*b^2 - b^2\*d - 4\*a\*c\*d + 4\*c\*d^2)\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*b\*c + sqrt(a\*b^2 - b^2\*d - 4\*a\*c\*d + 4\*c\*d^2)\*b^2\*sqrt(c))/sqrt(a\*b^2 - b^2\*d - 4\*a\*c\*d + 4\*c\*d^2) + log(abs(-(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))^2\*b^2\*c - 4\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))^2\*a\*c^2 + 8\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))^2\*c^2\*d - (sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*b^3\*sqrt(c) - 4\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*a\*b\*c^(3/2) + 8\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*b\*c^(3/2)\*d - 3\*a\*b^2\*c + 4\*sqrt(a\*b^2 - b^2\*d - 4\*a\*c\*d + 4\*c\*d^2)\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))^2\*c^(3/2) + 4\*a^2\*c^2 + 2\*b^2\*c\*d + 4\*sqrt(a\*b^2 - b^2\*d - 4\*a\*c\*d + 4\*c\*d^2)\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*b\*c + sqrt(a\*b^2 - b^2\*d - 4\*a\*c\*d + 4\*c\*d^2)\*b^2\*sqrt(c)))/sqrt(a\*b^2 - b^2\*d - 4\*a\*c\*d + 4\*c\*d^2) + log(abs(-(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))^2\*b^2\*c - 4\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))^2\*a\*c^2 + 8\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))^2\*c^2\*d - (sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*b^3\*sqrt(c) - 4\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*a\*b\*c^(3/2) + 8\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*b\*c^(3/2)\*d - 3\*a\*b^2\*c + 4\*sqrt(a\*b^2 - b^2\*d - 4\*a\*c\*d + 4\*c\*d^2)\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))^2\*c^(3/2) + 4\*a^2\*c^2 + 2\*b^2\*c\*d + 4\*sqrt(a\*b^2 - b^2\*d - 4\*a\*c\*d + 4\*c\*d^2)\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*b\*c + sqrt(a\*b^2 - b^2\*d - 4\*a\*c\*d + 4\*c\*d^2)\*b^2\*sqrt(c)))/sqrt(a\*b^2 - b^2\*d - 4\*a\*c\*d + 4\*c\*d^2)

$$\begin{aligned} & ))^2*c^2*d - (\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*b^3*\text{sqrt}(c) - 4*(\text{sqrt}(c)*x \\ & - \text{sqrt}(c*x^2 + b*x + a))*a*b*c^{(3/2)} + 8*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a) \\ & ))*b*c^{(3/2)}*d - 3*a*b^2*c - 4*\text{sqrt}(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*c^{(3/2)} + 4*a^2*c^2 + 2*b^2*c*d - 4*\text{sqrt}(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*b*c \\ & - \text{sqrt}(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*\text{sqrt}(c))/\text{sqrt}(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2) \end{aligned}$$

**maple [B]** time = 0.03, size = 307, normalized size = 4.65

$$\frac{\ln\left(\frac{2a-2d+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\sqrt{a+\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)^2}}{x-\frac{-b+\sqrt{b^2-4cd}}{2c}}\right)}{\sqrt{b^2-4cd}\sqrt{a-d}} + \frac{\ln\left(\frac{2a-2d-\sqrt{b^2-4cd}\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\sqrt{a+\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)^2}}{x+\frac{b+\sqrt{b^2-4cd}}{2c}}\right)}{\sqrt{b^2-4cd}\sqrt{a-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x^2+b\*x+d)/(c\*x^2+b\*x+a)^(1/2), x)

[Out]  $1/(b^2-4*c*d)^{(1/2)}/(a-d)^{(1/2)}*\ln((2*a-2*d-(b^2-4*c*d)^{(1/2)}*(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)+2*(a-d)^{(1/2)}*((x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)^2*c-(b^2-4*c*d)^{(1/2)}*(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)+a-d)^{(1/2)})/(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c))-1/(b^2-4*c*d)^{(1/2)}/(a-d)^{(1/2)}*\ln((2*a-2*d+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)}))/c)+2*(a-d)^{(1/2)}*((x-1/2*(-b+(b^2-4*c*d)^{(1/2)}))/c)^2*c+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)}))/c)+a-d)^{(1/2)})/(x-1/2*(-b+(b^2-4*c*d)^{(1/2)}))/c)$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)/(c\*x^2+b\*x+a)^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*c\*d-b^2>0)', see 'assume?' for more details) Is 4\*c\*d-b^2 positive, negative or zero?

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (cx^2 + bx + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + b\*x + c\*x^2)), x)

[Out] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + b\*x + c\*x^2)), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx + cx^2} (bx + cx^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x\*\*2+b\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] Integral(1/(sqrt(a + b\*x + c\*x\*\*2)\*(b\*x + c\*x\*\*2 + d)), x)

$$3.4 \quad \int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)^2} dx$$

**Optimal.** Leaf size=129

$$\frac{(4c(a-2d)+b^2) \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}(b^2-4cd)^{3/2}} - \frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(bx+cx^2+d)}$$

**Rubi [A]** time = 0.17, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {974, 12, 982, 208}

$$\frac{(4c(a-2d)+b^2) \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}(b^2-4cd)^{3/2}} - \frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(bx+cx^2+d)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)^2), x]

[Out] -(((b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/((a - d)\*(b^2 - 4\*c\*d)\*(d + b\*x + c\*x^2))) + ((b^2 + 4\*c\*(a - 2\*d))\*ArcTanh[(Sqrt[a - d]\*(b + 2\*c\*x))/(Sqrt[b^2 - 4\*c\*d]\*Sqrt[a + b\*x + c\*x^2]])/((a - d)^(3/2)\*(b^2 - 4\*c\*d)^(3/2)))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

#### Rule 982

Int[1/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2\*e, Subst[Int[1/(e\*(b\*e - 4\*a\*f) - (b\*d - a\*e)\*x^2), x], x, (e + 2\*f\*x)/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && EqQ[c\*e - b\*f, 0]

]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)^2} dx &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} + \frac{\int -\frac{c^2(b^2+4c(a-2d))(a-d)}{2\sqrt{a+bx+cx^2}(d+bx+cx^2)} dx}{c^2(a-d)^2(b^2-4cd)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} - \frac{(b^2+4c(a-2d)) \int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)} dx}{2(a-d)(b^2-4cd)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} + \frac{(b(b^2+4c(a-2d))) \operatorname{Subst}\left(\int \frac{1}{b\sqrt{a+bx+cx^2}} dx, b, d+bx+cx^2\right)}{(a-d)(b^2-4cd)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} + \frac{(b^2+4c(a-2d)) \tanh^{-1}\left(\frac{\sqrt{a+bx+cx^2}}{\sqrt{b^2-4cd}}\right)}{(a-d)^{3/2}(b^2-4cd)^{3/2}}
\end{aligned}$$

**Mathematica [B]** time = 0.92, size = 296, normalized size = 2.29

$$\frac{1}{2} \left( \frac{8c(b+2cx)\sqrt{a+x(b+cx)}}{(a-d)(4cd-b^2)(\sqrt{b^2-4cd}-b-2cx)(\sqrt{b^2-4cd}+b+2cx)} - \frac{(4c(a-2d)+b^2) \tanh^{-1}\left(\frac{4ac-2cx\sqrt{b^2-4cd}-b(\sqrt{b^2-4cd}+b)}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)}{(a-d)^{3/2}(b^2-4cd)^{3/2}} - \frac{(4c(a-2d)+b^2) \tanh^{-1}\left(\frac{-2c(2a+x\sqrt{b^2-4cd})-b\sqrt{b^2-4cd}+b^2}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)}{(a-d)^{3/2}(b^2-4cd)^{3/2}} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2), x]`

```
[Out] ((-8*c*(b + 2*c*x)*Sqrt[a + x*(b + c*x)])/((a - d)*(-b^2 + 4*c*d)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x)*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)) - ((b^2 + 4*c*(a - 2*d))*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])/((a - d)^(3/2)*(b^2 - 4*c*d)^(3/2)) - ((b^2 + 4*c*(a - 2*d))*ArcTanh[(b^2 - b*Sqrt[b^2 - 4*c*d] - 2*c*(2*a + Sqrt[b^2 - 4*c*d]*x))/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])/((a - d)^(3/2)*(b^2 - 4*c*d)^(3/2)))/2
```

**IntegrateAlgebraic [C]** time = 0.86, size = 239, normalized size = 1.85

$$\frac{(-4ac - b^2 + 8cd) \operatorname{RootSum}\left[\#1^4c - 2\#1^3b\sqrt{c} - 2\#1^2ac + \#1^2b^2 + 4\#1^2cd + 2\#1ab\sqrt{c} - 4\#1b\sqrt{c}d + a^2c - ab^2 + b^2d\&, \frac{\log\left(\frac{-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}}{\#1^2(-\sqrt{c}) + \#1b + a\sqrt{c} - 2\sqrt{cd}}\right) \& 1}{\#1^2(-\sqrt{c}) + \#1b + a\sqrt{c} - 2\sqrt{cd}}\right]}{2(a-d)(4cd-b^2)} - \frac{(-b-2cx)\sqrt{a+bx+cx^2}}{(a-d)(4cd-b^2)(bx+cx^2+d)}$$

Antiderivative was successfully verified.

`[In] IntegrateAlgebraic[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2), x]`

```
[Out] -(((b - 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(-b^2 + 4*c*d)*(d + b*x + c*x^2))) - (((-b^2 - 4*a*c + 8*c*d)*RootSum[-(a*b^2) + a^2*c + b^2*d + 2*a*b*Sqrt[c]*#1 - 4*b*Sqrt[c]*d*#1 + b^2*#1^2 - 2*a*c*#1^2 + 4*c*d*#1^2 - 2*b*Sqrt[c]*#1^3 + c*#1^4 &, Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]/(a*Sqrt[c] - 2*Sqrt[c]*d + b*#1 - Sqrt[c]*#1^2) & ])/(2*(a - d)*(-b^2 + 4*c*d))
```

**fricas [B]** time = 0.84, size = 1544, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2), x, algorithm="fricas")`

```
[Out] [1/4*(sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d - (b^3 + 4*a*b*c - 8*b*c*d)*x)*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2 + d^2)) - 4*(a*b^3 + 4*b*c*d^2 - (b^3 + 4*a*b*c)*d + 2*(a*b^2*c + 4*c^2*d^2 - (b^2*c + 4*a*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*d + 16*c^2*d^5 - 8*(b^2*c + 4*a*c^2)*d^4 + (b^4 + 16*a*b^2*c + 16*a^2*c^2)*d^3 - 2*(a*b^4 + 4*a^2*b^2*c)*d^2 + (a^2*b^4*c + 16*c^3*d^4 - 8*(b^2*c^2 + 4*a*c^3)*d^3 + (b^4*c + 16*a*b^2*c^2 + 16*a^2*c^3)*d^2 - 2*(a*b^4*c + 4*a^2*b^2*c^2)*d)*x^2 + (a^2*b^5 + 16*b*c^2*d^4 - 8*(b^3*c + 4*a*b*c^2)*d^3 + (b^5 + 16*a*b^3*c + 16*a^2*b*c^2)*d^2 - 2*(a*b^5 + 4*a^2*b^3*c)*d)*x), -1/2*(sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d - (b^3 + 4*a*b*c - 8*b*c*d)*x)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2 + 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3*c + 4*a*b*c^2)*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(b^2*c + 2*a*c^2)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(a*b^3 + 4*b*c*d^2 - (b^3 + 4*a*b*c)*d + 2*(a*b^2*c + 4*c^2*d^2 - (b^2*c + 4*a*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*d + 16*c^2*d^5 - 8*(b^2*c + 4*a*c^2)*d^4 + (b^4 + 16*a*b^2*c + 16*a^2*c^2)*d^3 - 2*(a*b^4 + 4*a^2*b^2*c)*d^2 + (a^2*b^4*c + 16*c^3*d^4 - 8*(b^2*c^2 + 4*a*c^3)*d^3 + (b^4*c + 16*a*b^2*c^2 + 16*a^2*c^3)*d^2 - 2*(a*b^4*c + 4*a^2*b^2*c^2)*d)*x^2 + (a^2*b^5 + 16*b*c^2*d^4 - 8*(b^3*c + 4*a*b*c^2)*d^3 + (b^5 + 16*a*b^3*c + 16*a^2*b*c^2)*d^2 - 2*(a*b^5 + 4*a^2*b^3*c)*d)*x)]
```

**giac [B]** time = 2.27, size = 1166, normalized size = 9.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*((b^2 + 4*a*c - 8*c*d)*log(abs((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^2*d + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^(3/2)*d + 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) - 4*a^2*c^2 - 2*b^2*c*d + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c + sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2) - (b^2 + 4*a*c - 8*c*d)*log(abs((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^2*d + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^(3/2)*d + 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) - 4*a^2*c^2 - 2*b^2*c*d - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c - sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2))/(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2) + ((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c) + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2) - 8*(sqrt(c)*x
```

$$-\sqrt{cx^2 + bx + a} \cdot c^{3/2} \cdot d + (\sqrt{c} \cdot x - \sqrt{cx^2 + bx + a}) \cdot b^3 + 4(\sqrt{c} \cdot x - \sqrt{cx^2 + bx + a}) \cdot a \cdot b \cdot c - 8(\sqrt{c} \cdot x - \sqrt{cx^2 + bx + a}) \cdot b \cdot c \cdot d + 3a \cdot b^2 \cdot \sqrt{c} - 4a^2 \cdot c^{3/2} - 2b^2 \cdot \sqrt{c} \cdot d / ((\sqrt{c} \cdot x - \sqrt{cx^2 + bx + a})^4 \cdot c + 2(\sqrt{c} \cdot x - \sqrt{cx^2 + bx + a})^3 \cdot b \cdot \sqrt{c} + (\sqrt{c} \cdot x - \sqrt{cx^2 + bx + a})^2 \cdot b^2 - 2(\sqrt{c} \cdot x - \sqrt{cx^2 + bx + a})^2 \cdot a \cdot c + 4(\sqrt{c} \cdot x - \sqrt{cx^2 + bx + a})^2 \cdot c \cdot d - 2(\sqrt{c} \cdot x - \sqrt{cx^2 + bx + a}) \cdot a \cdot b \cdot \sqrt{c} + 4(\sqrt{c} \cdot x - \sqrt{cx^2 + bx + a}) \cdot b \cdot \sqrt{c} \cdot d - a \cdot b^2 + a^2 \cdot c + b^2 \cdot d) \cdot (a \cdot b^2 - b^2 \cdot d - 4a \cdot c \cdot d + 4c \cdot d^2)$$

**maple [B]** time = 0.03, size = 829, normalized size = 6.43

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x^2+b\*x+d)^2/(c\*x^2+b\*x+a)^(1/2),x)

[Out] 
$$-2/(b^2-4cd)^{3/2} \cdot c/(a-d)^{1/2} \cdot \ln((2a-2d-(b^2-4cd)^{1/2}) \cdot (x+1/2 \cdot (b+(b^2-4cd)^{1/2})/c) + 2(a-d)^{1/2} \cdot (a+(x+1/2 \cdot (b+(b^2-4cd)^{1/2})/c))^2 \cdot c - d - (b^2-4cd)^{1/2} \cdot (x+1/2 \cdot (b+(b^2-4cd)^{1/2})/c))^{1/2}) / (x+1/2 \cdot (b+(b^2-4cd)^{1/2})/c) + 2/(b^2-4cd)^{3/2} \cdot c/(a-d)^{1/2} \cdot \ln((2a-2d+(b^2-4cd)^{1/2}) \cdot (x-1/2 \cdot (-b+(b^2-4cd)^{1/2})/c) + 2(a-d)^{1/2} \cdot (a+(x-1/2 \cdot (-b+(b^2-4cd)^{1/2})/c))^2 \cdot c - d + (b^2-4cd)^{1/2} \cdot (x-1/2 \cdot (-b+(b^2-4cd)^{1/2})/c))^{1/2}) / (x-1/2 \cdot (-b+(b^2-4cd)^{1/2})/c) - 1/(b^2-4cd) \cdot (a-d) / (x-1/2 \cdot c \cdot (b^2-4cd)^{1/2} + 1/2 \cdot c \cdot b) \cdot (a+(x-1/2 \cdot (-b+(b^2-4cd)^{1/2})/c))^2 \cdot c - d + (b^2-4cd)^{1/2} \cdot (x-1/2 \cdot (-b+(b^2-4cd)^{1/2})/c))^{1/2} + 1/2 \cdot (b^2-4cd)^{1/2} / (a-d)^{3/2} \cdot \ln((2a-2d+(b^2-4cd)^{1/2}) \cdot (x-1/2 \cdot (-b+(b^2-4cd)^{1/2})/c) + 2(a-d)^{1/2} \cdot (a+(x-1/2 \cdot (-b+(b^2-4cd)^{1/2})/c))^2 \cdot c - d + (b^2-4cd)^{1/2} \cdot (x-1/2 \cdot (-b+(b^2-4cd)^{1/2})/c))^{1/2}) / (x-1/2 \cdot (-b+(b^2-4cd)^{1/2})/c) - 1/(b^2-4cd) \cdot (a-d) / (x+1/2 \cdot c \cdot (b^2-4cd)^{1/2} + 1/2 \cdot c \cdot b) \cdot (a+(x+1/2 \cdot (b+(b^2-4cd)^{1/2})/c))^2 \cdot c - d - (b^2-4cd)^{1/2} \cdot (x+1/2 \cdot (b+(b^2-4cd)^{1/2})/c))^{1/2}) / (x+1/2 \cdot (b+(b^2-4cd)^{1/2})/c)$$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (cx^2 + bx + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)^2/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(c\*x^2 + b\*x + d)^2), x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (cx^2 + bx + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + b\*x + c\*x^2)^2),x)

[Out] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + b\*x + c\*x^2)^2), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx + cx^2} (bx + cx^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(b*x + c*x**2 + d)**2), x)
```



$$3.5 \quad \int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)^3} dx$$

**Optimal.** Leaf size=224

$$\frac{(16c^2(3a^2 - 8ad + 8d^2) + 8b^2c(a - 4d) + 3b^4) \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right) + \frac{3(b+2cx)(4c(a-2d) + b^2)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(bx+cx^2+d)}}{4(a-d)^{5/2}(b^2-4cd)^{5/2}}$$

**Rubi [A]** time = 0.43, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {974, 1060, 12, 982, 208}

$$\frac{(16c^2(3a^2 - 8ad + 8d^2) + 8b^2c(a - 4d) + 3b^4) \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right) + \frac{3(b+2cx)(4c(a-2d) + b^2)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(bx+cx^2+d)} - \frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(bx+cx^2+d)^2}}{4(a-d)^{5/2}(b^2-4cd)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)^3), x]

[Out] -((b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(2\*(a - d)\*(b^2 - 4\*c\*d)\*(d + b\*x + c\*x^2)^2) + (3\*(b^2 + 4\*c\*(a - 2\*d))\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(4\*(a - d)^2\*(b^2 - 4\*c\*d)^2\*(d + b\*x + c\*x^2)) - ((3\*b^4 + 8\*b^2\*c\*(a - 4\*d) + 16\*c^2\*(3\*a^2 - 8\*a\*d + 8\*d^2))\*ArcTanh[(Sqrt[a - d]\*(b + 2\*c\*x))/(Sqrt[b^2 - 4\*c\*d]\*Sqrt[a + b\*x + c\*x^2])])/(4\*(a - d)^(5/2)\*(b^2 - 4\*c\*d)^(5/2))

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^(q+1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), Int[(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p+1) - c\*d\*(p+2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p+q+2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p+q+2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p+1) - c\*e\*(2\*p+q+4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p+2\*q+5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

Rule 982

Int[1/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2\*e, Subst[Int[1/(e\*(b\*e - 4\*a\*f) - (b\*d - a\*e)\*x^2), x], x, (e + 2\*f\*x)/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && EqQ[c\*e - b\*f, 0]

]

Rule 1060

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)^3} dx &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{\int \frac{-\frac{1}{2}c^2(a-d)(3b^2+12ac-16cd)-4bc^3(a-d)x}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^2} dx}{2c^2(a-d)^2(b^2-4cd)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(d+bx+cx^2)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(d+bx+cx^2)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(d+bx+cx^2)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(d+bx+cx^2)}
\end{aligned}$$

Mathematica [B] time = 6.38, size = 1746, normalized size = 7.79

result too large to display

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)^3), x]

[Out] (-2\*c^2\*(a + b\*x + c\*x^2))/((a - d)\*(b^2 - 4\*c\*d)^(3/2)\*(b - Sqrt[b^2 - 4\*c\*d] + 2\*c\*x)^2\*Sqrt[a + x\*(b + c\*x)]) + (6\*c^2\*(a + b\*x + c\*x^2))/((a - d)\*

$$\begin{aligned}
& (b^2 - 4cd)^2(b - \sqrt{b^2 - 4cd} + 2cx)\sqrt{a + x(b + cx)} + (2c^2(a + bx + cx^2))/((a - d)(b^2 - 4cd)^{3/2}(b + \sqrt{b^2 - 4cd} \\
& + 2cx)^2\sqrt{a + x(b + cx)}) + (6c^2(a + bx + cx^2))/((a - d)(b^2 - 4cd)^2(b + \sqrt{b^2 - 4cd} + 2cx)\sqrt{a + x(b + cx)}) + (6c^2 \\
& 2\sqrt{a + bx + cx^2}\operatorname{ArcTanh}[(b^2 - 4ac - b\sqrt{b^2 - 4cd} - 2c\sqrt{b^2 - 4cd}x)/(4c\sqrt{a - d}\sqrt{a + bx + cx^2})])/( \sqrt{a - d}(b^2 - 4cd)^{5/2}\sqrt{a + x(b + cx)}) + (3c\sqrt{a + bx + cx^2}\operatorname{ArcTanh}[(b^2 - 4ac - b\sqrt{b^2 - 4cd} - 2c\sqrt{b^2 - 4cd}x)/(4c\sqrt{a - d}\sqrt{a + bx + cx^2})])/(2(a - d)^{3/2}(b^2 - 4cd)^{3/2}\sqrt{a + x(b + cx)}) + (6c^2\sqrt{a + bx + cx^2}\operatorname{ArcTanh}[(4ac - b(b + \sqrt{b^2 - 4cd}) - 2c\sqrt{b^2 - 4cd}x)/(4c\sqrt{a - d}\sqrt{a + bx + cx^2})])/( \sqrt{a - d}(b^2 - 4cd)^{5/2}\sqrt{a + x(b + cx)}) + (3c\sqrt{a + bx + cx^2}\operatorname{ArcTanh}[(4ac - b(b + \sqrt{b^2 - 4cd}) - 2c\sqrt{b^2 - 4cd}x)/(4c\sqrt{a - d}\sqrt{a + bx + cx^2})])/(2(a - d)^{3/2}(b^2 - 4cd)^{3/2}\sqrt{a + x(b + cx)}) + (4c^3\sqrt{a + bx + cx^2}(((-2c^2(-b + \sqrt{b^2 - 4cd}) - 2c^2(b + 2\sqrt{b^2 - 4cd}))\sqrt{a + bx + cx^2})/((4ac^2 + 2bc(-b + \sqrt{b^2 - 4cd}) + c(-b + \sqrt{b^2 - 4cd})^2)(-b + \sqrt{b^2 - 4cd} - 2cx)) + (4c\sqrt{a - d}(b(-2c^2(-b + \sqrt{b^2 - 4cd}) + 2c^2(b + 2\sqrt{b^2 - 4cd})) - 2(4ac^3 - c^2(-b + \sqrt{b^2 - 4cd}))(b + 2\sqrt{b^2 - 4cd})))\operatorname{ArcTanh}[( -4ac - b(-b + \sqrt{b^2 - 4cd}) - (2bc + 2c(-b + \sqrt{b^2 - 4cd}))x)/(4c\sqrt{a - d}\sqrt{a + bx + cx^2})])/((4ac^2 + 2bc(-b + \sqrt{b^2 - 4cd}) + c(-b + \sqrt{b^2 - 4cd})^2)(16ac^2 + 8bc(-b + \sqrt{b^2 - 4cd}) + 4c(-b + \sqrt{b^2 - 4cd})^2))))/( (b^2 - 4cd)^{3/2}(4ac^2 + 2bc(-b + \sqrt{b^2 - 4cd}) + c(-b + \sqrt{b^2 - 4cd})^2)\sqrt{a + x(b + cx)}) + (4c^3\sqrt{a + bx + cx^2}(((-2c^2(b - 2\sqrt{b^2 - 4cd}) + 2c^2(b + \sqrt{b^2 - 4cd}))\sqrt{a + bx + cx^2})/((4ac^2 - 2bc(b + \sqrt{b^2 - 4cd}) + c(b + \sqrt{b^2 - 4cd})^2)(b + \sqrt{b^2 - 4cd} + 2cx)) + (4c\sqrt{a - d}(b(2c^2(b - 2\sqrt{b^2 - 4cd}) + 2c^2(b + \sqrt{b^2 - 4cd})) - 2(4ac^3 + c^2(b - 2\sqrt{b^2 - 4cd}))(b + \sqrt{b^2 - 4cd})))\operatorname{ArcTanh}[(4ac - b(b + \sqrt{b^2 - 4cd}) - (-2bc + 2c(b + \sqrt{b^2 - 4cd}))x)/(4c\sqrt{a - d}\sqrt{a + bx + cx^2})])/((4ac^2 - 2bc(b + \sqrt{b^2 - 4cd}) + c(b + \sqrt{b^2 - 4cd})^2)(16ac^2 - 8bc(b + \sqrt{b^2 - 4cd}) + 4c(b + \sqrt{b^2 - 4cd})^2))))/( (b^2 - 4cd)^{3/2}(4ac^2 - 2bc(b + \sqrt{b^2 - 4cd}) + c(b + \sqrt{b^2 - 4cd})^2)\sqrt{a + x(b + cx)})
\end{aligned}$$

**IntegrateAlgebraic [F]** time = 180.01, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)^3),x]

[Out] \$Aborted

**fricas [B]** time = 4.82, size = 3818, normalized size = 17.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)^3/(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] [1/16\*((128\*c^2\*d^4 + (3\*b^4\*c^2 + 8\*a\*b^2\*c^3 + 48\*a^2\*c^4 + 128\*c^4\*d^2 - 32\*(b^2\*c^3 + 4\*a\*c^4)\*d)\*x^4 - 32\*(b^2\*c + 4\*a\*c^2)\*d^3 + 2\*(3\*b^5\*c + 8\*a\*b^3\*c^2 + 48\*a^2\*b\*c^3 + 128\*b\*c^3\*d^2 - 32\*(b^3\*c^2 + 4\*a\*b\*c^3)\*d)\*x^3 + (3\*b^4 + 8\*a\*b^2\*c + 48\*a^2\*c^2)\*d^2 + (3\*b^6 + 8\*a\*b^4\*c + 48\*a^2\*b^2\*c^2 + 256\*c^3\*d^3 + 64\*(b^2\*c^2 - 4\*a\*c^3)\*d^2 - 2\*(13\*b^4\*c + 56\*a\*b^2\*c^2 - 48\*a^2\*c^3)\*d)\*x^2 + 2\*(128\*b\*c^2\*d^3 - 32\*(b^3\*c + 4\*a\*b\*c^2)\*d^2 + (3\*b^5 + 8\*a\*b^3\*c + 48\*a^2\*b\*c^2)\*d)\*x)\*sqrt(a\*b^2 + 4\*c\*d^2 - (b^2 + 4\*a\*c)\*d)

```

*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 - 32*(
b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*
c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)
*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 -
2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2
+ 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4
*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c
*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d +
2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3
*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2
+ d^2)) - 4*(2*a^2*b^5 + 128*b*c^2*d^4 - 52*(b^3*c + 4*a*b*c^2)*d^3 - 6*(a
*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 + 12*(b^2*c^3 + 4*a*c^4)*d^2 - (b^4*c
^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3 + 5*(b^5 + 16*a*b^3*c + 16*a^2*b*c^2
)*d^2 - 9*(a*b^5*c + 4*a^2*b^3*c^2 - 32*b*c^3*d^3 + 12*(b^3*c^2 + 4*a*b*c^3
)*d^2 - (b^5*c + 16*a*b^3*c^2 + 16*a^2*b*c^3)*d)*x^2 - 7*(a*b^5 + 4*a^2*b^3
*c)*d - (3*a*b^6 + 8*a^2*b^4*c - 256*c^3*d^4 + 8*(b^2*c^2 + 52*a*c^3)*d^3 +
2*(13*b^4*c - 8*a*b^2*c^2 - 80*a^2*c^3)*d^2 - (3*b^6 + 34*a*b^4*c - 8*a^2*
b^2*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^3*b^6*d^2 + 64*c^3*d^8 - 48*(b^2*c
^2 + 4*a*c^3)*d^7 + 12*(b^4*c + 12*a*b^2*c^2 + 16*a^2*c^3)*d^6 - (b^6 + 36*
a*b^4*c + 144*a^2*b^2*c^2 + 64*a^3*c^3)*d^5 + 3*(a*b^6 + 12*a^2*b^4*c + 16*
a^3*b^2*c^2)*d^4 + (a^3*b^6*c^2 + 64*c^5*d^6 - 48*(b^2*c^4 + 4*a*c^5)*d^5 +
12*(b^4*c^3 + 12*a*b^2*c^4 + 16*a^2*c^5)*d^4 - (b^6*c^2 + 36*a*b^4*c^3 + 1
44*a^2*b^2*c^4 + 64*a^3*c^5)*d^3 + 3*(a*b^6*c^2 + 12*a^2*b^4*c^3 + 16*a^3*b
^2*c^4)*d^2 - 3*(a^2*b^6*c^2 + 4*a^3*b^4*c^3)*d)*x^4 - 3*(a^2*b^6 + 4*a^3*b
^4*c)*d^3 + 2*(a^3*b^7*c + 64*b*c^4*d^6 - 48*(b^3*c^3 + 4*a*b*c^4)*d^5 + 12
*(b^5*c^2 + 12*a*b^3*c^3 + 16*a^2*b*c^4)*d^4 - (b^7*c + 36*a*b^5*c^2 + 144*
a^2*b^3*c^3 + 64*a^3*b*c^4)*d^3 + 3*(a*b^7*c + 12*a^2*b^5*c^2 + 16*a^3*b^3*
c^3)*d^2 - 3*(a^2*b^7*c + 4*a^3*b^5*c^2)*d)*x^3 + (a^3*b^8 + 128*c^4*d^7 -
32*(b^2*c^3 + 12*a*c^4)*d^6 - 24*(b^4*c^2 - 4*a*b^2*c^3 - 16*a^2*c^4)*d^5 +
2*(5*b^6*c + 36*a*b^4*c^2 - 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^4 - (b^8 + 30*a
*b^6*c + 72*a^2*b^4*c^2 - 32*a^3*b^2*c^3)*d^3 + 3*(a*b^8 + 10*a^2*b^6*c + 8
*a^3*b^4*c^2)*d^2 - (3*a^2*b^8 + 10*a^3*b^6*c)*d)*x^2 + 2*(a^3*b^7*d + 64*b
*c^3*d^7 - 48*(b^3*c^2 + 4*a*b*c^3)*d^6 + 12*(b^5*c + 12*a*b^3*c^2 + 16*a^2
*b*c^3)*d^5 - (b^7 + 36*a*b^5*c + 144*a^2*b^3*c^2 + 64*a^3*b*c^3)*d^4 + 3*(
a*b^7 + 12*a^2*b^5*c + 16*a^3*b^3*c^2)*d^3 - 3*(a^2*b^7 + 4*a^3*b^5*c)*d^2)
*x), -1/8*((128*c^2*d^4 + (3*b^4*c^2 + 8*a*b^2*c^3 + 48*a^2*c^4 + 128*c^4*d
^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 - 32*(b^2*c + 4*a*c^2)*d^3 + 2*(3*b^5*c
+ 8*a*b^3*c^2 + 48*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*
x^3 + (3*b^4 + 8*a*b^2*c + 48*a^2*c^2)*d^2 + (3*b^6 + 8*a*b^4*c + 48*a^2*b^
2*c^2 + 256*c^3*d^3 + 64*(b^2*c^2 - 4*a*c^3)*d^2 - 2*(13*b^4*c + 56*a*b^2*c
^2 - 48*a^2*c^3)*d)*x^2 + 2*(128*b*c^2*d^3 - 32*(b^3*c + 4*a*b*c^2)*d^2 + (
3*b^5 + 8*a*b^3*c + 48*a^2*b*c^2)*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*
c)*d)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c
)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)
)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2 + 4*c^3*d^2 -
(b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3*c + 4*a*b*c^2)
*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(b^2*c + 2*a*c^2)
)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(2*a^2*b^5 + 128*b*c^2*d^4
- 52*(b^3*c + 4*a*b*c^2)*d^3 - 6*(a*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 +
12*(b^2*c^3 + 4*a*c^4)*d^2 - (b^4*c^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3
+ 5*(b^5 + 16*a*b^3*c + 16*a^2*b*c^2)*d^2 - 9*(a*b^5*c + 4*a^2*b^3*c^2 - 32
*b*c^3*d^3 + 12*(b^3*c^2 + 4*a*b*c^3)*d^2 - (b^5*c + 16*a*b^3*c^2 + 16*a^2*
b*c^3)*d)*x^2 - 7*(a*b^5 + 4*a^2*b^3*c)*d - (3*a*b^6 + 8*a^2*b^4*c - 256*c^
3*d^4 + 8*(b^2*c^2 + 52*a*c^3)*d^3 + 2*(13*b^4*c - 8*a*b^2*c^2 - 80*a^2*c^3
)*d^2 - (3*b^6 + 34*a*b^4*c - 8*a^2*b^2*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(
a^3*b^6*d^2 + 64*c^3*d^8 - 48*(b^2*c^2 + 4*a*c^3)*d^7 + 12*(b^4*c + 12*a*b^
2*c^2 + 16*a^2*c^3)*d^6 - (b^6 + 36*a*b^4*c + 144*a^2*b^2*c^2 + 64*a^3*c^3)
*d^5 + 3*(a*b^6 + 12*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (a^3*b^6*c^2 + 64*c^
5*d^6 - 48*(b^2*c^4 + 4*a*c^5)*d^5 + 12*(b^4*c^3 + 12*a*b^2*c^4 + 16*a^2*c^

```

$$\begin{aligned}
& 5)d^4 - (b^6c^2 + 36ab^4c^3 + 144a^2b^2c^4 + 64a^3c^5)d^3 + 3*(a \\
& *b^6c^2 + 12a^2b^4c^3 + 16a^3b^2c^4)d^2 - 3*(a^2b^6c^2 + 4a^3b^ \\
& 4c^3)*d)*x^4 - 3*(a^2b^6 + 4a^3b^4c)*d^3 + 2*(a^3b^7c + 64b^6c^4d^6 \\
& - 48*(b^3c^3 + 4a*b^4c^4)*d^5 + 12*(b^5c^2 + 12a*b^3c^3 + 16a^2b^2c^4 \\
& )d^4 - (b^7c + 36a*b^5c^2 + 144a^2b^3c^3 + 64a^3b^2c^4)*d^3 + 3*(a \\
& b^7c + 12a^2b^5c^2 + 16a^3b^3c^3)*d^2 - 3*(a^2b^7c + 4a^3b^5c^2 \\
& )d)*x^3 + (a^3b^8 + 128c^4d^7 - 32*(b^2c^3 + 12a*c^4)*d^6 - 24*(b^4c \\
& ^2 - 4a*b^2c^3 - 16a^2c^4)*d^5 + 2*(5b^6c + 36a*b^4c^2 - 48a^2b^2 \\
& *c^3 - 64a^3c^4)*d^4 - (b^8 + 30a*b^6c + 72a^2b^4c^2 - 32a^3b^2c^ \\
& 3)*d^3 + 3*(a*b^8 + 10a^2b^6c + 8a^3b^4c^2)*d^2 - (3a^2b^8 + 10a^3 \\
& *b^6c)*d)*x^2 + 2*(a^3b^7d + 64b^6c^3d^7 - 48*(b^3c^2 + 4a*b^4c^3)*d^6 \\
& + 12*(b^5c + 12a*b^3c^2 + 16a^2b^2c^3)*d^5 - (b^7 + 36a*b^5c + 144a \\
& ^2b^3c^2 + 64a^3b^2c^3)*d^4 + 3*(a*b^7 + 12a^2b^5c + 16a^3b^3c^2)* \\
& d^3 - 3*(a^2b^7 + 4a^3b^5c)*d^2)*x]
\end{aligned}$$

**giac [B]** time = 4.87, size = 2986, normalized size = 13.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)^3/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out] 
$$\begin{aligned}
& -1/8*((3b^4 + 8a*b^2c + 48a^2c^2 - 32b^2cd - 128ac^2d + 128c^2 \\
& d^2)*\log(\text{abs}(-(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*b^2c - 4*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + b*x + a))^2*a*c^2 + 8*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*c^ \\
& 2*d - (\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*b^3*\text{sqrt}(c) - 4*(\text{sqrt}(c)*x - \text{sqrt} \\
& (c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*b*c^ \\
& (3/2)*d - 3*a*b^2*c + 4*\text{sqrt}(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2))*(\text{sqrt}(c)*x \\
& - \text{sqrt}(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d + 4*\text{sqrt}(a*b^2 - \\
& b^2*d - 4*a*c*d + 4*c*d^2))*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*b*c + \text{sqrt}( \\
& a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2))*b^2*\text{sqrt}(c)))/\text{sqrt}(a*b^2 - b^2*d - 4*a*c \\
& *d + 4*c*d^2) - (3b^4 + 8a*b^2c + 48a^2c^2 - 32b^2cd - 128ac^2d \\
& + 128c^2d^2)*\log(\text{abs}(-(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*b^2c - 4*(\text{sq} \\
& \text{rt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*a*c^2 + 8*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x \\
& + a))^2*c^2*d - (\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*b^3*\text{sqrt}(c) - 4*(\text{sqrt}(c \\
& )*x - \text{sqrt}(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x \\
& + a))*b*c^(3/2)*d - 3*a*b^2*c - 4*\text{sqrt}(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2))* \\
& (\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d - 4*\text{sq} \\
& \text{rt}(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2))*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*b \\
& *c - \text{sqrt}(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2))*b^2*\text{sqrt}(c)))/\text{sqrt}(a*b^2 - b^2 \\
& *d - 4*a*c*d + 4*c*d^2))/(a^2*b^4 - 2a*b^4*d - 8a^2*b^2*c*d + b^4*d^2 + 1 \\
& 6a*b^2*c*d^2 + 16a^2*c^2*d^2 - 8b^2*c*d^3 - 32a*c^2*d^3 + 16c^2*d^4) - \\
& 1/4*(3*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^6*b^4c^(3/2) + 8*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + b*x + a))^6*a*b^2c^(5/2) + 48*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + \\
& a))^6*a^2c^(7/2) - 32*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^6*b^2c^(5/2)*d \\
& - 128*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^6*a*c^(7/2)*d + 128*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + b*x + a))^6*c^(7/2)*d^2 + 9*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a \\
& ))^5*b^5c + 24*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^5*a*b^3c^2 + 144*(\text{sqrt} \\
& (c)*x - \text{sqrt}(c*x^2 + b*x + a))^5*a^2b^2c^3 - 96*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b \\
& *x + a))^5*b^3c^2*d - 384*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^5*a*b^2c^3*d \\
& + 384*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^5*b^2c^3*d^2 + 9*(\text{sqrt}(c)*x - \text{sqrt} \\
& (c*x^2 + b*x + a))^4*b^6*\text{sqrt}(c) + 15*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4 \\
& *a*b^4c^(3/2) + 120*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a^2b^2c^(5/2) \\
& - 144*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a^3c^(7/2) - 78*(\text{sqrt}(c)*x - s \\
& \text{qrt}(c*x^2 + b*x + a))^4*b^4c^(3/2)*d - 240*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + \\
& a))^4*a*b^2c^(5/2)*d + 672*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a^2c^(7 \\
& /2)*d + 192*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*b^2c^(5/2)*d^2 - 1152*(s \\
& \text{qrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a*c^(7/2)*d^2 + 768*(\text{sqrt}(c)*x - \text{sqrt}(c \\
& *x^2 + b*x + a))^4*c^(7/2)*d^3 + 3*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*b^ \\
& 7 - 10*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a*b^5c - 288*(\text{sqrt}(c)*x - \text{sq}
\end{aligned}$$

$$\begin{aligned} & t(c*x^2 + b*x + a))^{3*a^3*b*c^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^{3*b} \\ & ^5*c*d + 160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^{3*a*b^3*c^2*d + 1344*(sqrt \\ & (c)*x - sqrt(c*x^2 + b*x + a))^{3*a^2*b^3*c^2*d} - 256*(sqrt(c)*x - sqrt(c*x^2 \\ & + b*x + a))^{3*b^3*c^2*d^2} - 2304*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^{3*a*b^3 \\ & c^3*d^2 + 1536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^{3*b*c^3*d^3} - 14*(sqrt(c) \\ & )*x - sqrt(c*x^2 + b*x + a))^{2*a*b^6*sqrt(c)} - 71*(sqrt(c)*x - sqrt(c*x^2 + \\ & b*x + a))^{2*a^2*b^4*c^{(3/2)} - 200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^{2*a^ \\ & 3*b^2*c^{(5/2)} + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^{2*a^4*c^{(7/2)} + 23* \\ & (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^{2*b^6*sqrt(c)*d} + 280*(sqrt(c)*x - sqrt \\ & (c*x^2 + b*x + a))^{2*a*b^4*c^{(3/2)}*d} + 1168*(sqrt(c)*x - sqrt(c*x^2 + b*x + \\ & a))^{2*a^2*b^2*c^{(5/2)}*d} - 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^{2*a^3*c^ \\ & (7/2)*d} - 272*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^{2*b^4*c^{(3/2)}*d^2} - 2048* \\ & (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^{2*a*b^2*c^{(5/2)}*d^2} + 640*(sqrt(c)*x - \\ & sqrt(c*x^2 + b*x + a))^{2*a^2*c^{(7/2)}*d^2} + 1152*(sqrt(c)*x - sqrt(c*x^2 + b \\ & *x + a))^{2*b^2*c^{(5/2)}*d^3} - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^7 - \\ & 47*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^5*c - 56*(sqrt(c)*x - sqrt(c*x \\ & ^2 + b*x + a))*a^3*b^3*c^2 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b* \\ & c^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^7*d + 136*(sqrt(c)*x - sqrt(c \\ & *x^2 + b*x + a))*a*b^5*c*d + 496*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^ \\ & 3*c^2*d - 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c^3*d - 80*(sqrt(c) \\ & *x - sqrt(c*x^2 + b*x + a))*b^5*c*d^2 - 896*(sqrt(c)*x - sqrt(c*x^2 + b*x + \\ & a))*a*b^3*c^2*d^2 + 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^3*d^2 \\ & + 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^2*d^3 - 11*a^2*b^6*sqrt(c) \\ & - 11*a^3*b^4*c^{(3/2)} + 72*a^4*b^2*c^{(5/2)} - 48*a^5*c^{(7/2)} + 17*a*b^6*sqrt(c) \\ & *d + 118*a^2*b^4*c^{(3/2)}*d - 256*a^3*b^2*c^{(5/2)}*d + 96*a^4*c^{(7/2)}*d - 6 \\ & *b^6*sqrt(c)*d^2 - 152*a*b^4*c^{(3/2)}*d^2 + 160*a^2*b^2*c^{(5/2)}*d^2 + 48*b^4 \\ & *c^{(3/2)}*d^3)/((a^2*b^4 - 2*a*b^4*d - 8*a^2*b^2*c*d + b^4*d^2 + 16*a*b^2*c* \\ & d^2 + 16*a^2*c^2*d^2 - 8*b^2*c*d^3 - 32*a*c^2*d^3 + 16*c^2*d^4))*((sqrt(c)*x \\ & - sqrt(c*x^2 + b*x + a))^{4*c} + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^{3*b*s \\ & qrt(c)} + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^{2*b^2} - 2*(sqrt(c)*x - sqrt(c* \\ & x^2 + b*x + a))^{2*a*c} + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^{2*c*d} - 2*(sq \\ & rt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*sqrt(c) + 4*(sqrt(c)*x - sqrt(c*x^2 + \\ & b*x + a))*b*sqrt(c)*d - a*b^2 + a^2*c + b^2*d)^2) \end{aligned}$$

**maple [B]** time = 0.03, size = 1884, normalized size = 8.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x^2+b\*x+d)^3/(c\*x^2+b\*x+a)^(1/2), x)

$$\begin{aligned} \text{[Out]} & -1/2/(b^2-4*c*d)^{(3/2)}/(a-d)/(x+1/2*b/c-1/2*(b^2-4*c*d)^{(1/2)}/c)^2*(a+(x-1/ \\ & 2*(-b+(b^2-4*c*d)^{(1/2)}/c))^2*c-d+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^ \\ & (1/2))/c))^{(1/2)}+3/4/(b^2-4*c*d)/(a-d)^2/(x+1/2*b/c-1/2*(b^2-4*c*d)^{(1/2)}/c \\ & )*(a+(x-1/2*(-b+(b^2-4*c*d)^{(1/2)}/c))^2*c-d+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b \\ & ^2-4*c*d)^{(1/2)}/c))^{(1/2)}-3/8/(b^2-4*c*d)^{(3/2)}/(a-d)^{(5/2)}*ln((2*a-2*d+(b \\ & ^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)}/c))+2*(a-d)^{(1/2)}*(a+(x-1/2*(- \\ & b+(b^2-4*c*d)^{(1/2)}/c))^2*c-d+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2) \\ & ))/c))^{(1/2)})/(x-1/2*(-b+(b^2-4*c*d)^{(1/2)}/c))*b^2+3/2/(b^2-4*c*d)^{(3/2)}/( \\ & a-d)^{(5/2)}*ln((2*a-2*d+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)}/c))+2 \\ & *(a-d)^{(1/2)}*(a+(x-1/2*(-b+(b^2-4*c*d)^{(1/2)}/c))^2*c-d+(b^2-4*c*d)^{(1/2)}*(x \\ & -1/2*(-b+(b^2-4*c*d)^{(1/2)}/c))^{(1/2)})/(x-1/2*(-b+(b^2-4*c*d)^{(1/2)}/c))*c* \\ & d-1/(b^2-4*c*d)^{(3/2)}*c/(a-d)^{(3/2)}*ln((2*a-2*d+(b^2-4*c*d)^{(1/2)}*(x-1/2*(- \\ & b+(b^2-4*c*d)^{(1/2)}/c))+2*(a-d)^{(1/2)}*(a+(x-1/2*(-b+(b^2-4*c*d)^{(1/2)}/c))^2 \\ & *c-d+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)}/c))^{(1/2)})/(x-1/2*(-b+ \\ & (b^2-4*c*d)^{(1/2)}/c))+1/2/(b^2-4*c*d)^{(3/2)}/(a-d)/(x+1/2*b/c+1/2*(b^2-4*c* \\ & d)^{(1/2)}/c)^2*(a+(x+1/2*(b+(b^2-4*c*d)^{(1/2)}/c))^2*c-d-(b^2-4*c*d)^{(1/2)}*(x \\ & +1/2*(b+(b^2-4*c*d)^{(1/2)}/c))^{(1/2)}+3/4/(b^2-4*c*d)/(a-d)^2/(x+1/2*b/c+1/2 \\ & *(b^2-4*c*d)^{(1/2)}/c)*(a+(x+1/2*(b+(b^2-4*c*d)^{(1/2)}/c))^2*c-d-(b^2-4*c*d)^ \\ & (1/2)*(x+1/2*(b+(b^2-4*c*d)^{(1/2)}/c))^{(1/2)}+3/8/(b^2-4*c*d)^{(3/2)}/(a-d)^{(5} \end{aligned}$$

$$\begin{aligned} & /2) * \ln((2*a-2*d-(b^2-4*c*d)^{(1/2)} * (x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c) + 2*(a-d)^{(1/2)} * (a+(x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c))^2 * c-d-(b^2-4*c*d)^{(1/2)} * (x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c))^{(1/2)}) / (x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c) * b^2-3/2/(b^2-4*c*d)^{(3/2)} / (a-d)^{(5/2)} * \ln((2*a-2*d-(b^2-4*c*d)^{(1/2)} * (x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c) + 2*(a-d)^{(1/2)} * (a+(x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c))^2 * c-d-(b^2-4*c*d)^{(1/2)} * (x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c))^{(1/2)}) / (x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c) * c*d + 1/(b^2-4*c*d)^{(3/2)} * c / (a-d)^{(3/2)} * \ln((2*a-2*d-(b^2-4*c*d)^{(1/2)} * (x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c) + 2*(a-d)^{(1/2)} * (a+(x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c))^2 * c-d-(b^2-4*c*d)^{(1/2)} * (x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c))^{(1/2)}) / (x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c) + 6*c^2 / (b^2-4*c*d)^{(5/2)} / (a-d)^{(1/2)} * \ln((2*a-2*d-(b^2-4*c*d)^{(1/2)} * (x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c) + 2*(a-d)^{(1/2)} * (a+(x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c))^2 * c-d-(b^2-4*c*d)^{(1/2)} * (x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c))^{(1/2)}) / (x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c) - 6*c^2 / (b^2-4*c*d)^{(5/2)} / (a-d)^{(1/2)} * \ln((2*a-2*d+(b^2-4*c*d)^{(1/2)} * (x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c) + 2*(a-d)^{(1/2)} * (a+(x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c))^2 * c-d+(b^2-4*c*d)^{(1/2)} * (x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c))^{(1/2)}) / (x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c) + 3/(b^2-4*c*d)^2 * c / (a-d) / (x+1/2*b/c-1/2*(b^2-4*c*d)^{(1/2)}/c) * (a+(x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c))^2 * c-d+(b^2-4*c*d)^{(1/2)} * (x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c))^{(1/2)} + 3/(b^2-4*c*d)^2 * c / (a-d) / (x+1/2*b/c+1/2*(b^2-4*c*d)^{(1/2)}/c) * (a+(x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c))^2 * c-d-(b^2-4*c*d)^{(1/2)} * (x+1/2*(b+(b^2-4*c*d)^{(1/2)))/c))^{(1/2)} \end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (cx^2 + bx + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)^3/(c\*x^2+b\*x+a)^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(c\*x^2 + b\*x + d)^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (cx^2 + bx + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + b\*x + c\*x^2)^3), x)

[Out] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + b\*x + c\*x^2)^3), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x\*\*2+b\*x+d)\*\*3/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] Timed out

$$3.6 \quad \int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)^4} dx$$

**Optimal.** Leaf size=328

$$\frac{(b+2cx)(16c^2(15a^2-44ad+44d^2)+8b^2c(7a-22d)+15b^4)\sqrt{a+bx+cx^2}}{24(a-d)^3(b^2-4cd)^3(bx+cx^2+d)} + \frac{(4c(a-2d)+b^2)(16c^2(5a^2-8ad+4d^2)+8b^2c(7a-22d)+15b^4)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(bx+cx^2+d)^3}$$

**Rubi [A]** time = 0.97, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, number of rules / integrand size = 0.185, Rules used = {974, 1060, 12, 982, 208}

$$\frac{(b+2cx)(16c^2(15a^2-44ad+44d^2)+8b^2c(7a-22d)+15b^4)\sqrt{a+bx+cx^2}}{24(a-d)^3(b^2-4cd)^3(bx+cx^2+d)} + \frac{(4c(a-2d)+b^2)(16c^2(5a^2-8ad+4d^2)+8b^2c(7a-22d)+15b^4)\sqrt{a+bx+cx^2}}{8(a-d)^2(b^2-4cd)^{7/2}} + \frac{5(b+2cx)(4c(a-2d)+b^2)\sqrt{a+bx+cx^2}}{12(a-d)(b^2-4cd)^2(bx+cx^2+d)^2} - \frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(bx+cx^2+d)^3}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)^4), x]

[Out] -((b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(3\*(a - d)\*(b^2 - 4\*c\*d)\*(d + b\*x + c\*x^2)^3) + (5\*(b^2 + 4\*c\*(a - 2\*d))\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(12\*(a - d)^2\*(b^2 - 4\*c\*d)^2\*(d + b\*x + c\*x^2)^2) - ((15\*b^4 + 8\*b^2\*c\*(7\*a - 2\*d) + 16\*c^2\*(15\*a^2 - 44\*a\*d + 44\*d^2))\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(24\*(a - d)^3\*(b^2 - 4\*c\*d)^3\*(d + b\*x + c\*x^2)) + ((b^2 + 4\*c\*(a - 2\*d))\*(5\*b^4 - 8\*b^2\*c\*(a + 4\*d) + 16\*c^2\*(5\*a^2 - 8\*a\*d + 8\*d^2))\*ArcTanh[(Sqrt[a - d]\*(b + 2\*c\*x))/(Sqrt[b^2 - 4\*c\*d]\*Sqrt[a + b\*x + c\*x^2])])/(8\*(a - d)^(7/2)\*(b^2 - 4\*c\*d)^(7/2))

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

### Rule 982

Int[1/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2\*e, Subst[Int[1/(e\*(b\*e - 4\*a\*f) - (b\*d - a\*e)



$x^2$ ), x], x, (e + 2\*f\*x)/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && EqQ[c\*e - b\*f, 0]

### Rule 1060

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((A\_) + (B\_)\*(x\_) + (C\_)\*(x\_)^2)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1)\*((A\*c - a\*C)\*(2\*a\*c\*e - b\*(c\*d + a\*f)) + (A\*b - a\*B)\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)) + c\*(A\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)) - B\*(b\*c\*d - 2\*a\*c\*e + a\*b\*f) + C\*(b^2\*d - a\*b\*e - 2\*a\*(c\*d - a\*f)))\*x)/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] + Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[(b\*B - 2\*A\*c - 2\*a\*C)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) + (b^2\*(C\*d + A\*f) - b\*(B\*c\*d + A\*c\*e + a\*C\*e + a\*B\*f) + 2\*(A\*c\*(c\*d - a\*f) - a\*(c\*C\*d - B\*c\*e - a\*C\*f)))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*((A\*c - a\*C)\*(2\*a\*c\*e - b\*(c\*d + a\*f)) + (A\*b - a\*B)\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)))\*(p + q + 2) - (2\*f\*((A\*c - a\*C)\*(2\*a\*c\*e - b\*(c\*d + a\*f)) + (A\*b - a\*B)\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)))\*(p + q + 2) - (b^2\*(C\*d + A\*f) - b\*(B\*c\*d + A\*c\*e + a\*C\*e + a\*B\*f) + 2\*(A\*c\*(c\*d - a\*f) - a\*(c\*C\*d - B\*c\*e - a\*C\*f)))\* (b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x - c\*f\*(b^2\*(C\*d + A\*f) - b\*(B\*c\*d + A\*c\*e + a\*C\*e + a\*B\*f) + 2\*(A\*c\*(c\*d - a\*f) - a\*(c\*C\*d - B\*c\*e - a\*C\*f)))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

### Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)^4} dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{\int \frac{-\frac{1}{2}c^2(a-d)(5b^2+20ac-24cd)-8bc^3(a-d)}{\sqrt{a+bx+cx^2}(d+bx+cx^2)} dx}{3c^2(a-d)^2(b^2-4cd)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)}$$

**Mathematica [B]** time = 6.61, size = 3382, normalized size = 10.31

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)^4),x]

[Out] 
$$\begin{aligned} & (-8c^3(a + bx + cx^2))/(3(a - d)(b^2 - 4cd)^2(b - \sqrt{b^2 - 4cd} \\ & + 2cx)^3\sqrt{a + x(b + cx)}) + (8c^3(a + bx + cx^2))/((a - d)(b \\ & ^2 - 4cd)^{5/2}(b - \sqrt{b^2 - 4cd} + 2cx)^2\sqrt{a + x(b + cx)}) \\ & - (20c^3(a + bx + cx^2))/((a - d)(b^2 - 4cd)^3(b - \sqrt{b^2 - 4cd} \\ & + 2cx)\sqrt{a + x(b + cx)}) - (8c^3(a + bx + cx^2))/(3(a - d)(b \\ & ^2 - 4cd)^2(b + \sqrt{b^2 - 4cd} + 2cx)^3\sqrt{a + x(b + cx)}) - (8 \\ & c^3(a + bx + cx^2))/((a - d)(b^2 - 4cd)^{5/2}(b + \sqrt{b^2 - 4cd} \\ & + 2cx)^2\sqrt{a + x(b + cx)}) - (20c^3(a + bx + cx^2))/((a - d)(b \\ & ^2 - 4cd)^3(b + \sqrt{b^2 - 4cd} + 2cx)\sqrt{a + x(b + cx)}) - (20c \\ & ^3\sqrt{a + bx + cx^2}\text{ArcTanh}[(b^2 - 4ac - b\sqrt{b^2 - 4cd} - 2c\sqrt{b^2 - 4cd}x)/(4c\sqrt{a - d}\sqrt{a + bx + cx^2}))/(\sqrt{a - d} \\ & *(b^2 - 4cd)^{7/2}\sqrt{a + x(b + cx)}) - (5c^2\sqrt{a + bx + cx^2} \\ & \text{ArcTanh}[(b^2 - 4ac - b\sqrt{b^2 - 4cd} - 2c\sqrt{b^2 - 4cd}x)/(4c\sqrt{a - d}\sqrt{a + bx + cx^2}))/((a - d)^{3/2}(b^2 - 4cd)^{5/2}\sqrt{a + x(b + cx)}) - (20c^3\sqrt{a + bx + cx^2}\text{ArcTanh}[(4ac - b(b + \sqrt{b^2 - 4cd}) - 2c\sqrt{b^2 - 4cd}x)/(4c\sqrt{a - d}\sqrt{a + bx + cx^2}))/(\sqrt{a - d}(b^2 - 4cd)^{7/2}\sqrt{a + x(b + cx)}) - (5c^2\sqrt{a + bx + cx^2}\text{ArcTanh}[(4ac - b(b + \sqrt{b^2 - 4cd}) - 2c\sqrt{b^2 - 4cd}x)/(4c\sqrt{a - d}\sqrt{a + bx + cx^2}))/((a - d)^{3/2}(b^2 - 4cd)^{5/2}\sqrt{a + x(b + cx)}) - (16c^4\sqrt{a + bx + cx^2}*((-2c^2(-b + \sqrt{b^2 - 4cd}) - 2c^2(b + 2\sqrt{b^2 - 4cd}))\sqrt{a + bx + cx^2}))/((4ac^2 + 2b*c*(-b + \sqrt{b^2 - 4cd}) + c*(-b + \sqrt{b^2 - 4cd})^2)*(-b + \sqrt{b^2 - 4cd} - 2cx)) + (4c\sqrt{a - d}*(b*(-2c^2(-b + \sqrt{b^2 - 4cd}) + 2c^2(b + 2\sqrt{b^2 - 4cd}))) - 2*(4ac^3 - c^2(-b + \sqrt{b^2 - 4cd}))(b + 2\sqrt{b^2 - 4cd})))\text{ArcTanh}((-4ac - b(-b + \sqrt{b^2 - 4cd}) - (2bc + 2c(-b + \sqrt{b^2 - 4cd}))x)/(4c\sqrt{a - d}\sqrt{a + bx + cx^2}))/((4ac^2 + 2b*c*(-b + \sqrt{b^2 - 4cd}) + c*(-b + \sqrt{b^2 - 4cd})^2)*(16ac^2 + 8b*c*(-b + \sqrt{b^2 - 4cd}) + 4c*(-b + \sqrt{b^2 - 4cd})^2)))/((b^2 - 4cd)^{5/2}*(4ac^2 + 2b*c*(-b + \sqrt{b^2 - 4cd}) + c*(-b + \sqrt{b^2 - 4cd})^2)\sqrt{a + x(b + cx)}) - (16c^4\sqrt{a + bx + cx^2}*(-1/2*((4c^2(-b + \sqrt{b^2 - 4cd}) + 2c^2(2b + 3\sqrt{b^2 - 4cd}))\sqrt{a + bx + cx^2}))/((4ac^2 + 2b*c*(-b + \sqrt{b^2 - 4cd}) + c*(-b + \sqrt{b^2 - 4cd})^2)*(-b + \sqrt{b^2 - 4cd} - 2cx)^2 - (((10c^3\sqrt{b^2 - 4cd}*(-b + \sqrt{b^2 - 4cd}) + 2c^3(10b^2 - 16ac - 24cd + 5b\sqrt{b^2 - 4cd}))\sqrt{a + bx + cx^2}))/((4ac^2 + 2b*c*(-b + \sqrt{b^2 - 4cd}) + c*(-b + \sqrt{b^2 - 4cd})^2)*(-b + \sqrt{b^2 - 4cd} - 2cx)) + (4c\sqrt{a - d}*(b*(10c^3\sqrt{b^2 - 4cd}*(-b + \sqrt{b^2 - 4cd}) - 2c^3(10b^2 - 16ac - 24cd + 5b\sqrt{b^2 - 4cd}))) - 2*(-20ac^4\sqrt{b^2 - 4cd} + c^3(-b + \sqrt{b^2 - 4cd})*(10b^2 - 16ac - 24cd + 5b\sqrt{b^2 - 4cd})))\text{ArcTanh}((-4ac - b(-b + \sqrt{b^2 - 4cd}) - (2bc + 2c(-b + \sqrt{b^2 - 4cd}))x)/(4c\sqrt{a - d}\sqrt{a + bx + cx^2}))/((4ac^2 + 2b*c*(-b + \sqrt{b^2 - 4cd}) + c*(-b + \sqrt{b^2 - 4cd})^2)*(16ac^2 + 8b*c*(-b + \sqrt{b^2 - 4cd}) + 4c*(-b + \sqrt{b^2 - 4cd})^2)))/((2*(4ac^2 + 2b*c*(-b + \sqrt{b^2 - 4cd}) + c*(-b + \sqrt{b^2 - 4cd})^2)))/((3*(b^2 - 4cd)^2*(4ac^2 + 2b*c*(-b + \sqrt{b^2 - 4cd}) + c*(-b + \sqrt{b^2 - 4cd})^2)\sqrt{a + x(b + cx)}) - (16c^4\sqrt{a + bx + cx^2}*((-2c^2(b - 2\sqrt{b^2 - 4cd}) + 2c^2(b + \sqrt{b^2 - 4cd}))\sqrt{a + bx + cx^2}))/((4ac^2 - 2b*c*(b + \sqrt{b^2 - 4cd}) + c*(b + \sqrt{b^2 - 4cd})^2)*(b + \sqrt{b^2 - 4cd} + 2cx)) + (4c\sqrt{a - d}*(b*(2c^2(b - 2\sqrt{b^2 - 4cd}) + 2c^2(b + \sqrt{b^2 - 4cd}))) - 2*(4ac^3 + c^2(b - 2\sqrt{b^2 - 4cd}))(b + \sqrt{b^2 - 4cd})))\text{ArcTanh}[(4ac - b(b + \sqrt{b^2 - 4cd}) - (-2bc + 2c*(b + \sqrt{b^2 - 4cd}))x)/(4c\sqrt{a - d}\sqrt{a + bx + cx^2}))/((4ac^2 - 2b*c*(b + \sqrt{b^2 - 4cd}) + c*(b + \sqrt{b^2 - 4cd})^2)*(16ac^2 - 8b*c*(b + \sqrt{b^2 - 4cd}) + 4c*(b + \sqrt{b^2 - 4cd})^2)))/((b^2 - 4cd)^{5/2}*(4ac^2 - 2b*c*(b$$

$$\begin{aligned}
& + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2*\text{Sqrt}[a + x*(b + c*x)] \\
& - (16*c^4*\text{Sqrt}[a + b*x + c*x^2]*(-1/2*((2*c^2*(2*b - 3*\text{Sqrt}[b^2 - 4*c*d]) \\
& - 4*c^2*(b + \text{Sqrt}[b^2 - 4*c*d]))*\text{Sqrt}[a + b*x + c*x^2])/((4*a*c^2 - 2*b*c*( \\
& b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*(b + \text{Sqrt}[b^2 - 4*c*d] \\
& ] + 2*c*x)^2) - (((-10*c^3*\text{Sqrt}[b^2 - 4*c*d]*(b + \text{Sqrt}[b^2 - 4*c*d]) - 2*c^ \\
& 3*(10*b^2 - 16*a*c - 24*c*d - 5*b*\text{Sqrt}[b^2 - 4*c*d]))*\text{Sqrt}[a + b*x + c*x^2] \\
& ))/(4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)* \\
& (b + \text{Sqrt}[b^2 - 4*c*d] + 2*c*x)) + (4*c*\text{Sqrt}[a - d]*(b*(-10*c^3*\text{Sqrt}[b^2 - \\
& 4*c*d]*(b + \text{Sqrt}[b^2 - 4*c*d]) + 2*c^3*(10*b^2 - 16*a*c - 24*c*d - 5*b*\text{Sqrt} \\
& [b^2 - 4*c*d])) - 2*(-20*a*c^4*\text{Sqrt}[b^2 - 4*c*d] + c^3*(b + \text{Sqrt}[b^2 - 4*c* \\
& d])*(10*b^2 - 16*a*c - 24*c*d - 5*b*\text{Sqrt}[b^2 - 4*c*d])))*\text{ArcTanh}[(4*a*c - b \\
& *(b + \text{Sqrt}[b^2 - 4*c*d]) - (-2*b*c + 2*c*(b + \text{Sqrt}[b^2 - 4*c*d]))*x)/(4*c*\text{S} \\
& \text{qrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2]))/(((4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d] \\
& ] + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*(16*a*c^2 - 8*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) \\
& + 4*c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)))/(2*(4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c \\
& *d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)))/(3*(b^2 - 4*c*d)^2*(4*a*c^2 - 2*b*c* \\
& (b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*\text{Sqrt}[a + x*(b + c*x) \\
& ])
\end{aligned}$$

**IntegrateAlgebraic [F]** time = 180.02, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(Sqrt[a + b\*x + c\*x^2]\*(d + b\*x + c\*x^2)^4),x]

[Out] \$Aborted

**fricas [B]** time = 20.45, size = 8134, normalized size = 24.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)^4/(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] [1/96\*(3\*(1024\*c^3\*d^6 - (5\*b^6\*c^3 + 12\*a\*b^4\*c^4 + 48\*a^2\*b^2\*c^5 + 320\*a^3\*c^6 - 1024\*c^6\*d^3 + 384\*(b^2\*c^5 + 4\*a\*c^6)\*d^2 - 24\*(3\*b^4\*c^4 + 8\*a\*b^2\*c^5 + 48\*a^2\*c^6)\*d)\*x^6 - 384\*(b^2\*c^2 + 4\*a\*c^3)\*d^5 - 3\*(5\*b^7\*c^2 + 12\*a\*b^5\*c^3 + 48\*a^2\*b^3\*c^4 + 320\*a^3\*b\*c^5 - 1024\*b\*c^5\*d^3 + 384\*(b^3\*c^4 + 4\*a\*b\*c^5)\*d^2 - 24\*(3\*b^5\*c^3 + 8\*a\*b^3\*c^4 + 48\*a^2\*b\*c^5)\*d)\*x^5 + 24\*(3\*b^4\*c + 8\*a\*b^2\*c^2 + 48\*a^2\*c^3)\*d^4 - 3\*(5\*b^8\*c + 12\*a\*b^6\*c^2 + 48\*a^2\*b^4\*c^3 + 320\*a^3\*b^2\*c^4 - 1024\*c^5\*d^4 - 128\*(5\*b^2\*c^4 - 12\*a\*c^5)\*d^3 + 24\*(13\*b^4\*c^3 + 56\*a\*b^2\*c^4 - 48\*a^2\*c^5)\*d^2 - (67\*b^6\*c^2 + 180\*a\*b^4\*c^3 + 1104\*a^2\*b^2\*c^4 - 320\*a^3\*c^5)\*d)\*x^4 - (5\*b^6 + 12\*a\*b^4\*c + 48\*a^2\*b^2\*c^2 + 320\*a^3\*c^3)\*d^3 - (5\*b^9 + 12\*a\*b^7\*c + 48\*a^2\*b^5\*c^2 + 320\*a^3\*b^3\*c^3 - 6144\*b\*c^4\*d^4 + 256\*(5\*b^3\*c^3 + 36\*a\*b\*c^4)\*d^3 - 48\*(b^5\*c^2 - 8\*a\*b^3\*c^3 + 144\*a^2\*b\*c^4)\*d^2 - 6\*(7\*b^7\*c + 20\*a\*b^5\*c^2 + 144\*a^2\*b^3\*c^3 - 320\*a^3\*b\*c^4)\*d)\*x^3 + 3\*(1024\*c^4\*d^5 + 128\*(5\*b^2\*c^3 - 12\*a\*c^4)\*d^4 - 24\*(13\*b^4\*c^2 + 56\*a\*b^2\*c^3 - 48\*a^2\*c^4)\*d^3 + (67\*b^6\*c + 180\*a\*b^4\*c^2 + 1104\*a^2\*b^2\*c^3 - 320\*a^3\*c^4)\*d^2 - (5\*b^8 + 12\*a\*b^6\*c + 48\*a^2\*b^4\*c^2 + 320\*a^3\*b^2\*c^3)\*d)\*x^2 + 3\*(1024\*b\*c^3\*d^5 - 384\*(b^3\*c^2 + 4\*a\*b\*c^3)\*d^4 + 24\*(3\*b^5\*c + 8\*a\*b^3\*c^2 + 48\*a^2\*b\*c^3)\*d^3 - (5\*b^7 + 12\*a\*b^5\*c + 48\*a^2\*b^3\*c^2 + 320\*a^3\*b\*c^3)\*d^2)\*x)\*sqrt(a\*b^2 + 4\*c\*d^2 - (b^2 + 4\*a\*c)\*d)\*log((8\*a^2\*b^4 + (b^4\*c^2 + 24\*a\*b^2\*c^3 + 16\*a^2\*c^4 + 128\*c^4\*d^2 - 32\*(b^2\*c^3 + 4\*a\*c^4)\*d)\*x^4 + 2\*(b^5\*c + 24\*a\*b^3\*c^2 + 16\*a^2\*b\*c^3 + 128\*b\*c^3\*d^2 - 32\*(b^3\*c^2 + 4\*a\*b\*c^3)\*d)\*x^3 + (b^4 + 24\*a\*b^2\*c + 16\*a^2\*c^2)\*d^2 + (b^6 + 32\*a\*b^4\*c + 48\*a^2\*b^2\*c^2 + 32\*(5\*b^2\*c^2 + 4\*a\*c^3)\*d^2 - 2\*(19\*b^4\*c + 104\*a\*b^2\*c^2 + 48\*a^2\*c^3)\*d)\*x^2 - 4\*(2\*a\*b^3 + 2\*(b^2\*c^2 + 4\*a\*c^3 - 8\*c^3\*d)\*x^3 + 3\*(b^3\*c + 4\*a\*b\*c^2 - 8\*b\*c^2\*d)\*x^2 - (b^3 + 4\*a\*b\*c)\*d + (b^4 + 8\*a\*b^2\*c - 2\*(5\*b^2\*c + 4\*a\*c^2)\*

$$\begin{aligned}
& d) * x) * \sqrt{a * b^2 + 4 * c * d^2 - (b^2 + 4 * a * c) * d} * \sqrt{c * x^2 + b * x + a} - 8 * (a * \\
& b^4 + 4 * a^2 * b^2 * c) * d + 2 * (4 * a * b^5 + 16 * a^2 * b^3 * c + 16 * (b^3 * c + 4 * a * b * c^2) * d \\
& ^2 - (3 * b^5 + 40 * a * b^3 * c + 48 * a^2 * b * c^2) * d) * x) / (c^2 * x^4 + 2 * b * c * x^3 + 2 * b * d \\
& * x + (b^2 + 2 * c * d) * x^2 + d^2) - 4 * (8 * a^3 * b^7 + 4608 * b * c^3 * d^6 - 2592 * (b^3 * \\
& c^2 + 4 * a * b * c^3) * d^5 + 2 * (15 * a * b^6 * c^3 + 56 * a^2 * b^4 * c^4 + 240 * a^3 * b^2 * c^5 + \\
& 2816 * c^6 * d^4 - 1408 * (b^2 * c^5 + 4 * a * c^6) * d^3 + 4 * (59 * b^4 * c^4 + 584 * a * b^2 * c^5 \\
& + 944 * a^2 * c^6) * d^2 - (15 * b^6 * c^3 + 292 * a * b^4 * c^4 + 1168 * a^2 * b^2 * c^5 + 960 \\
& * a^3 * c^6) * d) * x^5 + 4 * (123 * b^5 * c + 1352 * a * b^3 * c^2 + 1968 * a^2 * b * c^3) * d^4 + 5 * \\
& (15 * a * b^7 * c^2 + 56 * a^2 * b^5 * c^3 + 240 * a^3 * b^3 * c^4 + 2816 * b * c^5 * d^4 - 1408 * (b \\
& ^3 * c^4 + 4 * a * b * c^5) * d^3 + 4 * (59 * b^5 * c^3 + 584 * a * b^3 * c^4 + 944 * a^2 * b * c^5) * d^2 \\
& - (15 * b^7 * c^2 + 292 * a * b^5 * c^3 + 1168 * a^2 * b^3 * c^4 + 960 * a^3 * b * c^5) * d) * x^4 \\
& - (33 * b^7 + 940 * a * b^5 * c + 3760 * a^2 * b^3 * c^2 + 2112 * a^3 * b * c^3) * d^3 + 4 * (15 * a * \\
& b^8 * c + 51 * a^2 * b^6 * c^2 + 220 * a^3 * b^4 * c^3 + 3456 * c^5 * d^5 + 16 * (63 * b^2 * c^4 - \\
& 452 * a * c^5) * d^4 - 4 * (273 * b^4 * c^3 + 584 * a * b^2 * c^4 - 1264 * a^2 * c^5) * d^3 + 8 * (27 \\
& * b^6 * c^2 + 233 * a * b^4 * c^3 + 236 * a^2 * b^2 * c^4 - 160 * a^3 * c^5) * d^2 - (15 * b^8 * c + \\
& 267 * a * b^6 * c^2 + 992 * a^2 * b^4 * c^3 + 560 * a^3 * b^2 * c^4) * d) * x^3 + (59 * a * b^7 + 58 \\
& 4 * a^2 * b^5 * c + 944 * a^3 * b^3 * c^2) * d^2 + (15 * a * b^9 + 26 * a^2 * b^7 * c + 120 * a^3 * b^5 \\
& * c^2 + 20736 * b * c^4 * d^5 - 32 * (251 * b^3 * c^3 + 1356 * a * b * c^4) * d^4 + 8 * (61 * b^5 * c^2 \\
& + 1768 * a * b^3 * c^3 + 3792 * a^2 * b * c^4) * d^3 + 4 * (29 * b^7 * c - 124 * a * b^5 * c^2 - 18 \\
& 88 * a^2 * b^3 * c^3 - 1920 * a^3 * b * c^4) * d^2 - (15 * b^9 + 142 * a * b^7 * c + 112 * a^2 * b^5 * \\
& c^2 - 1440 * a^3 * b^3 * c^3) * d) * x^2 - 34 * (a^2 * b^7 + 4 * a^3 * b^5 * c) * d - 2 * (5 * a^2 * b^8 \\
& + 12 * a^3 * b^6 * c - 4608 * c^4 * d^6 - 864 * (b^2 * c^3 - 12 * a * c^4) * d^5 + 4 * (329 * b^4 \\
& * c^2 + 456 * a * b^2 * c^3 - 1968 * a^2 * c^4) * d^4 - (283 * b^6 * c + 2356 * a * b^4 * c^2 + 12 \\
& 96 * a^2 * b^2 * c^3 - 2112 * a^3 * c^4) * d^3 + (20 * b^8 + 413 * a * b^6 * c + 1304 * a^2 * b^4 * c \\
& ^2 + 336 * a^3 * b^2 * c^3) * d^2 - (25 * a * b^8 + 142 * a^2 * b^6 * c + 264 * a^3 * b^4 * c^2) * d) \\
& * x) * \sqrt{c * x^2 + b * x + a}) / (a^4 * b^8 * d^3 + 256 * c^4 * d^11 - 256 * (b^2 * c^3 + 4 * a \\
& * c^4) * d^10 + 32 * (3 * b^4 * c^2 + 32 * a * b^2 * c^3 + 48 * a^2 * c^4) * d^9 - 16 * (b^6 * c + 2 \\
& 4 * a * b^4 * c^2 + 96 * a^2 * b^2 * c^3 + 64 * a^3 * c^4) * d^8 + (b^8 + 64 * a * b^6 * c + 576 * a^2 \\
& * b^4 * c^2 + 1024 * a^3 * b^2 * c^3 + 256 * a^4 * c^4) * d^7 - 4 * (a * b^8 + 24 * a^2 * b^6 * c + \\
& 96 * a^3 * b^4 * c^2 + 64 * a^4 * b^2 * c^3) * d^6 + (a^4 * b^8 * c^3 + 256 * c^7 * d^8 - 256 * (b \\
& ^2 * c^6 + 4 * a * c^7) * d^7 + 32 * (3 * b^4 * c^5 + 32 * a * b^2 * c^6 + 48 * a^2 * c^7) * d^6 - 16 \\
& * (b^6 * c^4 + 24 * a * b^4 * c^5 + 96 * a^2 * b^2 * c^6 + 64 * a^3 * c^7) * d^5 + (b^8 * c^3 + 64 \\
& * a * b^6 * c^4 + 576 * a^2 * b^4 * c^5 + 1024 * a^3 * b^2 * c^6 + 256 * a^4 * c^7) * d^4 - 4 * (a * b \\
& ^8 * c^3 + 24 * a^2 * b^6 * c^4 + 96 * a^3 * b^4 * c^5 + 64 * a^4 * b^2 * c^6) * d^3 + 2 * (3 * a^2 * b \\
& ^8 * c^3 + 32 * a^3 * b^6 * c^4 + 48 * a^4 * b^4 * c^5) * d^2 - 4 * (a^3 * b^8 * c^3 + 4 * a^4 * b^6 * \\
& c^4) * d) * x^6 + 2 * (3 * a^2 * b^8 + 32 * a^3 * b^6 * c + 48 * a^4 * b^4 * c^2) * d^5 + 3 * (a^4 * b^ \\
& 9 * c^2 + 256 * b * c^6 * d^8 - 256 * (b^3 * c^5 + 4 * a * b * c^6) * d^7 + 32 * (3 * b^5 * c^4 + 32 * \\
& a * b^3 * c^5 + 48 * a^2 * b * c^6) * d^6 - 16 * (b^7 * c^3 + 24 * a * b^5 * c^4 + 96 * a^2 * b^3 * c^5 \\
& + 64 * a^3 * b * c^6) * d^5 + (b^9 * c^2 + 64 * a * b^7 * c^3 + 576 * a^2 * b^5 * c^4 + 1024 * a^3 \\
& * b^3 * c^5 + 256 * a^4 * b * c^6) * d^4 - 4 * (a * b^9 * c^2 + 24 * a^2 * b^7 * c^3 + 96 * a^3 * b^5 * \\
& c^4 + 64 * a^4 * b^3 * c^5) * d^3 + 2 * (3 * a^2 * b^9 * c^2 + 32 * a^3 * b^7 * c^3 + 48 * a^4 * b^5 * \\
& c^4) * d^2 - 4 * (a^3 * b^9 * c^2 + 4 * a^4 * b^7 * c^3) * d) * x^5 - 4 * (a^3 * b^8 + 4 * a^4 * b^6 * \\
& c) * d^4 + 3 * (a^4 * b^10 * c - 1024 * a * c^6 * d^8 + 256 * c^6 * d^9 - 32 * (5 * b^4 * c^4 - 48 * \\
& a^2 * c^6) * d^7 + 16 * (5 * b^6 * c^3 + 40 * a * b^4 * c^4 - 64 * a^3 * c^6) * d^6 - (15 * b^8 * c^2 \\
& + 320 * a * b^6 * c^3 + 960 * a^2 * b^4 * c^4 - 256 * a^4 * c^6) * d^5 + (b^10 * c + 60 * a * b^8 * \\
& c^2 + 480 * a^2 * b^6 * c^3 + 640 * a^3 * b^4 * c^4) * d^4 - 2 * (2 * a * b^10 * c + 45 * a^2 * b^8 * c \\
& ^2 + 160 * a^3 * b^6 * c^3 + 80 * a^4 * b^4 * c^4) * d^3 + 2 * (3 * a^2 * b^10 * c + 30 * a^3 * b^8 * c \\
& ^2 + 40 * a^4 * b^6 * c^3) * d^2 - (4 * a^3 * b^10 * c + 15 * a^4 * b^8 * c^2) * d) * x^4 + (a^4 * b^ \\
& 11 + 1536 * b * c^5 * d^9 - 256 * (5 * b^3 * c^4 + 24 * a * b * c^5) * d^8 + 64 * (5 * b^5 * c^3 + 80 \\
& * a * b^3 * c^4 + 144 * a^2 * b * c^5) * d^7 - 256 * (5 * a * b^5 * c^3 + 30 * a^2 * b^3 * c^4 + 24 * a^ \\
& 3 * b * c^5) * d^6 - 2 * (5 * b^9 * c - 960 * a^2 * b^5 * c^3 - 2560 * a^3 * b^3 * c^4 - 768 * a^4 * b * \\
& c^5) * d^5 + (b^11 + 40 * a * b^9 * c - 1280 * a^3 * b^5 * c^3 - 1280 * a^4 * b^3 * c^4) * d^4 - \\
& 4 * (a * b^11 + 15 * a^2 * b^9 * c - 80 * a^4 * b^5 * c^3) * d^3 + 2 * (3 * a^2 * b^11 + 20 * a^3 * b^9 \\
& * c) * d^2 - 2 * (2 * a^3 * b^11 + 5 * a^4 * b^9 * c) * d) * x^3 + 3 * (a^4 * b^10 * d - 1024 * a * c^5 * \\
& d^9 + 256 * c^5 * d^10 - 32 * (5 * b^4 * c^3 - 48 * a^2 * c^5) * d^8 + 16 * (5 * b^6 * c^2 + 40 * a \\
& * b^4 * c^3 - 64 * a^3 * c^5) * d^7 - (15 * b^8 * c + 320 * a * b^6 * c^2 + 960 * a^2 * b^4 * c^3 - \\
& 256 * a^4 * c^5) * d^6 + (b^10 + 60 * a * b^8 * c + 480 * a^2 * b^6 * c^2 + 640 * a^3 * b^4 * c^3) * \\
& d^5 - 2 * (2 * a * b^10 + 45 * a^2 * b^8 * c + 160 * a^3 * b^6 * c^2 + 80 * a^4 * b^4 * c^3) * d^4 + \\
& 2 * (3 * a^2 * b^10 + 30 * a^3 * b^8 * c + 40 * a^4 * b^6 * c^2) * d^3 - (4 * a^3 * b^10 + 15 * a^4 * b
\end{aligned}$$

$$\begin{aligned}
& ^8c) * d^2) * x^2 + 3 * (a^4 * b^9 * d^2 + 256 * b * c^4 * d^{10} - 256 * (b^3 * c^3 + 4 * a * b * c^4) \\
& ) * d^9 + 32 * (3 * b^5 * c^2 + 32 * a * b^3 * c^3 + 48 * a^2 * b * c^4) * d^8 - 16 * (b^7 * c + 24 * a \\
& * b^5 * c^2 + 96 * a^2 * b^3 * c^3 + 64 * a^3 * b * c^4) * d^7 + (b^9 + 64 * a * b^7 * c + 576 * a^2 \\
& * b^5 * c^2 + 1024 * a^3 * b^3 * c^3 + 256 * a^4 * b * c^4) * d^6 - 4 * (a * b^9 + 24 * a^2 * b^7 * c \\
& + 96 * a^3 * b^5 * c^2 + 64 * a^4 * b^3 * c^3) * d^5 + 2 * (3 * a^2 * b^9 + 32 * a^3 * b^7 * c + 48 * a^4 \\
& * b^5 * c^2) * d^4 - 4 * (a^3 * b^9 + 4 * a^4 * b^7 * c) * d^3) * x, -1/48 * (3 * (1024 * c^3 * d^6 \\
& - (5 * b^6 * c^3 + 12 * a * b^4 * c^4 + 48 * a^2 * b^2 * c^5 + 320 * a^3 * c^6 - 1024 * c^6 * d^3 \\
& + 384 * (b^2 * c^5 + 4 * a * c^6) * d^2 - 24 * (3 * b^4 * c^4 + 8 * a * b^2 * c^5 + 48 * a^2 * c^6) * d \\
& ) * x^6 - 384 * (b^2 * c^2 + 4 * a * c^3) * d^5 - 3 * (5 * b^7 * c^2 + 12 * a * b^5 * c^3 + 48 * a^2 * \\
& b^3 * c^4 + 320 * a^3 * b * c^5 - 1024 * b * c^5 * d^3 + 384 * (b^3 * c^4 + 4 * a * b * c^5) * d^2 - \\
& 24 * (3 * b^5 * c^3 + 8 * a * b^3 * c^4 + 48 * a^2 * b * c^5) * d) * x^5 + 24 * (3 * b^4 * c + 8 * a * b^2 * \\
& c^2 + 48 * a^2 * c^3) * d^4 - 3 * (5 * b^8 * c + 12 * a * b^6 * c^2 + 48 * a^2 * b^4 * c^3 + 320 * a^3 \\
& * b^2 * c^4 - 1024 * c^5 * d^4 - 128 * (5 * b^2 * c^4 - 12 * a * c^5) * d^3 + 24 * (13 * b^4 * c^3 \\
& + 56 * a * b^2 * c^4 - 48 * a^2 * c^5) * d^2 - (67 * b^6 * c^2 + 180 * a * b^4 * c^3 + 1104 * a^2 * b^2 \\
& * c^4 - 320 * a^3 * c^5) * d) * x^4 - (5 * b^6 + 12 * a * b^4 * c + 48 * a^2 * b^2 * c^2 + 320 * a^3 \\
& * c^3) * d^3 - (5 * b^9 + 12 * a * b^7 * c + 48 * a^2 * b^5 * c^2 + 320 * a^3 * b^3 * c^3 - 6144 \\
& * b * c^4 * d^4 + 256 * (5 * b^3 * c^3 + 36 * a * b * c^4) * d^3 - 48 * (b^5 * c^2 - 8 * a * b^3 * c^3 + \\
& 144 * a^2 * b * c^4) * d^2 - 6 * (7 * b^7 * c + 20 * a * b^5 * c^2 + 144 * a^2 * b^3 * c^3 - 320 * a^3 \\
& * b * c^4) * d) * x^3 + 3 * (1024 * c^4 * d^5 + 128 * (5 * b^2 * c^3 - 12 * a * c^4) * d^4 - 24 * (13 * \\
& b^4 * c^2 + 56 * a * b^2 * c^3 - 48 * a^2 * c^4) * d^3 + (67 * b^6 * c + 180 * a * b^4 * c^2 + 1104 \\
& * a^2 * b^2 * c^3 - 320 * a^3 * c^4) * d^2 - (5 * b^8 + 12 * a * b^6 * c + 48 * a^2 * b^4 * c^2 + 32 \\
& 0 * a^3 * b^2 * c^3) * d) * x^2 + 3 * (1024 * b * c^3 * d^5 - 384 * (b^3 * c^2 + 4 * a * b * c^3) * d^4 + \\
& 24 * (3 * b^5 * c + 8 * a * b^3 * c^2 + 48 * a^2 * b * c^3) * d^3 - (5 * b^7 + 12 * a * b^5 * c + 48 * a^2 \\
& * b^3 * c^2 + 320 * a^3 * b * c^3) * d^2) * x) * \text{sqrt}(-a * b^2 - 4 * c * d^2 + (b^2 + 4 * a * c) * d) \\
& ) * \text{arctan}(-1/2 * (2 * a * b^2 + (b^2 * c + 4 * a * c^2 - 8 * c^2 * d) * x^2 - (b^2 + 4 * a * c) * d \\
& + (b^3 + 4 * a * b * c - 8 * b * c * d) * x) * \text{sqrt}(-a * b^2 - 4 * c * d^2 + (b^2 + 4 * a * c) * d) * \text{sqrt} \\
& t(c * x^2 + b * x + a) / (a^2 * b^3 + 4 * a * b * c * d^2 + 2 * (a * b^2 * c^2 + 4 * c^3 * d^2 - (b^2 \\
& * c^2 + 4 * a * c^3) * d) * x^3 + 3 * (a * b^3 * c + 4 * b * c^2 * d^2 - (b^3 * c + 4 * a * b * c^2) * d) * \\
& x^2 - (a * b^3 + 4 * a^2 * b * c) * d + (a * b^4 + 2 * a^2 * b^2 * c + 4 * (b^2 * c + 2 * a * c^2) * d^2 \\
& - (b^4 + 6 * a * b^2 * c + 8 * a^2 * c^2) * d) * x)) + 2 * (8 * a^3 * b^7 + 4608 * b * c^3 * d^6 - \\
& 2592 * (b^3 * c^2 + 4 * a * b * c^3) * d^5 + 2 * (15 * a * b^6 * c^3 + 56 * a^2 * b^4 * c^4 + 240 * a^3 \\
& * b^2 * c^5 + 2816 * c^6 * d^4 - 1408 * (b^2 * c^5 + 4 * a * c^6) * d^3 + 4 * (59 * b^4 * c^4 + 58 \\
& 4 * a * b^2 * c^5 + 944 * a^2 * c^6) * d^2 - (15 * b^6 * c^3 + 292 * a * b^4 * c^4 + 1168 * a^2 * b^2 \\
& * c^5 + 960 * a^3 * c^6) * d) * x^5 + 4 * (123 * b^5 * c + 1352 * a * b^3 * c^2 + 1968 * a^2 * b * c^3 \\
& ) * d^4 + 5 * (15 * a * b^7 * c^2 + 56 * a^2 * b^5 * c^3 + 240 * a^3 * b^3 * c^4 + 2816 * b * c^5 * d^4 \\
& - 1408 * (b^3 * c^4 + 4 * a * b * c^5) * d^3 + 4 * (59 * b^5 * c^3 + 584 * a * b^3 * c^4 + 944 * a^2 \\
& * b * c^5) * d^2 - (15 * b^7 * c^2 + 292 * a * b^5 * c^3 + 1168 * a^2 * b^3 * c^4 + 960 * a^3 * b * c^5 \\
& ) * d) * x^4 - (33 * b^7 + 940 * a * b^5 * c + 3760 * a^2 * b^3 * c^2 + 2112 * a^3 * b * c^3) * d^3 \\
& + 4 * (15 * a * b^8 * c + 51 * a^2 * b^6 * c^2 + 220 * a^3 * b^4 * c^3 + 3456 * c^5 * d^5 + 16 * (63 * \\
& b^2 * c^4 - 452 * a * c^5) * d^4 - 4 * (273 * b^4 * c^3 + 584 * a * b^2 * c^4 - 1264 * a^2 * c^5) * d^3 \\
& + 8 * (27 * b^6 * c^2 + 233 * a * b^4 * c^3 + 236 * a^2 * b^2 * c^4 - 160 * a^3 * c^5) * d^2 - ( \\
& 15 * b^8 * c + 267 * a * b^6 * c^2 + 992 * a^2 * b^4 * c^3 + 560 * a^3 * b^2 * c^4) * d) * x^3 + (59 * \\
& a * b^7 + 584 * a^2 * b^5 * c + 944 * a^3 * b^3 * c^2) * d^2 + (15 * a * b^9 + 26 * a^2 * b^7 * c + 1 \\
& 20 * a^3 * b^5 * c^2 + 20736 * b * c^4 * d^5 - 32 * (251 * b^3 * c^3 + 1356 * a * b * c^4) * d^4 + 8 * \\
& (61 * b^5 * c^2 + 1768 * a * b^3 * c^3 + 3792 * a^2 * b * c^4) * d^3 + 4 * (29 * b^7 * c - 124 * a * b^5 \\
& * c^2 - 1888 * a^2 * b^3 * c^3 - 1920 * a^3 * b * c^4) * d^2 - (15 * b^9 + 142 * a * b^7 * c + 11 \\
& 2 * a^2 * b^5 * c^2 - 1440 * a^3 * b^3 * c^3) * d) * x^2 - 34 * (a^2 * b^7 + 4 * a^3 * b^5 * c) * d - 2 \\
& * (5 * a^2 * b^8 + 12 * a^3 * b^6 * c - 4608 * c^4 * d^6 - 864 * (b^2 * c^3 - 12 * a * c^4) * d^5 + \\
& 4 * (329 * b^4 * c^2 + 456 * a * b^2 * c^3 - 1968 * a^2 * c^4) * d^4 - (283 * b^6 * c + 2356 * a * b^4 \\
& * c^2 + 1296 * a^2 * b^2 * c^3 - 2112 * a^3 * c^4) * d^3 + (20 * b^8 + 413 * a * b^6 * c + 1304 \\
& * a^2 * b^4 * c^2 + 336 * a^3 * b^2 * c^3) * d^2 - (25 * a * b^8 + 142 * a^2 * b^6 * c + 264 * a^3 * b^4 \\
& * c^2) * d) * x) * \text{sqrt}(c * x^2 + b * x + a) / (a^4 * b^8 * d^3 + 256 * c^4 * d^{11} - 256 * (b^2 \\
& * c^3 + 4 * a * c^4) * d^{10} + 32 * (3 * b^4 * c^2 + 32 * a * b^2 * c^3 + 48 * a^2 * c^4) * d^9 - 16 * \\
& (b^6 * c + 24 * a * b^4 * c^2 + 96 * a^2 * b^2 * c^3 + 64 * a^3 * c^4) * d^8 + (b^8 + 64 * a * b^6 * \\
& c + 576 * a^2 * b^4 * c^2 + 1024 * a^3 * b^2 * c^3 + 256 * a^4 * c^4) * d^7 - 4 * (a * b^8 + 24 * a^2 \\
& * b^6 * c + 96 * a^3 * b^4 * c^2 + 64 * a^4 * b^2 * c^3) * d^6 + (a^4 * b^8 * c^3 + 256 * c^7 * d^8 \\
& - 256 * (b^2 * c^6 + 4 * a * c^7) * d^7 + 32 * (3 * b^4 * c^5 + 32 * a * b^2 * c^6 + 48 * a^2 * c^7) \\
& ) * d^6 - 16 * (b^6 * c^4 + 24 * a * b^4 * c^5 + 96 * a^2 * b^2 * c^6 + 64 * a^3 * c^7) * d^5 + (b^8 \\
& * c^3 + 64 * a * b^6 * c^4 + 576 * a^2 * b^4 * c^5 + 1024 * a^3 * b^2 * c^6 + 256 * a^4 * c^7) * d^4
\end{aligned}$$

$$\begin{aligned}
& 4 - 4*(a*b^8*c^3 + 24*a^2*b^6*c^4 + 96*a^3*b^4*c^5 + 64*a^4*b^2*c^6)*d^3 + \\
& 2*(3*a^2*b^8*c^3 + 32*a^3*b^6*c^4 + 48*a^4*b^4*c^5)*d^2 - 4*(a^3*b^8*c^3 + \\
& 4*a^4*b^6*c^4)*d)*x^6 + 2*(3*a^2*b^8 + 32*a^3*b^6*c + 48*a^4*b^4*c^2)*d^5 + \\
& 3*(a^4*b^9*c^2 + 256*b*c^6*d^8 - 256*(b^3*c^5 + 4*a*b*c^6)*d^7 + 32*(3*b^5 \\
& *c^4 + 32*a*b^3*c^5 + 48*a^2*b*c^6)*d^6 - 16*(b^7*c^3 + 24*a*b^5*c^4 + 96*a \\
& ^2*b^3*c^5 + 64*a^3*b*c^6)*d^5 + (b^9*c^2 + 64*a*b^7*c^3 + 576*a^2*b^5*c^4 \\
& + 1024*a^3*b^3*c^5 + 256*a^4*b*c^6)*d^4 - 4*(a*b^9*c^2 + 24*a^2*b^7*c^3 + 9 \\
& 6*a^3*b^5*c^4 + 64*a^4*b^3*c^5)*d^3 + 2*(3*a^2*b^9*c^2 + 32*a^3*b^7*c^3 + 4 \\
& 8*a^4*b^5*c^4)*d^2 - 4*(a^3*b^9*c^2 + 4*a^4*b^7*c^3)*d)*x^5 - 4*(a^3*b^8 + \\
& 4*a^4*b^6*c)*d^4 + 3*(a^4*b^10*c - 1024*a*c^6*d^8 + 256*c^6*d^9 - 32*(5*b^4 \\
& *c^4 - 48*a^2*c^6)*d^7 + 16*(5*b^6*c^3 + 40*a*b^4*c^4 - 64*a^3*c^6)*d^6 - ( \\
& 15*b^8*c^2 + 320*a*b^6*c^3 + 960*a^2*b^4*c^4 - 256*a^4*c^6)*d^5 + (b^10*c + \\
& 60*a*b^8*c^2 + 480*a^2*b^6*c^3 + 640*a^3*b^4*c^4)*d^4 - 2*(2*a*b^10*c + 45 \\
& *a^2*b^8*c^2 + 160*a^3*b^6*c^3 + 80*a^4*b^4*c^4)*d^3 + 2*(3*a^2*b^10*c + 30 \\
& *a^3*b^8*c^2 + 40*a^4*b^6*c^3)*d^2 - (4*a^3*b^10*c + 15*a^4*b^8*c^2)*d)*x^4 \\
& + (a^4*b^11 + 1536*b*c^5*d^9 - 256*(5*b^3*c^4 + 24*a*b*c^5)*d^8 + 64*(5*b^ \\
& 5*c^3 + 80*a*b^3*c^4 + 144*a^2*b*c^5)*d^7 - 256*(5*a*b^5*c^3 + 30*a^2*b^3*c \\
& ^4 + 24*a^3*b*c^5)*d^6 - 2*(5*b^9*c - 960*a^2*b^5*c^3 - 2560*a^3*b^3*c^4 - \\
& 768*a^4*b*c^5)*d^5 + (b^11 + 40*a*b^9*c - 1280*a^3*b^5*c^3 - 1280*a^4*b^3*c \\
& ^4)*d^4 - 4*(a*b^11 + 15*a^2*b^9*c - 80*a^4*b^5*c^3)*d^3 + 2*(3*a^2*b^11 + \\
& 20*a^3*b^9*c)*d^2 - 2*(2*a^3*b^11 + 5*a^4*b^9*c)*d)*x^3 + 3*(a^4*b^10*d - 1 \\
& 024*a*c^5*d^9 + 256*c^5*d^10 - 32*(5*b^4*c^3 - 48*a^2*c^5)*d^8 + 16*(5*b^6* \\
& c^2 + 40*a*b^4*c^3 - 64*a^3*c^5)*d^7 - (15*b^8*c + 320*a*b^6*c^2 + 960*a^2* \\
& b^4*c^3 - 256*a^4*c^5)*d^6 + (b^10 + 60*a*b^8*c + 480*a^2*b^6*c^2 + 640*a^3 \\
& *b^4*c^3)*d^5 - 2*(2*a*b^10 + 45*a^2*b^8*c + 160*a^3*b^6*c^2 + 80*a^4*b^4*c \\
& ^3)*d^4 + 2*(3*a^2*b^10 + 30*a^3*b^8*c + 40*a^4*b^6*c^2)*d^3 - (4*a^3*b^10 \\
& + 15*a^4*b^8*c)*d^2)*x^2 + 3*(a^4*b^9*d^2 + 256*b*c^4*d^10 - 256*(b^3*c^3 + \\
& 4*a*b*c^4)*d^9 + 32*(3*b^5*c^2 + 32*a*b^3*c^3 + 48*a^2*b*c^4)*d^8 - 16*(b^ \\
& 7*c + 24*a*b^5*c^2 + 96*a^2*b^3*c^3 + 64*a^3*b*c^4)*d^7 + (b^9 + 64*a*b^7*c \\
& + 576*a^2*b^5*c^2 + 1024*a^3*b^3*c^3 + 256*a^4*b*c^4)*d^6 - 4*(a*b^9 + 24* \\
& a^2*b^7*c + 96*a^3*b^5*c^2 + 64*a^4*b^3*c^3)*d^5 + 2*(3*a^2*b^9 + 32*a^3*b^ \\
& 7*c + 48*a^4*b^5*c^2)*d^4 - 4*(a^3*b^9 + 4*a^4*b^7*c)*d^3)*x]
\end{aligned}$$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)^4/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.04, size = 3695, normalized size = 11.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x^2+b\*x+d)^4/(c\*x^2+b\*x+a)^(1/2),x)

[Out] 
$$\begin{aligned}
& 5/2/(b^2-4*c*d)^2/(a-d)^3/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x+1/2*(b+ \\
& (b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/ \\
& c))^(1/2)*c*d+5/2/(b^2-4*c*d)^2/(a-d)^3/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c) \\
& *(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^ \\
& 2-4*c*d)^(1/2))/c))^(1/2)*c*d-3/4/(b^2-4*c*d)^(3/2)/(a-d)^(5/2)*c*\ln((2*a-2 \\
& *d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1 \\
& /2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d) \\
& ^2)/(c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))-1/3/(b^2-4*c*d)^2/(a-d) \\
& )/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/c)^3*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^ \\
& 2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)-3/(b^2-4*c*d
\end{aligned}$$



$$-1/2*(-b+(b^2-4*c*d)^{(1/2)})/c)^2*c-d+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)})/c))^{(1/2)}+5/4/(b^2-4*c*d)^{(3/2)}/(a-d)^{(7/2)}*\ln((2*a-2*d-(b^2-4*c*d)^{(1/2)}*(x+1/2*(b+(b^2-4*c*d)^{(1/2)})/c)+2*(a-d)^{(1/2)}*(a+(x+1/2*(b+(b^2-4*c*d)^{(1/2)})/c))^2*c-d-(b^2-4*c*d)^{(1/2)}*(x+1/2*(b+(b^2-4*c*d)^{(1/2)})/c))^{(1/2)})/(x+1/2*(b+(b^2-4*c*d)^{(1/2)})/c))*c*d-5/4/(b^2-4*c*d)^{(3/2)}/(a-d)^{(7/2)}*\ln((2*a-2*d+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)})/c)+2*(a-d)^{(1/2)}*(a+(x-1/2*(-b+(b^2-4*c*d)^{(1/2)})/c))^2*c-d+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)})/c))^{(1/2)})/(x-1/2*(-b+(b^2-4*c*d)^{(1/2)})/c))*c*d$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (cx^2 + bx + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+d)^4/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(c\*x^2 + b\*x + d)^4), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (cx^2 + bx + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + b\*x + c\*x^2)^4),x)

[Out] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + b\*x + c\*x^2)^4), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x\*\*2+b\*x+d)\*\*4/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Timed out



$$3.7 \quad \int \frac{1}{\sqrt{d+ex+fx^2} (ae+bex+bf x^2)^2} dx$$

**Optimal.** Leaf size=162

$$\frac{(8aef - b(4df + e^2)) \tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}} - \frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)}$$

**Rubi [A]** time = 0.31, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {974, 12, 982, 208}

$$\frac{(8aef - b(4df + e^2)) \tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}} - \frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d + e\*x + f\*x^2]\*(a\*e + b\*e\*x + b\*f\*x^2)^2), x]

[Out] -((b\*(e + 2\*f\*x)\*Sqrt[d + e\*x + f\*x^2])/(e\*(b\*d - a\*e)\*(b\*e - 4\*a\*f)\*(a\*e + b\*e\*x + b\*f\*x^2))) - ((8\*a\*e\*f - b\*(e^2 + 4\*d\*f))\*ArcTanh[(Sqrt[b\*d - a\*e]\*(e + 2\*f\*x))/(Sqrt[e]\*Sqrt[b\*e - 4\*a\*f]\*Sqrt[d + e\*x + f\*x^2])])/(e^(3/2)\*(b\*d - a\*e)^(3/2)\*(b\*e - 4\*a\*f)^(3/2))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

#### Rule 982

Int[1/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2\*e, Subst[Int[1/(e\*(b\*e - 4\*a\*f) - (b\*d - a\*e)\*x^2), x], x, (e + 2\*f\*x)/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d,

e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && EqQ[c\*e - b\*f, 0]

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+fx^2} (ae+bex+bf x^2)^2} dx = -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)} + \frac{\int \frac{b(bd-ae)f^2(8aef-b(e^2+4df))}{2\sqrt{d+ex+fx^2}(ae+bex+bf x^2)} dx}{be(bd-ae)^2 f^2 (be-4af)}$$

$$= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)} + \frac{(8aef-b(e^2+4df)) \int \frac{1}{\sqrt{d+ex+fx^2}} dx}{2e(bd-ae)}$$

$$= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)} - \frac{(8aef-b(e^2+4df)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{d+ex+fx^2}} dx, x, \frac{e+fx}{b}\right)}{2e(bd-ae)}$$

$$= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)} - \frac{(8aef-b(e^2+4df)) \operatorname{atanh}\left(\frac{e+fx}{\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}}$$

Mathematica [B] time = 1.96, size = 490, normalized size = 3.02

$$2f \left( \frac{(b(4df+e^2)-8aef) \operatorname{tanh}^{-1}\left(\frac{-\sqrt{e+2fx}\sqrt{be-4af}-\sqrt{b}(e^2-4df)}{4f\sqrt{bd-ae}\sqrt{d+ex+fx^2}}\right)}{4f(bd-ae)^{3/2}(be-4af)^{3/2}} - \frac{e \operatorname{tanh}^{-1}\left(\frac{\sqrt{b}(e^2-4df)-\sqrt{e+2fx}\sqrt{be-4af}}{4f\sqrt{bd-ae}\sqrt{d+ex+fx^2}}\right)}{4f(bd-ae)^{3/2}\sqrt{be-4af}} + \frac{\operatorname{tanh}^{-1}\left(\frac{\sqrt{e+2fx}\sqrt{be-4af}-\sqrt{b}(e^2-4df)}{4f\sqrt{bd-ae}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}(be-4af)^{3/2}} - \frac{\sqrt{b}\sqrt{d+ex+fx^2}}{(bd-ae)(be-4af)(\sqrt{e+2fx}-\sqrt{e}\sqrt{be-4af})} - \frac{\sqrt{e}\sqrt{d+ex+fx^2}}{(bd-ae)(be-4af)(\sqrt{e}\sqrt{be-4af}+\sqrt{e+2fx})} \right) e^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d + e\*x + f\*x^2]\*(a\*e + b\*e\*x + b\*f\*x^2)^2), x]

[Out] (2\*f\*(-((Sqrt[b]\*Sqrt[e]\*Sqrt[d + x\*(e + f\*x)])/((b\*d - a\*e)\*(b\*e - 4\*a\*f)\*(-Sqrt[e]\*Sqrt[b\*e - 4\*a\*f]) + Sqrt[b]\*(e + 2\*f\*x)))) - (Sqrt[b]\*Sqrt[e]\*Sqrt[d + x\*(e + f\*x)])/((b\*d - a\*e)\*(b\*e - 4\*a\*f)\*(Sqrt[e]\*Sqrt[b\*e - 4\*a\*f] + Sqrt[b]\*(e + 2\*f\*x))) - ((-8\*a\*e\*f + b\*(e^2 + 4\*d\*f))\*ArcTanh[(-Sqrt[b]\*(e^2 - 4\*d\*f)) - Sqrt[e]\*Sqrt[b\*e - 4\*a\*f]\*(e + 2\*f\*x)]/(4\*Sqrt[b\*d - a\*e]\*f\*Sqrt[d + x\*(e + f\*x)])]/(4\*(b\*d - a\*e)^(3/2)\*f\*(b\*e - 4\*a\*f)^(3/2)) - (e\*ArcTanh[(Sqrt[b]\*(e^2 - 4\*d\*f) - Sqrt[e]\*Sqrt[b\*e - 4\*a\*f]\*(e + 2\*f\*x))/(4\*Sqrt[b\*d - a\*e]\*f\*Sqrt[d + x\*(e + f\*x)])]/(4\*(b\*d - a\*e)^(3/2)\*f\*Sqrt[b\*e - 4\*a\*f]) + ArcTanh[(-Sqrt[b]\*(e^2 - 4\*d\*f)) + Sqrt[e]\*Sqrt[b\*e - 4\*a\*f]\*(e + 2\*f\*x)]/(4\*Sqrt[b\*d - a\*e]\*f\*Sqrt[d + x\*(e + f\*x)])]/(Sqrt[b\*d - a\*e]\*(b\*e - 4\*a\*f)^(3/2)))/e^(3/2)

IntegrateAlgebraic [C] time = 1.34, size = 266, normalized size = 1.64

$$\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(ae-bd)(be-4af)(ae+bex+bf x^2)} - \frac{(8aef-4bdf-b^2)\operatorname{RootSum}\left[\#1^4 b f - 2\#1^3 b e \sqrt{f} + 4\#1^2 a e f - 2\#1^2 b d f + \#1^2 b e^2 - 4\#1 a e^2 \sqrt{f} + 2\#1 b d e \sqrt{f} + a e^3 + b d^2 f - b d e^2 \&, \frac{\log(-\#1 + \sqrt{d+ex+fx^2} - \sqrt{f} x)}{\#1^2(-b)\sqrt{f} + \#1 b e - 2 a e \sqrt{f} + b d \sqrt{f}}\right]}{2e(ae-bd)(be-4af)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[d + e\*x + f\*x^2]\*(a\*e + b\*e\*x + b\*f\*x^2)^2), x]

[Out] (b\*(e + 2\*f\*x)\*Sqrt[d + e\*x + f\*x^2])/(e\*(-(b\*d) + a\*e)\*(b\*e - 4\*a\*f)\*(a\*e + b\*e\*x + b\*f\*x^2)) - (((-b\*e^2) - 4\*b\*d\*f + 8\*a\*e\*f)\*RootSum[-(b\*d\*e^2) + a\*e^3 + b\*d^2\*f + 2\*b\*d\*e\*Sqrt[f]\*#1 - 4\*a\*e^2\*Sqrt[f]\*#1 + b\*e^2\*#1^2 - 2\*b\*d\*f\*#1^2 + 4\*a\*e\*f\*#1^2 - 2\*b\*e\*Sqrt[f]\*#1^3 + b\*f\*#1^4 &, Log[-(Sqrt[f]\*x) + Sqrt[d + e\*x + f\*x^2] - #1]/(b\*d\*Sqrt[f] - 2\*a\*e\*Sqrt[f] + b\*e\*#1 - b\*Sqrt[f]\*#1^2) & ])/(2\*e\*(-(b\*d) + a\*e)\*(b\*e - 4\*a\*f))

fricas [B] time = 1.57, size = 2005, normalized size = 12.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(b^2*d*e^2 - a*b*e^3 - 4*(a*b*d*e - a^2*e^2)*f)*(a*b*e^3 + (b^2*
e^2*f + 4*(b^2*d - 2*a*b*e)*f^2)*x^2 + 4*(a*b*d*e - 2*a^2*e^2)*f + (b^2*e^3
+ 4*(b^2*d*e - 2*a*b*e^2)*f)*x)*log(((8*b^2*d^2*e^4 - 8*a*b*d*e^5 + a^2*e^6
+ 16*a^2*d^2*e^2*f^2 + (b^2*e^4*f^2 + 16*(b^2*d^2 - 8*a*b*d*e + 8*a^2*e^2)
*f^4 + 8*(3*b^2*d*e^2 - 4*a*b*e^3)*f^3)*x^4 + 2*(b^2*e^5*f + 16*(b^2*d^2*e
- 8*a*b*d*e^2 + 8*a^2*e^3)*f^3 + 8*(3*b^2*d*e^3 - 4*a*b*e^4)*f^2)*x^3 + (b^
2*e^6 - 32*(3*a*b*d^2*e - 4*a^2*d*e^2)*f^3 + 16*(3*b^2*d^2*e^2 - 13*a*b*d*e
^3 + 10*a^2*e^4)*f^2 + 2*(16*b^2*d*e^4 - 19*a*b*e^5)*f)*x^2 - 4*sqrt(b^2*d*
e^2 - a*b*e^3 - 4*(a*b*d*e - a^2*e^2)*f)*(2*b*d*e^3 - a*e^4 - 4*a*d*e^2*f +
2*(b*e^2*f^2 + 4*(b*d - 2*a*e)*f^3)*x^3 + 3*(b*e^3*f + 4*(b*d*e - 2*a*e^2)
*f^2)*x^2 + (b*e^4 - 8*a*d*e*f^2 + 2*(4*b*d*e^2 - 5*a*e^3)*f)*x)*sqrt(f*x^2
+ e*x + d) - 8*(4*a*b*d^2*e^3 - 3*a^2*d*e^4)*f + 2*(4*b^2*d*e^5 - 3*a*b*e^
6 - 16*(3*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^2 + 8*(2*b^2*d^2*e^3 - 5*a*b*d*e^4 +
2*a^2*e^5)*f)*x)/(b^2*f^2*x^4 + 2*b^2*e*f*x^3 + 2*a*b*e^2*x + a^2*e^2 + (b
^2*e^2 + 2*a*b*e*f)*x^2)) + 4*(b^3*d*e^3 - a*b^2*e^4 - 4*(a*b^2*d*e^2 - a^2
*b*e^3)*f - 2*(4*(a*b^2*d*e - a^2*b*e^2)*f^2 - (b^3*d*e^2 - a*b^2*e^3)*f)*x
)*sqrt(f*x^2 + e*x + d)/(a*b^4*d^2*e^5 - 2*a^2*b^3*d*e^6 + a^3*b^2*e^7 + 1
6*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5)*f^2 + (16*(a^2*b^3*d^2*e^2 -
2*a^3*b^2*d*e^3 + a^4*b*e^4)*f^3 - 8*(a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 + a^3
*b^2*e^5)*f^2 + (b^5*d^2*e^4 - 2*a*b^4*d*e^5 + a^2*b^3*e^6)*f)*x^2 - 8*(a^2
*b^3*d^2*e^4 - 2*a^3*b^2*d*e^5 + a^4*b*e^6)*f + (b^5*d^2*e^5 - 2*a*b^4*d*e^
6 + a^2*b^3*e^7 + 16*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e^4 + a^4*b*e^5)*f^2 -
8*(a*b^4*d^2*e^4 - 2*a^2*b^3*d*e^5 + a^3*b^2*e^6)*f)*x), 1/2*(sqrt(-b^2*d*e
^2 + a*b*e^3 + 4*(a*b*d*e - a^2*e^2)*f)*(a*b*e^3 + (b^2*e^2*f + 4*(b^2*d -
2*a*b*e)*f^2)*x^2 + 4*(a*b*d*e - 2*a^2*e^2)*f + (b^2*e^3 + 4*(b^2*d*e - 2*a
*b*e^2)*f)*x)*arctan(-1/2*sqrt(-b^2*d*e^2 + a*b*e^3 + 4*(a*b*d*e - a^2*e^2)
*f)*(2*b*d*e^2 - a*e^3 - 4*a*d*e*f + (b*e^2*f + 4*(b*d - 2*a*e)*f^2)*x^2 +
(b*e^3 + 4*(b*d*e - 2*a*e^2)*f)*x)*sqrt(f*x^2 + e*x + d)/(b^2*d^2*e^3 - a*b
*d*e^4 - 2*(4*(a*b*d*e - a^2*e^2)*f^3 - (b^2*d*e^2 - a*b*e^3)*f^2)*x^3 - 3*
(4*(a*b*d*e^2 - a^2*e^3)*f^2 - (b^2*d*e^3 - a*b*e^4)*f)*x^2 - 4*(a*b*d^2*e^
2 - a^2*d*e^3)*f + (b^2*d*e^4 - a*b*e^5 - 8*(a*b*d^2*e - a^2*d*e^2)*f^2 + 2
*(b^2*d^2*e^2 - 3*a*b*d*e^3 + 2*a^2*e^4)*f)*x)) - 2*(b^3*d*e^3 - a*b^2*e^4
- 4*(a*b^2*d*e^2 - a^2*b*e^3)*f - 2*(4*(a*b^2*d*e - a^2*b*e^2)*f^2 - (b^3*d
*e^2 - a*b^2*e^3)*f)*x)*sqrt(f*x^2 + e*x + d))/(a*b^4*d^2*e^5 - 2*a^2*b^3*d
*e^6 + a^3*b^2*e^7 + 16*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5)*f^2 + (
16*(a^2*b^3*d^2*e^2 - 2*a^3*b^2*d*e^3 + a^4*b*e^4)*f^3 - 8*(a*b^4*d^2*e^3 -
2*a^2*b^3*d*e^4 + a^3*b^2*e^5)*f^2 + (b^5*d^2*e^4 - 2*a*b^4*d*e^5 + a^2*b^
3*e^6)*f)*x^2 - 8*(a^2*b^3*d^2*e^4 - 2*a^3*b^2*d*e^5 + a^4*b*e^6)*f + (b^5*
d^2*e^5 - 2*a*b^4*d*e^6 + a^2*b^3*e^7 + 16*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e
^4 + a^4*b*e^5)*f^2 - 8*(a*b^4*d^2*e^4 - 2*a^2*b^3*d*e^5 + a^3*b^2*e^6)*f)*
x)]
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to divide, perhaps due to rounding er
ror%{1, [2]}%, [8, 2, 0, 0, 0]}%+%%{%%{%%{-4, [1]}%, 0}: [1, 0, %%{-1,
[1]}%}%}, [7, 2, 1, 0, 0]}%+%%{%%{%%{6, [1]}%}, [6, 2, 2, 0, 0]}%+%%{%%{-4, [2
]}%}, [6, 2, 0, 0, 1]}%+%%{%%{8, [2]}%}, [6, 1, 1, 1, 0]}%+%%{%%{-4, 0}: [1, 0,
```

```

%%{-1, [1]%%}%%}, [5, 2, 3, 0, 0]%%}+%%{%%{[%%{-12, [1]%%}, 0] : [1, 0, %%{-1, [1]
]%%}%%}, [5, 2, 1, 0, 1]%%}+%%{%%{[%%{-24, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}%%}
}, [5, 1, 2, 1, 0]%%}+%%{1, [4, 2, 4, 0, 0]%%}+%%{%%{-14, [1]%%}, [4, 2, 2, 0, 1]%%}
+%%{%%{-6, [2]%%}, [4, 2, 0, 0, 2]%%}+%%{%%{-26, [1]%%}, [4, 1, 3, 1, 0]%%}+%%{%%
{-16, [2]%%}, [4, 1, 1, 1, 1]%%}+%%{%%{-16, [2]%%}, [4, 0, 2, 2, 0]%%}+%%{%%{-8
, 0] : [1, 0, %%{-1, [1]%%}%%}, [3, 2, 3, 0, 1]%%}+%%{%%{[%%{-12, [1]%%}, 0] : [1, 0
, %%{-1, [1]%%}%%}, [3, 2, 1, 0, 2]%%}+%%{%%{-12, 0] : [1, 0, %%{-1, [1]%%}%%},
[3, 1, 4, 1, 0]%%}+%%{%%{[%%{-32, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}%%}, [3, 1, 2, 1,
1]%%}+%%{%%{[%%{-32, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}%%}, [3, 0, 3, 2, 0]%%}+%%
{-2, [2, 2, 4, 0, 1]%%}+%%{%%{-10, [1]%%}, [2, 2, 2, 0, 2]%%}+%%{%%{-4, [2]%%}
, [2, 2, 0, 0, 3]%%}+%%{2, [2, 1, 5, 1, 0]%%}+%%{%%{-28, [1]%%}, [2, 1, 3, 1, 1]%%}+
%%{%%{-8, [2]%%}, [2, 1, 1, 1, 2]%%}+%%{%%{-24, [1]%%}, [2, 0, 4, 2, 0]%%}+%%{%%
{-4, 0] : [1, 0, %%{-1, [1]%%}%%}, [1, 2, 3, 0, 2]%%}+%%{%%{[%%{-4, [1]%%}, 0] : [1
, 0, %%{-1, [1]%%}%%}, [1, 2, 1, 0, 3]%%}+%%{%%{[12, 0] : [1, 0, %%{-1, [1]%%}%%}
, [1, 1, 4, 1, 1]%%}+%%{%%{[%%{-8, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}%%}, [1, 1, 2, 1
, 2]%%}+%%{%%{[-8, 0] : [1, 0, %%{-1, [1]%%}%%}, [1, 0, 5, 2, 0]%%}+%%{1, [0, 2, 4,
0, 2]%%}+%%{%%{-2, [1]%%}, [0, 2, 2, 0, 3]%%}+%%{%%{-1, [2]%%}, [0, 2, 0, 0, 4]%%
}+%%{-2, [0, 1, 5, 1, 1]%%}+%%{%%{-2, [1]%%}, [0, 1, 3, 1, 2]%%}+%%{1, [0, 0, 6, 2,
0]%%} / %%{%%{-1, [3]%%}, [8, 2, 0, 0, 0]%%}+%%{%%{poly1 [%%{-4, [2]%%}, 0] : [
1, 0, %%{-1, [1]%%}%%}, [7, 2, 1, 0, 0]%%}+%%{%%{-6, [2]%%}, [6, 2, 2, 0, 0]%%}+%%
{%%{-4, [3]%%}, [6, 2, 0, 0, 1]%%}+%%{%%{-8, [3]%%}, [6, 1, 1, 1, 0]%%}+%%{%%{p
oly1 [%%{-4, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}%%}, [5, 2, 3, 0, 0]%%}+%%{%%{poly1
[%%{-12, [2]%%}, 0] : [1, 0, %%{-1, [1]%%}%%}, [5, 2, 1, 0, 1]%%}+%%{%%{poly1 [%%
{-24, [2]%%}, 0] : [1, 0, %%{-1, [1]%%}%%}, [5, 1, 2, 1, 0]%%}+%%{%%{-1, [1]%%}, [
4, 2, 4, 0, 0]%%}+%%{%%{-14, [2]%%}, [4, 2, 2, 0, 1]%%}+%%{%%{-6, [3]%%}, [4, 2, 0
, 0, 2]%%}+%%{%%{-26, [2]%%}, [4, 1, 3, 1, 0]%%}+%%{%%{-16, [3]%%}, [4, 1, 1, 1, 1
]%%}+%%{%%{-16, [3]%%}, [4, 0, 2, 2, 0]%%}+%%{%%{poly1 [%%{-8, [1]%%}, 0] : [1, 0
, %%{-1, [1]%%}%%}, [3, 2, 3, 0, 1]%%}+%%{%%{poly1 [%%{-12, [2]%%}, 0] : [1, 0, %%
{-1, [1]%%}%%}, [3, 2, 1, 0, 2]%%}+%%{%%{poly1 [%%{-12, [1]%%}, 0] : [1, 0, %%{-1
, [1]%%}%%}, [3, 1, 4, 1, 0]%%}+%%{%%{poly1 [%%{-32, [2]%%}, 0] : [1, 0, %%{-1, [1]
}%%}%%}, [3, 1, 2, 1, 1]%%}+%%{%%{poly1 [%%{-32, [2]%%}, 0] : [1, 0, %%{-1, [1]%%
}%%}%%}, [3, 0, 3, 2, 0]%%}+%%{%%{-2, [1]%%}, [2, 2, 4, 0, 1]%%}+%%{%%{-10, [2]%%}
}, [2, 2, 2, 0, 2]%%}+%%{%%{-4, [3]%%}, [2, 2, 0, 0, 3]%%}+%%{%%{-2, [1]%%}, [2, 1
, 5, 1, 0]%%}+%%{%%{-28, [2]%%}, [2, 1, 3, 1, 1]%%}+%%{%%{-8, [3]%%}, [2, 1, 1, 1,
2]%%}+%%{%%{-24, [2]%%}, [2, 0, 4, 2, 0]%%}+%%{%%{poly1 [%%{-4, [1]%%}, 0] : [1
, 0, %%{-1, [1]%%}%%}, [1, 2, 3, 0, 2]%%}+%%{%%{poly1 [%%{-4, [2]%%}, 0] : [1, 0, %%
{-1, [1]%%}%%}, [1, 2, 1, 0, 3]%%}+%%{%%{poly1 [%%{-12, [1]%%}, 0] : [1, 0, %%{-1
, [1]%%}%%}, [1, 1, 4, 1, 1]%%}+%%{%%{poly1 [%%{-8, [2]%%}, 0] : [1, 0, %%{-1, [1]
}%%}%%}, [1, 1, 2, 1, 2]%%}+%%{%%{poly1 [%%{-8, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}
]%%}, [1, 0, 5, 2, 0]%%}+%%{%%{-1, [1]%%}, [0, 2, 4, 0, 2]%%}+%%{%%{-2, [2]%%}, [
0, 2, 2, 0, 3]%%}+%%{%%{-1, [3]%%}, [0, 2, 0, 0, 4]%%}+%%{%%{-2, [1]%%}, [0, 1, 5,
1, 1]%%}+%%{%%{-2, [2]%%}, [0, 1, 3, 1, 2]%%}+%%{%%{-1, [1]%%}, [0, 0, 6, 2, 0]%%}
} Error: Bad Argument Value

```

**maple [B]** time = 0.03, size = 1377, normalized size = 8.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^{(1/2)}, x)$

[Out] 
$$-2/(4*a*f-b*e)/e*f/(-4*a*f-b*e)*b*e)^{(1/2)/(-a*e-b*d)/b)^{(1/2)}*\ln((-2*(a*e-b*d)/b+(-4*a*f-b*e)*b*e)^{(1/2)}*(x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)/b+2*(-a*e-b*d)/b)^{(1/2)}*((x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)^2*f+(-4*a*f-b*e)*b*e)^{(1/2)}*(x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)/b-(a*e-b*d)/b)^{(1/2)}/(x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)-1/(4*a*f-b*e)/e/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-4*a*f-b*e)*b*e)^{(1/2)}*((x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)^2*f+(-4*a*f-b*e)*b*e)^{(1/2)}*(x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)/b-(a*e-b*d)/b)^{(1/2)}+1/2/(4*a*f-b*e)/b/e*(-4*a*f-b*e)*b*e)^{(1/2)/(a*e-b*d)/(-a*e-b*d)/b)^{(1/2)}*\ln((-2*(a*e-b*d)/b+(-4*a*f-b*e)*b*e)^{(1/2)}/(-a*e-b*d)/b)^{(1/2)}*\ln((-2*(a*e-b*d)/b+(-4*a*f-b*e)*b*e)^{(1/2)}/(-a*e-b*d)/b)^{(1/2)}$$

$$\begin{aligned} & f-b*e)*b*e)^{(1/2)}*(x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)/b+2*(-(a*e-b*d)/b)^{(1/2)}*((x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)^2*f+(-4*a*f-b*e)* \\ & b*e)^{(1/2)}*(x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)/b-(a*e-b*d)/b)^{(1/2)} \\ & )/(x-1/2*(-b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)-1/(4*a*f-b*e)/e/(a*e-b*d)/(x \\ & +1/2/f*e+1/2/b/f*(-4*a*f-b*e)*b*e)^{(1/2)}*((x+1/2*(b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)^2*f- \\ & (-4*a*f-b*e)*b*e)^{(1/2)}*(x+1/2*(b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)/b-(a*e-b*d)/b)^{(1/2)}- \\ & 1/2/(4*a*f-b*e)/b/e*(-4*a*f-b*e)*b*e)^{(1/2)}/(a*e-b*d)/(-a*e-b*d)/b)^{(1/2)}* \\ & \ln((-2*(a*e-b*d)/b-(-4*a*f-b*e)*b*e)^{(1/2)}*(x+1/2*(b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)/b+2*(-(a*e-b*d)/b)^{(1/2)}* \\ & ((x+1/2*(b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)^2*f-(-4*a*f-b*e)*b*e)^{(1/2)}*(x+1/2*(b \\ & *e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f)/b-(a*e-b*d)/b)^{(1/2)})/(x+1/2*(b*e+(-4*a* \\ & f-b*e)*b*e)^{(1/2)})/b/f))+2/(4*a*f-b*e)/e*f/(-4*a*f-b*e)*b*e)^{(1/2)}/(-a*e- \\ & b*d)/b)^{(1/2)}*\ln((-2*(a*e-b*d)/b-(-4*a*f-b*e)*b*e)^{(1/2)}*(x+1/2*(b*e+(-4*a* \\ & a*f-b*e)*b*e)^{(1/2)})/b/f)/b+2*(-(a*e-b*d)/b)^{(1/2)}*((x+1/2*(b*e+(-4*a*f-b* \\ & e)*b*e)^{(1/2)})/b/f)^2*f-(-4*a*f-b*e)*b*e)^{(1/2)}*(x+1/2*(b*e+(-4*a*f-b*e)* \\ & b*e)^{(1/2)})/b/f)/b-(a*e-b*d)/b)^{(1/2)})/(x+1/2*(b*e+(-4*a*f-b*e)*b*e)^{(1/2)})/b/f) \end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bfx^2 + bex + ae)^2 \sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*f\*x^2+b\*e\*x+a\*e)^2/(f\*x^2+e\*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b\*f\*x^2 + b\*e\*x + a\*e)^2\*sqrt(f\*x^2 + e\*x + d)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(bfx^2 + bex + ae)^2 \sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a\*e + b\*e\*x + b\*f\*x^2)^2\*(d + e\*x + f\*x^2)^(1/2)),x)

[Out] int(1/((a\*e + b\*e\*x + b\*f\*x^2)^2\*(d + e\*x + f\*x^2)^(1/2)), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*f\*x\*\*2+b\*e\*x+a\*e)\*\*2/(f\*x\*\*2+e\*x+d)\*\*(1/2),x)

[Out] Timed out

$$3.8 \quad \int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx$$

**Optimal.** Leaf size=28

$$\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {982, 204}

$$\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((4 + 2\*x + x^2)\*Sqrt[5 + 2\*x + x^2]),x]

[Out] ArcTan[(1 + x)/(Sqrt[3]\*Sqrt[5 + 2\*x + x^2])]/Sqrt[3]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 982

Int[1/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] :> Dist[-2\*e, Subst[Int[1/(e\*(b\*e - 4\*a\*f) - (b\*d - a\*e)\*x^2), x], x, (e + 2\*f\*x)/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && EqQ[c\*e - b\*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx &= -\left(4 \text{Subst}\left(\int \frac{1}{-24-2x^2} dx, x, \frac{2+2x}{\sqrt{5+2x+x^2}}\right)\right) \\ &= \frac{\tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)}{\sqrt{3}} \end{aligned}$$

**Mathematica [C]** time = 0.07, size = 84, normalized size = 3.00

$$\frac{i\left(\tanh^{-1}\left(\frac{-i\sqrt{3}x-i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right) - \tanh^{-1}\left(\frac{i\sqrt{3}x+i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((4 + 2\*x + x^2)\*Sqrt[5 + 2\*x + x^2]),x]

[Out] ((-1/2\*I)\*(ArcTanh[(4 - I\*Sqrt[3] - I\*Sqrt[3]\*x)/Sqrt[5 + 2\*x + x^2]] - ArcTanh[(4 + I\*Sqrt[3] + I\*Sqrt[3]\*x)/Sqrt[5 + 2\*x + x^2]]))/Sqrt[3]

**IntegrateAlgebraic [A]** time = 0.28, size = 55, normalized size = 1.96

$$\frac{\tan^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{(x+1)\sqrt{x^2+2x+5}}{\sqrt{3}} + \frac{2x}{\sqrt{3}} + \frac{4}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((4 + 2\*x + x^2)\*Sqrt[5 + 2\*x + x^2]),x]

[Out] -(ArcTan[4/Sqrt[3] + (2\*x)/Sqrt[3] + x^2/Sqrt[3] - ((1 + x)\*Sqrt[5 + 2\*x + x^2])/Sqrt[3]]/Sqrt[3])

**fricas [A]** time = 0.40, size = 38, normalized size = 1.36

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}(x+1) - \frac{1}{3}\sqrt{3}(x^2+2x+4)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+2\*x+4)/(x^2+2\*x+5)^(1/2),x, algorithm="fricas")

[Out] 1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*sqrt(x^2 + 2\*x + 5)\*(x + 1) - 1/3\*sqrt(3)\*(x^2 + 2\*x + 4))

**giac [B]** time = 0.22, size = 52, normalized size = 1.86

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x - \sqrt{x^2+2x+5} + 2\right)\right) + \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x - \sqrt{x^2+2x+5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+2\*x+4)/(x^2+2\*x+5)^(1/2),x, algorithm="giac")

[Out] -1/3\*sqrt(3)\*arctan(-1/3\*sqrt(3)\*(x - sqrt(x^2 + 2\*x + 5) + 2)) + 1/3\*sqrt(3)\*arctan(-1/3\*sqrt(3)\*(x - sqrt(x^2 + 2\*x + 5)))

**maple [A]** time = 0.02, size = 27, normalized size = 0.96

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}(2x+2)}{6\sqrt{x^2+2x+5}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+2\*x+4)/(x^2+2\*x+5)^(1/2),x)

[Out] 1/3\*3^(1/2)\*arctan(1/6\*3^(1/2)/(x^2+2\*x+5)^(1/2)\*(2\*x+2))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+2x+5}(x^2+2x+4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+2\*x+4)/(x^2+2\*x+5)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^2 + 2\*x + 5)\*(x^2 + 2\*x + 4)), x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(x^2+2x+4)\sqrt{x^2+2x+5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)`

[Out] `int(1/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2 + 2x + 4)\sqrt{x^2 + 2x + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2+2*x+4)/(x**2+2*x+5)**(1/2), x)`

[Out] `Integral(1/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)`



$$3.9 \quad \int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}} dx$$

**Optimal.** Leaf size=48

$$\frac{\sqrt{x^2+1}\sqrt{x^2+2x+1}}{x+1} + \frac{\sqrt{x^2+2x+1}\sinh^{-1}(x)}{x+1}$$

**Rubi [A]** time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {970, 641, 215}

$$\frac{\sqrt{x^2+1}\sqrt{x^2+2x+1}}{x+1} + \frac{\sqrt{x^2+2x+1}\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + 2\*x + x^2]/Sqrt[1 + x^2], x]

[Out] (Sqrt[1 + x^2]\*Sqrt[1 + 2\*x + x^2])/(1 + x) + (Sqrt[1 + 2\*x + x^2]\*ArcSinh[x])/(1 + x)

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 970

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Dist[(a + b\*x + c\*x^2)^FracPart[p]/((4\*c)^IntPart[p]\*(b + 2\*c\*x)^(2\*FracPart[p])), Int[(b + 2\*c\*x)^(2\*p)\*(d + f\*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, p, q}, x] && EqQ[b^2 - 4\*a\*c, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}} dx &= \frac{\sqrt{1+2x+x^2} \int \frac{2+2x}{\sqrt{1+x^2}} dx}{2+2x} \\ &= \frac{\sqrt{1+x^2}\sqrt{1+2x+x^2}}{1+x} + \frac{(2\sqrt{1+2x+x^2}) \int \frac{1}{\sqrt{1+x^2}} dx}{2+2x} \\ &= \frac{\sqrt{1+x^2}\sqrt{1+2x+x^2}}{1+x} + \frac{\sqrt{1+2x+x^2}\sinh^{-1}(x)}{1+x} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 27, normalized size = 0.56

$$\frac{\sqrt{(x+1)^2} \left( \sqrt{x^2+1} + \sinh^{-1}(x) \right)}{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + 2\*x + x^2]/Sqrt[1 + x^2],x]

[Out] (Sqrt[(1 + x)^2]\*(Sqrt[1 + x^2] + ArcSinh[x]))/(1 + x)

**IntegrateAlgebraic [A]** time = 0.14, size = 41, normalized size = 0.85

$$\frac{\sqrt{(x+1)^2} \left( \sqrt{x^2+1} - \log\left(\sqrt{x^2+1} - x\right) \right)}{x+1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + 2\*x + x^2]/Sqrt[1 + x^2],x]

[Out] (Sqrt[(1 + x)^2]\*(Sqrt[1 + x^2] - Log[-x + Sqrt[1 + x^2]]))/(1 + x)

**fricas [A]** time = 0.40, size = 22, normalized size = 0.46

$$\sqrt{x^2+1} - \log\left(-x + \sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")

[Out] sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))

**giac [A]** time = 0.23, size = 49, normalized size = 1.02

$$-\left(\sqrt{2} - \log\left(\sqrt{2} + 1\right)\right)\operatorname{sgn}(x+1) - \log\left(-x + \sqrt{x^2+1}\right)\operatorname{sgn}(x+1) + \sqrt{x^2+1}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")

[Out] -(sqrt(2) - log(sqrt(2) + 1))\*sgn(x + 1) - log(-x + sqrt(x^2 + 1))\*sgn(x + 1) + sqrt(x^2 + 1)\*sgn(x + 1)

**maple [C]** time = 0.05, size = 16, normalized size = 0.33

$$\left(\operatorname{arcsinh}(x) + \sqrt{x^2+1}\right)\operatorname{csgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x)

[Out] csgn(1+x)\*(arcsinh(x)+(x^2+1)^(1/2))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt((x + 1)^2)/sqrt(x^2 + 1), x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x + 1)^2)^(1/2)/(x^2 + 1)^(1/2), x)`

[Out] `int(((x + 1)^2)^(1/2)/(x^2 + 1)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((1+x)**2)**(1/2)/(x**2+1)**(1/2), x)`

[Out] `Integral(sqrt((x + 1)**2)/sqrt(x**2 + 1), x)`

$$3.10 \quad \int \frac{1}{(-1+x^2)^2 \sqrt{-1+x+x^2}} dx$$

**Optimal.** Leaf size=70

$$\frac{\sqrt{x^2+x-1}}{2(1-x^2)} - \frac{1}{8} \tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right) - \frac{5}{8} \tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)$$

**Rubi [A]** time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {976, 1033, 724, 206, 204}

$$\frac{\sqrt{x^2+x-1}}{2(1-x^2)} - \frac{1}{8} \tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right) - \frac{5}{8} \tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/((-1 + x^2)^2\*Sqrt[-1 + x + x^2]),x]

[Out] Sqrt[-1 + x + x^2]/(2\*(1 - x^2)) - ArcTan[(3 + x)/(2\*Sqrt[-1 + x + x^2])]/8 - (5\*ArcTanh[(1 - 3\*x)/(2\*Sqrt[-1 + x + x^2])])/8

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 976

Int[((a\_.) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e + c\*(2\*c^2\*d - c\*(2\*a\*f))\*x)\*(a + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^(q+1))/((-4\*a\*c)\*(a\*c\*e^2 + (c\*d - a\*f)^2)\*(p+1)), x] - Dist[1/((-4\*a\*c)\*(a\*c\*e^2 + (c\*d - a\*f)^2)\*(p+1)), Int[(a + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (-(a\*e))\*(c\*e))\*(p+1) - (2\*c^2\*d - c\*(2\*a\*f))\*(a\*f\*(p+1) - c\*d\*(p+2)) - e\*(-2\*a\*c^2\*e)\*(p+q+2) + (2\*f\*(2\*a\*c^2\*e)\*(p+q+2) - (2\*c^2\*d - c\*(2\*a\*f))\*(-(c\*e\*(2\*p+q+4)))]\*x + c\*f\*(2\*c^2\*d - c\*(2\*a\*f))\*(2\*p+2\*q+5)\*x^2, x], x] /; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[a\*c\*e^2 + (c\*d - a\*f)^2, 0] && (!(IntegerQ[p] && ILtQ[q, -1])) && !IGtQ[q, 0]

#### Rule 1033

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[-(a\*c), 2]}, Dist[h/2 + (c\*g)/(2\*q

), Int[1/((-q + c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x] + Dist[h/2 - (c\*g)/(2\*q), Int[1/((q + c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4\*d\*f, 0] && PosQ[-(a\*c)]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-1+x^2)^2 \sqrt{-1+x+x^2}} dx &= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)} - \frac{1}{4} \int \frac{3+2x}{(-1+x^2) \sqrt{-1+x+x^2}} dx \\ &= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)} + \frac{1}{8} \int \frac{1}{(1+x) \sqrt{-1+x+x^2}} dx - \frac{5}{8} \int \frac{1}{(-1+x) \sqrt{-1+x+x^2}} dx \\ &= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)} - \frac{1}{4} \text{Subst} \left( \int \frac{1}{-4-x^2} dx, x, \frac{-3-x}{\sqrt{-1+x+x^2}} \right) + \frac{5}{4} \text{Subst} \left( \int \frac{1}{-4-x^2} dx, x, \frac{-3-x}{\sqrt{-1+x+x^2}} \right) \\ &= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)} - \frac{1}{8} \tan^{-1} \left( \frac{3+x}{2\sqrt{-1+x+x^2}} \right) - \frac{5}{8} \tanh^{-1} \left( \frac{1-3x}{2\sqrt{-1+x+x^2}} \right) \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 66, normalized size = 0.94

$$\frac{1}{8} \left( -\frac{4\sqrt{x^2+x-1}}{x^2-1} - \tan^{-1} \left( \frac{x+3}{2\sqrt{x^2+x-1}} \right) - 5 \tanh^{-1} \left( \frac{1-3x}{2\sqrt{x^2+x-1}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((-1 + x^2)^2\*Sqrt[-1 + x + x^2]),x]

[Out] ((-4\*Sqrt[-1 + x + x^2])/(-1 + x^2) - ArcTan[(3 + x)/(2\*Sqrt[-1 + x + x^2])] - 5\*ArcTanh[(1 - 3\*x)/(2\*Sqrt[-1 + x + x^2])])/8

**IntegrateAlgebraic [A]** time = 0.28, size = 62, normalized size = 0.89

$$-\frac{\sqrt{x^2+x-1}}{2(x^2-1)} - \frac{1}{4} \tan^{-1} \left( -\sqrt{x^2+x-1} + x + 1 \right) + \frac{5}{4} \tanh^{-1} \left( \sqrt{x^2+x-1} - x + 1 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((-1 + x^2)^2\*Sqrt[-1 + x + x^2]),x]

[Out] -1/2\*Sqrt[-1 + x + x^2]/(-1 + x^2) - ArcTan[1 + x - Sqrt[-1 + x + x^2]]/4 + (5\*ArcTanh[1 - x + Sqrt[-1 + x + x^2]])/4

**fricas [A]** time = 0.41, size = 82, normalized size = 1.17

$$\frac{2(x^2-1) \arctan(-x + \sqrt{x^2+x-1} - 1) + 5(x^2-1) \log(-x + \sqrt{x^2+x-1} + 2) - 5(x^2-1) \log(-x + \sqrt{x^2+x-1}) - 4\sqrt{x^2+x-1}}{8(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="fricas")

[Out] 1/8\*(2\*(x^2 - 1)\*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5\*(x^2 - 1)\*log(-x + sqrt(x^2 + x - 1) + 2) - 5\*(x^2 - 1)\*log(-x + sqrt(x^2 + x - 1)) - 4\*sqrt(x^2 + x - 1))/(x^2 - 1)

**giac [B]** time = 0.26, size = 143, normalized size = 2.04

$$\frac{2(x - \sqrt{x^2+x-1})^3 + 3(x - \sqrt{x^2+x-1})^2 - x + \sqrt{x^2+x-1} - 1}{2((x - \sqrt{x^2+x-1})^4 - 2(x - \sqrt{x^2+x-1})^2 - 4x + 4\sqrt{x^2+x-1})} + \frac{1}{4} \arctan(-x + \sqrt{x^2+x-1} - 1) + \frac{5}{8} \log(-x + \sqrt{x^2+x-1} + 2) - \frac{5}{8} \log(-x + \sqrt{x^2+x-1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{2} * (2 * (x - \sqrt{x^2 + x - 1})^3 + 3 * (x - \sqrt{x^2 + x - 1})^2 - x + \sqrt{x^2 + x - 1} - 1) / ((x - \sqrt{x^2 + x - 1})^4 - 2 * (x - \sqrt{x^2 + x - 1})^2 - 4 * x + 4 * \sqrt{x^2 + x - 1}) + \frac{1}{4} * \arctan(-x + \sqrt{x^2 + x - 1} - 1) + \frac{5}{8} * \log(\text{abs}(-x + \sqrt{x^2 + x - 1} + 2)) - \frac{5}{8} * \log(\text{abs}(-x + \sqrt{x^2 + x - 1}))$

**maple** [A] time = 0.02, size = 84, normalized size = 1.20

$$\frac{5 \operatorname{arctanh}\left(\frac{3x-1}{2\sqrt{3x+(x-1)^2-2}}\right)}{8} + \frac{\operatorname{arctan}\left(\frac{-x-3}{2\sqrt{-x+(x+1)^2-2}}\right)}{8} + \frac{\sqrt{-x+(x+1)^2-2}}{4x+4} - \frac{\sqrt{3x+(x-1)^2-2}}{4(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2-1)^2/(x^2+x-1)^(1/2),x)

[Out]  $\frac{1}{4} / (x+1) * ((x+1)^2 - 2 - x)^{(1/2)} + \frac{1}{8} * \arctan(1/2 * (-3-x) / ((x+1)^2 - 2 - x)^{(1/2)}) + \frac{5}{8} * \operatorname{arctanh}(1/2 * (-1+3*x) / ((x-1)^2 + 3*x - 2)^{(1/2)}) - \frac{1}{4} / (x-1) * ((x-1)^2 + 3*x - 2)^{(1/2)}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2 + x - 1} (x^2 - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^2 + x - 1)\*(x^2 - 1)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^2 - 1)^2 \sqrt{x^2 + x - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 - 1)^2\*(x + x^2 - 1)^(1/2)),x)

[Out] int(1/((x^2 - 1)^2\*(x + x^2 - 1)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-1)^2 (x+1)^2 \sqrt{x^2 + x - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x\*\*2-1)\*\*2/(x\*\*2+x-1)\*\*(1/2),x)

[Out] Integral(1/((x - 1)\*\*2\*(x + 1)\*\*2\*sqrt(x\*\*2 + x - 1)), x)

$$3.11 \quad \int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} dx$$

**Optimal.** Leaf size=98

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) - \frac{1}{2} \sin^{-1}(x+2)$$

**Rubi [A]** time = 0.21, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {989, 619, 216, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) - \frac{1}{2} \sin^{-1}(x+2)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-3 - 4\*x - x^2]/(3 + 4\*x + 2\*x^2), x]

[Out] -ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4\*x - x^2])/Sqrt[2]]/Sqrt[2] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4\*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh[x/Sqrt[-3 - 4\*x - x^2]]/2

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 216**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

**Rule 618**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 619**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol] := Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 - 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[g, Subst[Int[1/(a + (c*d - a*f)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := -Dist[(2*h*d - g*e)/e, Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))
```

Rubi steps



$$\begin{aligned}
\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} dx &= -\left(\frac{1}{2} \int \frac{1}{\sqrt{-3-4x-x^2}} dx\right) - \frac{1}{2} \int \frac{3+4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
&= \frac{1}{4} \text{Subst} \left( \int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -4-2x \right) + \frac{1}{2} \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx + \frac{3}{2} \int \frac{1}{3-3x^2} dx \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{1}{4} \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx + \frac{1}{4} \int -\frac{4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \tanh^{-1} \left( \frac{x}{\sqrt{-3-4x-x^2}} \right) + \frac{3}{2} \text{Subst} \left( \int \frac{1}{3-3x^2} dx, x, \frac{x}{\sqrt{-3-4x-x^2}} \right) \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1} \left( \frac{x}{\sqrt{-3-4x-x^2}} \right) - 8 \text{Subst} \left( \int \frac{1+3x^2}{-4-8x^2-36x^4} dx, x, \frac{x}{\sqrt{-3-4x-x^2}} \right) \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1} \left( \frac{x}{\sqrt{-3-4x-x^2}} \right) + \frac{1}{3} \text{Subst} \left( \int \frac{1}{\frac{1}{3} - \frac{2x}{3} + x^2} dx, x, \frac{x}{\sqrt{-3-4x-x^2}} \right) \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1} \left( \frac{x}{\sqrt{-3-4x-x^2}} \right) - \frac{2}{3} \text{Subst} \left( \int \frac{1}{-\frac{8}{9} - x^2} dx, x, \frac{2}{3} \left( -1 + \frac{x}{\sqrt{-3-4x-x^2}} \right) \right) \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{\tan^{-1} \left( \frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{\sqrt{2}} + \frac{\tan^{-1} \left( \frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1} \left( \frac{x}{\sqrt{-3-4x-x^2}} \right)
\end{aligned}$$

**Mathematica [C]** time = 0.39, size = 159, normalized size = 1.62

$$\frac{1}{4} \left( -i\sqrt{1-2i\sqrt{2}} \tanh^{-1} \left( \frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) + i\sqrt{1+2i\sqrt{2}} \tanh^{-1} \left( \frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) - 2\sin^{-1}(x+2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-3 - 4\*x - x^2]/(3 + 4\*x + 2\*x^2), x]

[Out] (-2\*ArcSin[2 + x] - I\*Sqrt[1 - (2\*I)\*Sqrt[2]]\*ArcTanh[(2 + (2\*I)\*Sqrt[2] + 2\*x + I\*Sqrt[2]\*x)/(Sqrt[2 - (4\*I)\*Sqrt[2]]\*Sqrt[-3 - 4\*x - x^2])] + I\*Sqrt[1 + (2\*I)\*Sqrt[2]]\*ArcTanh[(2 - (2\*I)\*Sqrt[2] + (2 - I\*Sqrt[2])\*x)/(Sqrt[2 + (4\*I)\*Sqrt[2]]\*Sqrt[-3 - 4\*x - x^2])])/4

**IntegrateAlgebraic [A]** time = 0.29, size = 80, normalized size = 0.82

$$\frac{\tan^{-1} \left( \frac{\sqrt{2}x + \frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}} \right)}{\sqrt{2}} + \tan^{-1} \left( \frac{\sqrt{-x^2-4x-3}}{x+3} \right) - \frac{1}{2} \tanh^{-1} \left( \frac{x}{\sqrt{-x^2-4x-3}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-3 - 4\*x - x^2]/(3 + 4\*x + 2\*x^2), x]

[Out] ArcTan[(3/Sqrt[2] + Sqrt[2]\*x)/Sqrt[-3 - 4\*x - x^2]]/Sqrt[2] + ArcTan[Sqrt[-3 - 4\*x - x^2]/(3 + x)] - ArcTanh[x/Sqrt[-3 - 4\*x - x^2]]/2

**fricas [A]** time = 0.42, size = 161, normalized size = 1.64

$$-\frac{1}{4} \sqrt{2} \arctan \left( \frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)} \right) - \frac{1}{4} \sqrt{2} \arctan \left( -\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)} \right) + \frac{1}{2} \arctan \left( \frac{\sqrt{-x^2-4x-3}(x+2)}{x^2+4x+3} \right) + \frac{1}{8} \log \left( -\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2} \right) - \frac{1}{8} \log \left( \frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-4\*x-3)^(1/2)/(2\*x^2+4\*x+3), x, algorithm="fricas")

[Out]  $-1/4*\sqrt{2}*\arctan(1/2*(\sqrt{2}*x + 3*\sqrt{2}*\sqrt{-x^2 - 4*x - 3})/(2*x + 3)) - 1/4*\sqrt{2}*\arctan(-1/2*(\sqrt{2}*x - 3*\sqrt{2}*\sqrt{-x^2 - 4*x - 3})/(2*x + 3)) + 1/2*\arctan(\sqrt{-x^2 - 4*x - 3}*(x + 2)/(x^2 + 4*x + 3)) + 1/8*\log(-(2*\sqrt{-x^2 - 4*x - 3}*x + 4*x + 3)/x^2) - 1/8*\log((2*\sqrt{-x^2 - 4*x - 3})*x - 4*x - 3)/x^2)$

**giac** [B] time = 0.21, size = 171, normalized size = 1.74

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2}\right)+1\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}\right)+1\right)-\frac{1}{2}\arcsin(x+2)-\frac{1}{4}\log\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2}+\frac{3(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2}+1\right)+\frac{1}{4}\log\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2}+\frac{(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-4\*x-3)^(1/2)/(2\*x^2+4\*x+3), x, algorithm="giac")

[Out]  $-1/2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(3*(\sqrt{-x^2 - 4*x - 3} - 1)/(x + 2) + 1)) - 1/2*\sqrt{2}*\arctan(1/2*\sqrt{2}*((\sqrt{-x^2 - 4*x - 3} - 1)/(x + 2) + 1)) - 1/2*\arcsin(x + 2) - 1/4*\log(2*(\sqrt{-x^2 - 4*x - 3} - 1)/(x + 2) + 3*(\sqrt{-x^2 - 4*x - 3} - 1)^2/(x + 2)^2 + 1) + 1/4*\log(2*(\sqrt{-x^2 - 4*x - 3} - 1)/(x + 2) + (\sqrt{-x^2 - 4*x - 3} - 1)^2/(x + 2)^2 + 3)$

**maple** [B] time = 0.03, size = 341, normalized size = 3.48

$$\frac{\arcsin(x+2)}{2} - \frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{(-x-3)^2}-12}\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{(-x-3)^2}-12}\sqrt{2}}{6}\right)}{3\sqrt{\frac{x^2}{(-x-3)^2}-4}\left(\frac{x}{-x-3}+1\right)} + \frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{(-x-3)^2}-12}\left(-\operatorname{arctanh}\left(\frac{3x}{(-x-3)\sqrt{\frac{3x^2}{(-x-3)^2}-12}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{(-x-3)^2}-12}\sqrt{2}}{6}\right)\right)}{12\sqrt{\frac{x^2}{(-x-3)^2}-4}\left(\frac{x}{-x-3}+1\right)} + \frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{(-x-3)^2}-12}\left(\operatorname{arctanh}\left(\frac{3x}{(-x-3)\sqrt{\frac{3x^2}{(-x-3)^2}-12}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{(-x-3)^2}-12}\sqrt{2}}{6}\right)\right)}{6\sqrt{\frac{x^2}{(-x-3)^2}-4}\left(\frac{x}{-x-3}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2-4\*x-3)^(1/2)/(2\*x^2+4\*x+3), x)

[Out]  $-1/2*\arcsin(2+x)+1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*\arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-\operatorname{arctanh}(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))-1/3*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*\arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*\arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+\operatorname{arctanh}(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2 - 4x - 3}}{2x^2 + 4x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-4\*x-3)^(1/2)/(2\*x^2+4\*x+3), x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 - 4\*x - 3)/(2\*x^2 + 4\*x + 3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-x^2 - 4x - 3}}{2x^2 + 4x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((- 4*x - x^2 - 3)^(1/2)/(4*x + 2*x^2 + 3), x)`

[Out] `int((- 4*x - x^2 - 3)^(1/2)/(4*x + 2*x^2 + 3), x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x+1)(x+3)}}{2x^2 + 4x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2-4*x-3)**(1/2)/(2*x**2+4*x+3), x)`

[Out] `Integral(sqrt(-(x + 1)*(x + 3))/(2*x**2 + 4*x + 3), x)`

$$3.12 \quad \int (3 - x + 2x^2)(2 + 3x + 5x^2)^4 dx$$

**Optimal.** Leaf size=68

$$\frac{1250x^{11}}{11} + \frac{475x^{10}}{2} + \frac{5075x^9}{9} + \frac{3415x^8}{4} + 1176x^7 + \frac{2377x^6}{2} + \frac{5099x^5}{5} + 656x^4 + \frac{1064x^3}{3} + 136x^2 + 48x$$

**Rubi [A]** time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {1657}

$$\frac{1250x^{11}}{11} + \frac{475x^{10}}{2} + \frac{5075x^9}{9} + \frac{3415x^8}{4} + 1176x^7 + \frac{2377x^6}{2} + \frac{5099x^5}{5} + 656x^4 + \frac{1064x^3}{3} + 136x^2 + 48x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^4, x]

[Out] 48\*x + 136\*x^2 + (1064\*x^3)/3 + 656\*x^4 + (5099\*x^5)/5 + (2377\*x^6)/2 + 1176\*x^7 + (3415\*x^8)/4 + (5075\*x^9)/9 + (475\*x^10)/2 + (1250\*x^11)/11

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)(2 + 3x + 5x^2)^4 dx &= \int (48 + 272x + 1064x^2 + 2624x^3 + 5099x^4 + 7131x^5 + 8232x^6 + 6830x^7 \\ &\quad + 48x + 136x^2 + \frac{1064x^3}{3} + 656x^4 + \frac{5099x^5}{5} + \frac{2377x^6}{2} + 1176x^7 + \frac{3415x^8}{4} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 68, normalized size = 1.00

$$\frac{1250x^{11}}{11} + \frac{475x^{10}}{2} + \frac{5075x^9}{9} + \frac{3415x^8}{4} + 1176x^7 + \frac{2377x^6}{2} + \frac{5099x^5}{5} + 656x^4 + \frac{1064x^3}{3} + 136x^2 + 48x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^4, x]

[Out] 48\*x + 136\*x^2 + (1064\*x^3)/3 + 656\*x^4 + (5099\*x^5)/5 + (2377\*x^6)/2 + 1176\*x^7 + (3415\*x^8)/4 + (5075\*x^9)/9 + (475\*x^10)/2 + (1250\*x^11)/11

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - x + 2x^2)(2 + 3x + 5x^2)^4 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^4, x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^4, x]

**fricas [A]** time = 0.34, size = 54, normalized size = 0.79

$$\frac{1250}{11}x^{11} + \frac{475}{2}x^{10} + \frac{5075}{9}x^9 + \frac{3415}{4}x^8 + 1176x^7 + \frac{2377}{2}x^6 + \frac{5099}{5}x^5 + 656x^4 + \frac{1064}{3}x^3 + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^4,x, algorithm="fricas")

[Out] 1250/11\*x^11 + 475/2\*x^10 + 5075/9\*x^9 + 3415/4\*x^8 + 1176\*x^7 + 2377/2\*x^6 + 5099/5\*x^5 + 656\*x^4 + 1064/3\*x^3 + 136\*x^2 + 48\*x

**giac** [A] time = 0.21, size = 54, normalized size = 0.79

$$\frac{1250}{11}x^{11} + \frac{475}{2}x^{10} + \frac{5075}{9}x^9 + \frac{3415}{4}x^8 + 1176x^7 + \frac{2377}{2}x^6 + \frac{5099}{5}x^5 + 656x^4 + \frac{1064}{3}x^3 + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^4,x, algorithm="giac")

[Out] 1250/11\*x^11 + 475/2\*x^10 + 5075/9\*x^9 + 3415/4\*x^8 + 1176\*x^7 + 2377/2\*x^6 + 5099/5\*x^5 + 656\*x^4 + 1064/3\*x^3 + 136\*x^2 + 48\*x

**maple** [A] time = 0.00, size = 55, normalized size = 0.81

$$\frac{1250}{11}x^{11} + \frac{475}{2}x^{10} + \frac{5075}{9}x^9 + \frac{3415}{4}x^8 + 1176x^7 + \frac{2377}{2}x^6 + \frac{5099}{5}x^5 + 656x^4 + \frac{1064}{3}x^3 + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^4,x)

[Out] 48\*x+136\*x^2+1064/3\*x^3+656\*x^4+5099/5\*x^5+2377/2\*x^6+1176\*x^7+3415/4\*x^8+5075/9\*x^9+475/2\*x^10+1250/11\*x^11

**maxima** [A] time = 0.46, size = 54, normalized size = 0.79

$$\frac{1250}{11}x^{11} + \frac{475}{2}x^{10} + \frac{5075}{9}x^9 + \frac{3415}{4}x^8 + 1176x^7 + \frac{2377}{2}x^6 + \frac{5099}{5}x^5 + 656x^4 + \frac{1064}{3}x^3 + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^4,x, algorithm="maxima")

[Out] 1250/11\*x^11 + 475/2\*x^10 + 5075/9\*x^9 + 3415/4\*x^8 + 1176\*x^7 + 2377/2\*x^6 + 5099/5\*x^5 + 656\*x^4 + 1064/3\*x^3 + 136\*x^2 + 48\*x

**mupad** [B] time = 0.06, size = 54, normalized size = 0.79

$$\frac{1250x^{11}}{11} + \frac{475x^{10}}{2} + \frac{5075x^9}{9} + \frac{3415x^8}{4} + 1176x^7 + \frac{2377x^6}{2} + \frac{5099x^5}{5} + 656x^4 + \frac{1064x^3}{3} + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)\*(3\*x + 5\*x^2 + 2)^4,x)

[Out] 48\*x + 136\*x^2 + (1064\*x^3)/3 + 656\*x^4 + (5099\*x^5)/5 + (2377\*x^6)/2 + 1176\*x^7 + (3415\*x^8)/4 + (5075\*x^9)/9 + (475\*x^10)/2 + (1250\*x^11)/11

**sympy** [A] time = 0.09, size = 65, normalized size = 0.96

$$\frac{1250x^{11}}{11} + \frac{475x^{10}}{2} + \frac{5075x^9}{9} + \frac{3415x^8}{4} + 1176x^7 + \frac{2377x^6}{2} + \frac{5099x^5}{5} + 656x^4 + \frac{1064x^3}{3} + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*(5\*x\*\*2+3\*x+2)\*\*4,x)

[Out] 1250\*x\*\*11/11 + 475\*x\*\*10/2 + 5075\*x\*\*9/9 + 3415\*x\*\*8/4 + 1176\*x\*\*7 + 2377\*x\*\*6/2 + 5099\*x\*\*5/5 + 656\*x\*\*4 + 1064\*x\*\*3/3 + 136\*x\*\*2 + 48\*x

$$3.13 \quad \int (3 - x + 2x^2)(2 + 3x + 5x^2)^3 dx$$

**Optimal.** Leaf size=56

$$\frac{250x^9}{9} + \frac{325x^8}{8} + \frac{720x^7}{7} + 134x^6 + \frac{876x^5}{5} + \frac{579x^4}{4} + \frac{322x^3}{3} + 50x^2 + 24x$$

**Rubi [A]** time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {1657}

$$\frac{250x^9}{9} + \frac{325x^8}{8} + \frac{720x^7}{7} + 134x^6 + \frac{876x^5}{5} + \frac{579x^4}{4} + \frac{322x^3}{3} + 50x^2 + 24x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] 24\*x + 50\*x^2 + (322\*x^3)/3 + (579\*x^4)/4 + (876\*x^5)/5 + 134\*x^6 + (720\*x^7)/7 + (325\*x^8)/8 + (250\*x^9)/9

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)(2 + 3x + 5x^2)^3 dx &= \int (24 + 100x + 322x^2 + 579x^3 + 876x^4 + 804x^5 + 720x^6 + 325x^7 + 250x^8 + 24x^9) dx \\ &= 24x + 50x^2 + \frac{322x^3}{3} + \frac{579x^4}{4} + \frac{876x^5}{5} + 134x^6 + \frac{720x^7}{7} + \frac{325x^8}{8} + \frac{250x^9}{9} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 56, normalized size = 1.00

$$\frac{250x^9}{9} + \frac{325x^8}{8} + \frac{720x^7}{7} + 134x^6 + \frac{876x^5}{5} + \frac{579x^4}{4} + \frac{322x^3}{3} + 50x^2 + 24x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] 24\*x + 50\*x^2 + (322\*x^3)/3 + (579\*x^4)/4 + (876\*x^5)/5 + 134\*x^6 + (720\*x^7)/7 + (325\*x^8)/8 + (250\*x^9)/9

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - x + 2x^2)(2 + 3x + 5x^2)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^3, x]

**fricas [A]** time = 0.34, size = 44, normalized size = 0.79

$$\frac{250}{9}x^9 + \frac{325}{8}x^8 + \frac{720}{7}x^7 + 134x^6 + \frac{876}{5}x^5 + \frac{579}{4}x^4 + \frac{322}{3}x^3 + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 250/9\*x^9 + 325/8\*x^8 + 720/7\*x^7 + 134\*x^6 + 876/5\*x^5 + 579/4\*x^4 + 322/3\*x^3 + 50\*x^2 + 24\*x

**giac** [A] time = 0.18, size = 44, normalized size = 0.79

$$\frac{250}{9}x^9 + \frac{325}{8}x^8 + \frac{720}{7}x^7 + 134x^6 + \frac{876}{5}x^5 + \frac{579}{4}x^4 + \frac{322}{3}x^3 + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 250/9\*x^9 + 325/8\*x^8 + 720/7\*x^7 + 134\*x^6 + 876/5\*x^5 + 579/4\*x^4 + 322/3\*x^3 + 50\*x^2 + 24\*x

**maple** [A] time = 0.00, size = 45, normalized size = 0.80

$$\frac{250}{9}x^9 + \frac{325}{8}x^8 + \frac{720}{7}x^7 + 134x^6 + \frac{876}{5}x^5 + \frac{579}{4}x^4 + \frac{322}{3}x^3 + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^3,x)

[Out] 24\*x+50\*x^2+322/3\*x^3+579/4\*x^4+876/5\*x^5+134\*x^6+720/7\*x^7+325/8\*x^8+250/9\*x^9

**maxima** [A] time = 0.45, size = 44, normalized size = 0.79

$$\frac{250}{9}x^9 + \frac{325}{8}x^8 + \frac{720}{7}x^7 + 134x^6 + \frac{876}{5}x^5 + \frac{579}{4}x^4 + \frac{322}{3}x^3 + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 250/9\*x^9 + 325/8\*x^8 + 720/7\*x^7 + 134\*x^6 + 876/5\*x^5 + 579/4\*x^4 + 322/3\*x^3 + 50\*x^2 + 24\*x

**mupad** [B] time = 0.03, size = 44, normalized size = 0.79

$$\frac{250x^9}{9} + \frac{325x^8}{8} + \frac{720x^7}{7} + 134x^6 + \frac{876x^5}{5} + \frac{579x^4}{4} + \frac{322x^3}{3} + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)\*(3\*x + 5\*x^2 + 2)^3,x)

[Out] 24\*x + 50\*x^2 + (322\*x^3)/3 + (579\*x^4)/4 + (876\*x^5)/5 + 134\*x^6 + (720\*x^7)/7 + (325\*x^8)/8 + (250\*x^9)/9

**sympy** [A] time = 0.08, size = 53, normalized size = 0.95

$$\frac{250x^9}{9} + \frac{325x^8}{8} + \frac{720x^7}{7} + 134x^6 + \frac{876x^5}{5} + \frac{579x^4}{4} + \frac{322x^3}{3} + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] 250\*x\*\*9/9 + 325\*x\*\*8/8 + 720\*x\*\*7/7 + 134\*x\*\*6 + 876\*x\*\*5/5 + 579\*x\*\*4/4 + 322\*x\*\*3/3 + 50\*x\*\*2 + 24\*x

$$3.14 \quad \int (3 - x + 2x^2)(2 + 3x + 5x^2)^2 dx$$

Optimal. Leaf size=44

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

**Rubi [A]** time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {1657}

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] 12\*x + 16\*x^2 + (83\*x^3)/3 + (85\*x^4)/4 + (103\*x^5)/5 + (35\*x^6)/6 + (50\*x^7)/7

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)(2 + 3x + 5x^2)^2 dx &= \int (12 + 32x + 83x^2 + 85x^3 + 103x^4 + 35x^5 + 50x^6) dx \\ &= 12x + 16x^2 + \frac{83x^3}{3} + \frac{85x^4}{4} + \frac{103x^5}{5} + \frac{35x^6}{6} + \frac{50x^7}{7} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 44, normalized size = 1.00

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] 12\*x + 16\*x^2 + (83\*x^3)/3 + (85\*x^4)/4 + (103\*x^5)/5 + (35\*x^6)/6 + (50\*x^7)/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - x + 2x^2)(2 + 3x + 5x^2)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^2, x]

**fricas [A]** time = 0.34, size = 34, normalized size = 0.77

$$\frac{50}{7}x^7 + \frac{35}{6}x^6 + \frac{103}{5}x^5 + \frac{85}{4}x^4 + \frac{83}{3}x^3 + 16x^2 + 12x$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 50/7\*x^7 + 35/6\*x^6 + 103/5\*x^5 + 85/4\*x^4 + 83/3\*x^3 + 16\*x^2 + 12\*x

**giac** [A] time = 0.21, size = 34, normalized size = 0.77

$$\frac{50}{7}x^7 + \frac{35}{6}x^6 + \frac{103}{5}x^5 + \frac{85}{4}x^4 + \frac{83}{3}x^3 + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 50/7\*x^7 + 35/6\*x^6 + 103/5\*x^5 + 85/4\*x^4 + 83/3\*x^3 + 16\*x^2 + 12\*x

**maple** [A] time = 0.00, size = 35, normalized size = 0.80

$$\frac{50}{7}x^7 + \frac{35}{6}x^6 + \frac{103}{5}x^5 + \frac{85}{4}x^4 + \frac{83}{3}x^3 + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^2,x)

[Out] 12\*x+16\*x^2+83/3\*x^3+85/4\*x^4+103/5\*x^5+35/6\*x^6+50/7\*x^7

**maxima** [A] time = 0.43, size = 34, normalized size = 0.77

$$\frac{50}{7}x^7 + \frac{35}{6}x^6 + \frac{103}{5}x^5 + \frac{85}{4}x^4 + \frac{83}{3}x^3 + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 50/7\*x^7 + 35/6\*x^6 + 103/5\*x^5 + 85/4\*x^4 + 83/3\*x^3 + 16\*x^2 + 12\*x

**mupad** [B] time = 0.03, size = 34, normalized size = 0.77

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)\*(3\*x + 5\*x^2 + 2)^2,x)

[Out] 12\*x + 16\*x^2 + (83\*x^3)/3 + (85\*x^4)/4 + (103\*x^5)/5 + (35\*x^6)/6 + (50\*x^7)/7

**sympy** [A] time = 0.08, size = 41, normalized size = 0.93

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*(5\*x\*\*2+3\*x+2)\*\*2,x)

[Out] 50\*x\*\*7/7 + 35\*x\*\*6/6 + 103\*x\*\*5/5 + 85\*x\*\*4/4 + 83\*x\*\*3/3 + 16\*x\*\*2 + 12\*x

### 3.15 $\int (3 - x + 2x^2)(2 + 3x + 5x^2) dx$

**Optimal.** Leaf size=30

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

**Rubi [A]** time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {1657}

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2), x]

[Out] 6\*x + (7\*x^2)/2 + (16\*x^3)/3 + x^4/4 + 2\*x^5

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)(2 + 3x + 5x^2) dx &= \int (6 + 7x + 16x^2 + x^3 + 10x^4) dx \\ &= 6x + \frac{7x^2}{2} + \frac{16x^3}{3} + \frac{x^4}{4} + 2x^5 \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 30, normalized size = 1.00

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2), x]

[Out] 6\*x + (7\*x^2)/2 + (16\*x^3)/3 + x^4/4 + 2\*x^5

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - x + 2x^2)(2 + 3x + 5x^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2), x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2), x]

**fricas [A]** time = 0.34, size = 24, normalized size = 0.80

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] 2\*x^5 + 1/4\*x^4 + 16/3\*x^3 + 7/2\*x^2 + 6\*x

**giac** [A] time = 0.18, size = 24, normalized size = 0.80

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] 2\*x^5 + 1/4\*x^4 + 16/3\*x^3 + 7/2\*x^2 + 6\*x

**maple** [A] time = 0.00, size = 25, normalized size = 0.83

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)\*(5\*x^2+3\*x+2),x)

[Out] 6\*x+7/2\*x^2+16/3\*x^3+1/4\*x^4+2\*x^5

**maxima** [A] time = 0.43, size = 24, normalized size = 0.80

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)\*(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] 2\*x^5 + 1/4\*x^4 + 16/3\*x^3 + 7/2\*x^2 + 6\*x

**mupad** [B] time = 0.02, size = 24, normalized size = 0.80

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)\*(3\*x + 5\*x^2 + 2),x)

[Out] 6\*x + (7\*x^2)/2 + (16\*x^3)/3 + x^4/4 + 2\*x^5

**sympy** [A] time = 0.06, size = 26, normalized size = 0.87

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*(5\*x\*\*2+3\*x+2),x)

[Out] 2\*x\*\*5 + x\*\*4/4 + 16\*x\*\*3/3 + 7\*x\*\*2/2 + 6\*x

$$3.16 \quad \int \frac{3-x+2x^2}{2+3x+5x^2} dx$$

**Optimal.** Leaf size=42

$$-\frac{11}{50} \log(5x^2 + 3x + 2) + \frac{2x}{5} + \frac{143 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

**Rubi [A]** time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {1657, 634, 618, 204, 628}

$$-\frac{11}{50} \log(5x^2 + 3x + 2) + \frac{2x}{5} + \frac{143 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2), x]

[Out] (2\*x)/5 + (143\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(25\*Sqrt[31]) - (11\*Log[2 + 3\*x + 5\*x^2])/50

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

#### Rubi steps

$$\begin{aligned}
\int \frac{3-x+2x^2}{2+3x+5x^2} dx &= \int \left( \frac{2}{5} + \frac{11(1-x)}{5(2+3x+5x^2)} \right) dx \\
&= \frac{2x}{5} + \frac{11}{5} \int \frac{1-x}{2+3x+5x^2} dx \\
&= \frac{2x}{5} - \frac{11}{50} \int \frac{3+10x}{2+3x+5x^2} dx + \frac{143}{50} \int \frac{1}{2+3x+5x^2} dx \\
&= \frac{2x}{5} - \frac{11}{50} \log(2+3x+5x^2) - \frac{143}{25} \operatorname{Subst} \left( \int \frac{1}{-31-x^2} dx, x, 3+10x \right) \\
&= \frac{2x}{5} + \frac{143 \tan^{-1} \left( \frac{3+10x}{\sqrt{31}} \right)}{25\sqrt{31}} - \frac{11}{50} \log(2+3x+5x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 42, normalized size = 1.00

$$-\frac{11}{50} \log(5x^2 + 3x + 2) + \frac{2x}{5} + \frac{143 \tan^{-1} \left( \frac{10x+3}{\sqrt{31}} \right)}{25\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2), x]

[Out] (2\*x)/5 + (143\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(25\*Sqrt[31]) - (11\*Log[2 + 3\*x + 5\*x^2])/50

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3-x+2x^2}{2+3x+5x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2), x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2), x]

**fricas [A]** time = 0.39, size = 33, normalized size = 0.79

$$\frac{143}{775} \sqrt{31} \arctan \left( \frac{1}{31} \sqrt{31} (10x + 3) \right) + \frac{2}{5} x - \frac{11}{50} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2), x, algorithm="fricas")

[Out] 143/775\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 2/5\*x - 11/50\*log(5\*x^2 + 3\*x + 2)

**giac [A]** time = 0.18, size = 33, normalized size = 0.79

$$\frac{143}{775} \sqrt{31} \arctan \left( \frac{1}{31} \sqrt{31} (10x + 3) \right) + \frac{2}{5} x - \frac{11}{50} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2), x, algorithm="giac")

[Out] 143/775\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 2/5\*x - 11/50\*log(5\*x^2 + 3\*x + 2)

**maple** [A] time = 0.00, size = 34, normalized size = 0.81

$$\frac{2x}{5} + \frac{143\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{775} - \frac{11 \ln(5x^2 + 3x + 2)}{50}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)/(5\*x^2+3\*x+2),x)

[Out] 2/5\*x-11/50\*ln(5\*x^2+3\*x+2)+143/775\*arctan(1/31\*(3+10\*x)\*31^(1/2))\*31^(1/2)

**maxima** [A] time = 0.97, size = 33, normalized size = 0.79

$$\frac{143}{775} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{2}{5} x - \frac{11}{50} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] 143/775\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 2/5\*x - 11/50\*log(5\*x^2 + 3\*x + 2)

**mupad** [B] time = 3.39, size = 35, normalized size = 0.83

$$\frac{2x}{5} - \frac{11 \ln(5x^2 + 3x + 2)}{50} + \frac{143 \sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{775}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)/(3\*x + 5\*x^2 + 2),x)

[Out] (2\*x)/5 - (11\*log(3\*x + 5\*x^2 + 2))/50 + (143\*31^(1/2)\*atan((10\*31^(1/2)\*x)/31 + (3\*31^(1/2))/31))/775

**sympy** [A] time = 0.14, size = 49, normalized size = 1.17

$$\frac{2x}{5} - \frac{11 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{50} + \frac{143\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{775}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)/(5\*x\*\*2+3\*x+2),x)

[Out] 2\*x/5 - 11\*log(x\*\*2 + 3\*x/5 + 2/5)/50 + 143\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/775

$$3.17 \quad \int \frac{3-x+2x^2}{(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=43

$$\frac{11(13x+7)}{155(5x^2+3x+2)} + \frac{82 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$ , Rules used = {1660, 12, 618, 204}

$$\frac{11(13x+7)}{155(5x^2+3x+2)} + \frac{82 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2)^2,x]

[Out] (11\*(7 + 13\*x))/(155\*(2 + 3\*x + 5\*x^2)) + (82\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(31\*Sqrt[31])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{3-x+2x^2}{(2+3x+5x^2)^2} dx &= \frac{11(7+13x)}{155(2+3x+5x^2)} + \frac{1}{31} \int \frac{41}{2+3x+5x^2} dx \\
&= \frac{11(7+13x)}{155(2+3x+5x^2)} + \frac{41}{31} \int \frac{1}{2+3x+5x^2} dx \\
&= \frac{11(7+13x)}{155(2+3x+5x^2)} - \frac{82}{31} \text{Subst} \left( \int \frac{1}{-31-x^2} dx, x, 3+10x \right) \\
&= \frac{11(7+13x)}{155(2+3x+5x^2)} + \frac{82 \tan^{-1} \left( \frac{3+10x}{\sqrt{31}} \right)}{31\sqrt{31}}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 43, normalized size = 1.00

$$\frac{11(13x+7)}{155(5x^2+3x+2)} + \frac{82 \tan^{-1} \left( \frac{10x+3}{\sqrt{31}} \right)}{31\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2)^2, x]

[Out] (11\*(7 + 13\*x))/(155\*(2 + 3\*x + 5\*x^2)) + (82\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(31\*Sqrt[31])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2)^2, x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2)^2, x]

**fricas [A]** time = 0.39, size = 45, normalized size = 1.05

$$\frac{410 \sqrt{31} (5x^2 + 3x + 2) \arctan \left( \frac{1}{31} \sqrt{31} (10x + 3) \right) + 4433x + 2387}{4805 (5x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 1/4805\*(410\*sqrt(31)\*(5\*x^2 + 3\*x + 2)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 4433\*x + 2387)/(5\*x^2 + 3\*x + 2)

**giac [A]** time = 0.21, size = 36, normalized size = 0.84

$$\frac{82}{961} \sqrt{31} \arctan \left( \frac{1}{31} \sqrt{31} (10x + 3) \right) + \frac{11(13x + 7)}{155(5x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2)^2,x, algorithm="giac")



[Out]  $82/961*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^2 + 3*x + 2)$

**maple** [A] time = 0.00, size = 34, normalized size = 0.79

$$\frac{82\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{961} + \frac{\frac{143x}{775} + \frac{77}{775}}{x^2 + \frac{3}{5}x + \frac{2}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)/(5*x^2+3*x+2)^2,x)`

[Out]  $(143/775*x+77/775)/(x^2+3/5*x+2/5)+82/961*31^{(1/2)}*\arctan(1/31*(10*x+3)*31^{(1/2)})$

**maxima** [A] time = 0.97, size = 36, normalized size = 0.84

$$\frac{82}{961} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{11(13x + 7)}{155(5x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")`

[Out]  $82/961*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^2 + 3*x + 2)$

**mupad** [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{\frac{143x}{775} + \frac{77}{775}}{x^2 + \frac{3x}{5} + \frac{2}{5}} + \frac{82\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{961}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2)^2,x)`

[Out]  $((143*x)/775 + 77/775)/((3*x)/5 + x^2 + 2/5) + (82*31^{(1/2)}*\operatorname{atan}((10*31^{(1/2)}*x)/31 + (3*31^{(1/2)})/31))/961$

**sympy** [A] time = 0.16, size = 42, normalized size = 0.98

$$\frac{143x + 77}{775x^2 + 465x + 310} + \frac{82\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{961}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)/(5*x**2+3*x+2)**2,x)`

[Out]  $(143*x + 77)/(775*x**2 + 465*x + 310) + 82*\sqrt{31}*\operatorname{atan}(10*\sqrt{31}*x/31 + 3*\sqrt{31}/31)/961$

$$3.18 \quad \int \frac{3-x+2x^2}{(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=64

$$\frac{553(10x+3)}{9610(5x^2+3x+2)} + \frac{11(13x+7)}{310(5x^2+3x+2)^2} + \frac{1106 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {1660, 12, 614, 618, 204}

$$\frac{553(10x+3)}{9610(5x^2+3x+2)} + \frac{11(13x+7)}{310(5x^2+3x+2)^2} + \frac{1106 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2)^3,x]

[Out] (11\*(7 + 13\*x))/(310\*(2 + 3\*x + 5\*x^2)^2) + (553\*(3 + 10\*x))/(9610\*(2 + 3\*x + 5\*x^2)) + (1106\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(961\*Sqrt[31])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 614

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] - Dist[(2\*c\*(2\*p + 3))/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4\*p]

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{3-x+2x^2}{(2+3x+5x^2)^3} dx &= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{1}{62} \int \frac{553}{5(2+3x+5x^2)^2} dx \\
&= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{553}{310} \int \frac{1}{(2+3x+5x^2)^2} dx \\
&= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{553(3+10x)}{9610(2+3x+5x^2)} + \frac{553}{961} \int \frac{1}{2+3x+5x^2} dx \\
&= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{553(3+10x)}{9610(2+3x+5x^2)} - \frac{1106}{961} \text{Subst} \left( \int \frac{1}{-31-x^2} dx, x, 3+10x \right) \\
&= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{553(3+10x)}{9610(2+3x+5x^2)} + \frac{1106 \tan^{-1} \left( \frac{3+10x}{\sqrt{31}} \right)}{961\sqrt{31}}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 53, normalized size = 0.83

$$\frac{31(5530x^3+4977x^2+4094x+1141)}{(5x^2+3x+2)^2} + 2212\sqrt{31} \tan^{-1} \left( \frac{10x+3}{\sqrt{31}} \right)$$

59582

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2)^3, x]

[Out] ((31\*(1141 + 4094\*x + 4977\*x^2 + 5530\*x^3))/(2 + 3\*x + 5\*x^2)^2 + 2212\*sqrt[31]\*ArcTan[(3 + 10\*x)/sqrt[31]])/59582

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2)^3, x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2)^3, x]

**fricas [A]** time = 0.39, size = 75, normalized size = 1.17

$$\frac{171430x^3 + 2212\sqrt{31}(25x^4 + 30x^3 + 29x^2 + 12x + 4) \arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + 154287x^2 + 126914x + 35371}{59582(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/59582\*(171430\*x^3 + 2212\*sqrt(31)\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 154287\*x^2 + 126914\*x + 35371)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)

**giac [A]** time = 0.17, size = 46, normalized size = 0.72

$$\frac{1106}{29791} \sqrt{31} \arctan \left( \frac{1}{31} \sqrt{31} (10x+3) \right) + \frac{5530x^3 + 4977x^2 + 4094x + 1141}{1922(5x^2 + 3x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 1106/29791\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 1/1922\*(5530\*x^3 + 4977\*x^2 + 4094\*x + 1141)/(5\*x^2 + 3\*x + 2)^2

**maple** [A] time = 0.00, size = 47, normalized size = 0.73

$$\frac{1106\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{29791} + \frac{\frac{2765}{961}x^3 + \frac{4977}{1922}x^2 + \frac{2047}{961}x + \frac{1141}{1922}}{(5x^2 + 3x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x)

[Out] 25\*(553/4805\*x^3+4977/48050\*x^2+2047/24025\*x+1141/48050)/(5\*x^2+3\*x+2)^2+1106/29791\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))

**maxima** [A] time = 0.96, size = 56, normalized size = 0.88

$$\frac{1106}{29791} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{5530x^3 + 4977x^2 + 4094x + 1141}{1922(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 1106/29791\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 1/1922\*(5530\*x^3 + 4977\*x^2 + 4094\*x + 1141)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)

**mupad** [B] time = 0.05, size = 55, normalized size = 0.86

$$\frac{1106 \sqrt{31} \operatorname{atan}\left(\frac{10 \sqrt{31} x}{31} + \frac{3 \sqrt{31}}{31}\right)}{29791} + \frac{\frac{553x^3}{4805} + \frac{4977x^2}{48050} + \frac{2047x}{24025} + \frac{1141}{48050}}{x^4 + \frac{6x^3}{5} + \frac{29x^2}{25} + \frac{12x}{25} + \frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)/(3\*x + 5\*x^2 + 2)^3,x)

[Out] (1106\*31^(1/2)\*atan((10\*31^(1/2)\*x)/31 + (3\*31^(1/2))/31))/29791 + ((2047\*x)/24025 + (4977\*x^2)/48050 + (553\*x^3)/4805 + 1141/48050)/((12\*x)/25 + (29\*x^2)/25 + (6\*x^3)/5 + x^4 + 4/25)

**sympy** [A] time = 0.18, size = 63, normalized size = 0.98

$$\frac{5530x^3 + 4977x^2 + 4094x + 1141}{48050x^4 + 57660x^3 + 55738x^2 + 23064x + 7688} + \frac{1106\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{29791}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)/(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] (5530\*x\*\*3 + 4977\*x\*\*2 + 4094\*x + 1141)/(48050\*x\*\*4 + 57660\*x\*\*3 + 55738\*x\*\*2 + 23064\*x + 7688) + 1106\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/29791

$$3.19 \quad \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^4 dx$$

**Optimal.** Leaf size=80

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3} + 384x^2 + 144x$$

**Rubi [A]** time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1657}

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3} + 384x^2 + 144x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] 144\*x + 384\*x^2 + (3016\*x^3)/3 + 1838\*x^4 + (14801\*x^5)/5 + (10771\*x^6)/3 + (27763\*x^7)/7 + 3315\*x^8 + (24859\*x^9)/9 + 1571\*x^10 + (11525\*x^11)/11 + (875\*x^12)/3 + (2500\*x^13)/13

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^4 dx &= \int (144 + 768x + 3016x^2 + 7352x^3 + 14801x^4 + 21542x^5 + 27763x^6 + 144x^7 + 384x^8 + 1838x^9 + 14801x^{10} + 10771x^{11} + 27763x^{12} + 2500x^{13}) dx \\ &= 144x + 384x^2 + \frac{3016x^3}{3} + 1838x^4 + \frac{14801x^5}{5} + \frac{10771x^6}{3} + \frac{27763x^7}{7} + \frac{3315x^8}{9} + \frac{11525x^9}{11} + 1571x^{10} + \frac{875x^{11}}{3} + \frac{2500x^{12}}{13} + \frac{2500x^{13}}{13} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 80, normalized size = 1.00

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3} + 384x^2 + 144x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] 144\*x + 384\*x^2 + (3016\*x^3)/3 + 1838\*x^4 + (14801\*x^5)/5 + (10771\*x^6)/3 + (27763\*x^7)/7 + 3315\*x^8 + (24859\*x^9)/9 + 1571\*x^10 + (11525\*x^11)/11 + (875\*x^12)/3 + (2500\*x^13)/13

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^4 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^4, x]

**fricas [A]** time = 0.35, size = 64, normalized size = 0.80

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571x^{10} + \frac{24859}{9}x^9 + 3315x^8 + \frac{27763}{7}x^7 + \frac{10771}{3}x^6 + \frac{14801}{5}x^5 + 1838x^4 + \frac{3016}{3}x^3 + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^4,x, algorithm="fricas")

[Out] 2500/13\*x^13 + 875/3\*x^12 + 11525/11\*x^11 + 1571\*x^10 + 24859/9\*x^9 + 3315\*x^8 + 27763/7\*x^7 + 10771/3\*x^6 + 14801/5\*x^5 + 1838\*x^4 + 3016/3\*x^3 + 384\*x^2 + 144\*x

**giac** [A] time = 0.20, size = 64, normalized size = 0.80

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571x^{10} + \frac{24859}{9}x^9 + 3315x^8 + \frac{27763}{7}x^7 + \frac{10771}{3}x^6 + \frac{14801}{5}x^5 + 1838x^4 + \frac{3016}{3}x^3 + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^4,x, algorithm="giac")

[Out] 2500/13\*x^13 + 875/3\*x^12 + 11525/11\*x^11 + 1571\*x^10 + 24859/9\*x^9 + 3315\*x^8 + 27763/7\*x^7 + 10771/3\*x^6 + 14801/5\*x^5 + 1838\*x^4 + 3016/3\*x^3 + 384\*x^2 + 144\*x

**maple** [A] time = 0.00, size = 65, normalized size = 0.81

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571x^{10} + \frac{24859}{9}x^9 + 3315x^8 + \frac{27763}{7}x^7 + \frac{10771}{3}x^6 + \frac{14801}{5}x^5 + 1838x^4 + \frac{3016}{3}x^3 + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^4,x)

[Out] 144\*x+384\*x^2+3016/3\*x^3+1838\*x^4+14801/5\*x^5+10771/3\*x^6+27763/7\*x^7+3315\*x^8+24859/9\*x^9+1571\*x^10+11525/11\*x^11+875/3\*x^12+2500/13\*x^13

**maxima** [A] time = 0.45, size = 64, normalized size = 0.80

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571x^{10} + \frac{24859}{9}x^9 + 3315x^8 + \frac{27763}{7}x^7 + \frac{10771}{3}x^6 + \frac{14801}{5}x^5 + 1838x^4 + \frac{3016}{3}x^3 + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^4,x, algorithm="maxima")

[Out] 2500/13\*x^13 + 875/3\*x^12 + 11525/11\*x^11 + 1571\*x^10 + 24859/9\*x^9 + 3315\*x^8 + 27763/7\*x^7 + 10771/3\*x^6 + 14801/5\*x^5 + 1838\*x^4 + 3016/3\*x^3 + 384\*x^2 + 144\*x

**mupad** [B] time = 0.08, size = 64, normalized size = 0.80

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3} + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^2\*(3\*x + 5\*x^2 + 2)^4,x)

[Out] 144\*x + 384\*x^2 + (3016\*x^3)/3 + 1838\*x^4 + (14801\*x^5)/5 + (10771\*x^6)/3 + (27763\*x^7)/7 + 3315\*x^8 + (24859\*x^9)/9 + 1571\*x^10 + (11525\*x^11)/11 + (875\*x^12)/3 + (2500\*x^13)/13

**sympy** [A] time = 0.10, size = 76, normalized size = 0.95

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3} + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*2\*(5\*x\*\*2+3\*x+2)\*\*4,x)

[Out] 2500\*x\*\*13/13 + 875\*x\*\*12/3 + 11525\*x\*\*11/11 + 1571\*x\*\*10 + 24859\*x\*\*9/9 + 3315\*x\*\*8 + 27763\*x\*\*7/7 + 10771\*x\*\*6/3 + 14801\*x\*\*5/5 + 1838\*x\*\*4 + 3016\*x\*\*3/3 + 384\*x\*\*2 + 144\*x

$$3.20 \quad \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^3 dx$$

**Optimal.** Leaf size=66

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

**Rubi [A]** time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1657}

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^3, x]

[Out] 72\*x + 138\*x^2 + (914\*x^3)/3 + (1615\*x^4)/4 + (2693\*x^5)/5 + 449\*x^6 + 444\*x^7 + (1863\*x^8)/8 + (1865\*x^9)/9 + 40\*x^10 + (500\*x^11)/11

**Rule 1657**

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\begin{aligned} \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^3 dx &= \int (72 + 276x + 914x^2 + 1615x^3 + 2693x^4 + 2694x^5 + 3108x^6 + 1863x^7 + 1865x^8 + 40x^9 + 40x^{10} + 500x^{11}) dx \\ &= 72x + 138x^2 + \frac{914x^3}{3} + \frac{1615x^4}{4} + \frac{2693x^5}{5} + 449x^6 + 444x^7 + \frac{1863x^8}{8} + \frac{1865x^9}{9} + 40x^{10} + \frac{500x^{11}}{11} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 66, normalized size = 1.00

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^3, x]

[Out] 72\*x + 138\*x^2 + (914\*x^3)/3 + (1615\*x^4)/4 + (2693\*x^5)/5 + 449\*x^6 + 444\*x^7 + (1863\*x^8)/8 + (1865\*x^9)/9 + 40\*x^10 + (500\*x^11)/11

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^3, x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^3, x]

**fricas [A]** time = 0.34, size = 54, normalized size = 0.82

$$\frac{500}{11}x^{11} + 40x^{10} + \frac{1865}{9}x^9 + \frac{1863}{8}x^8 + 444x^7 + 449x^6 + \frac{2693}{5}x^5 + \frac{1615}{4}x^4 + \frac{914}{3}x^3 + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 500/11\*x^11 + 40\*x^10 + 1865/9\*x^9 + 1863/8\*x^8 + 444\*x^7 + 449\*x^6 + 2693/5\*x^5 + 1615/4\*x^4 + 914/3\*x^3 + 138\*x^2 + 72\*x

**giac** [A] time = 0.21, size = 54, normalized size = 0.82

$$\frac{500}{11}x^{11} + 40x^{10} + \frac{1865}{9}x^9 + \frac{1863}{8}x^8 + 444x^7 + 449x^6 + \frac{2693}{5}x^5 + \frac{1615}{4}x^4 + \frac{914}{3}x^3 + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 500/11\*x^11 + 40\*x^10 + 1865/9\*x^9 + 1863/8\*x^8 + 444\*x^7 + 449\*x^6 + 2693/5\*x^5 + 1615/4\*x^4 + 914/3\*x^3 + 138\*x^2 + 72\*x

**maple** [A] time = 0.00, size = 55, normalized size = 0.83

$$\frac{500}{11}x^{11} + 40x^{10} + \frac{1865}{9}x^9 + \frac{1863}{8}x^8 + 444x^7 + 449x^6 + \frac{2693}{5}x^5 + \frac{1615}{4}x^4 + \frac{914}{3}x^3 + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^3,x)

[Out] 72\*x+138\*x^2+914/3\*x^3+1615/4\*x^4+2693/5\*x^5+449\*x^6+444\*x^7+1863/8\*x^8+1865/9\*x^9+40\*x^10+500/11\*x^11

**maxima** [A] time = 0.44, size = 54, normalized size = 0.82

$$\frac{500}{11}x^{11} + 40x^{10} + \frac{1865}{9}x^9 + \frac{1863}{8}x^8 + 444x^7 + 449x^6 + \frac{2693}{5}x^5 + \frac{1615}{4}x^4 + \frac{914}{3}x^3 + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 500/11\*x^11 + 40\*x^10 + 1865/9\*x^9 + 1863/8\*x^8 + 444\*x^7 + 449\*x^6 + 2693/5\*x^5 + 1615/4\*x^4 + 914/3\*x^3 + 138\*x^2 + 72\*x

**mupad** [B] time = 0.05, size = 54, normalized size = 0.82

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^2\*(3\*x + 5\*x^2 + 2)^3,x)

[Out] 72\*x + 138\*x^2 + (914\*x^3)/3 + (1615\*x^4)/4 + (2693\*x^5)/5 + 449\*x^6 + 444\*x^7 + (1863\*x^8)/8 + (1865\*x^9)/9 + 40\*x^10 + (500\*x^11)/11

**sympy** [A] time = 0.09, size = 63, normalized size = 0.95

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*2\*(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] 500\*x\*\*11/11 + 40\*x\*\*10 + 1865\*x\*\*9/9 + 1863\*x\*\*8/8 + 444\*x\*\*7 + 449\*x\*\*6 + 2693\*x\*\*5/5 + 1615\*x\*\*4/4 + 914\*x\*\*3/3 + 138\*x\*\*2 + 72\*x



$$3.21 \quad \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^2 dx$$

**Optimal.** Leaf size=54

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

**Rubi [A]** time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1657}

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^2, x]

[Out] 36\*x + 42\*x^2 + (241\*x^3)/3 + 59\*x^4 + 78\*x^5 + (86\*x^6)/3 + (321\*x^7)/7 + (5\*x^8)/2 + (100\*x^9)/9

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^2 dx &= \int (36 + 84x + 241x^2 + 236x^3 + 390x^4 + 172x^5 + 321x^6 + 20x^7 + 100x^8 + 36x^9) dx \\ &= 36x + 42x^2 + \frac{241x^3}{3} + 59x^4 + 78x^5 + \frac{86x^6}{3} + \frac{321x^7}{7} + \frac{5x^8}{2} + \frac{100x^9}{9} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 54, normalized size = 1.00

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^2, x]

[Out] 36\*x + 42\*x^2 + (241\*x^3)/3 + 59\*x^4 + 78\*x^5 + (86\*x^6)/3 + (321\*x^7)/7 + (5\*x^8)/2 + (100\*x^9)/9

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^2, x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^2, x]

**fricas [A]** time = 0.33, size = 44, normalized size = 0.81

$$\frac{100}{9}x^9 + \frac{5}{2}x^8 + \frac{321}{7}x^7 + \frac{86}{3}x^6 + 78x^5 + 59x^4 + \frac{241}{3}x^3 + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 100/9\*x^9 + 5/2\*x^8 + 321/7\*x^7 + 86/3\*x^6 + 78\*x^5 + 59\*x^4 + 241/3\*x^3 + 42\*x^2 + 36\*x

**giac** [A] time = 0.20, size = 44, normalized size = 0.81

$$\frac{100}{9}x^9 + \frac{5}{2}x^8 + \frac{321}{7}x^7 + \frac{86}{3}x^6 + 78x^5 + 59x^4 + \frac{241}{3}x^3 + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 100/9\*x^9 + 5/2\*x^8 + 321/7\*x^7 + 86/3\*x^6 + 78\*x^5 + 59\*x^4 + 241/3\*x^3 + 42\*x^2 + 36\*x

**maple** [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{100}{9}x^9 + \frac{5}{2}x^8 + \frac{321}{7}x^7 + \frac{86}{3}x^6 + 78x^5 + 59x^4 + \frac{241}{3}x^3 + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^2,x)

[Out] 36\*x+42\*x^2+241/3\*x^3+59\*x^4+78\*x^5+86/3\*x^6+321/7\*x^7+5/2\*x^8+100/9\*x^9

**maxima** [A] time = 0.44, size = 44, normalized size = 0.81

$$\frac{100}{9}x^9 + \frac{5}{2}x^8 + \frac{321}{7}x^7 + \frac{86}{3}x^6 + 78x^5 + 59x^4 + \frac{241}{3}x^3 + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 100/9\*x^9 + 5/2\*x^8 + 321/7\*x^7 + 86/3\*x^6 + 78\*x^5 + 59\*x^4 + 241/3\*x^3 + 42\*x^2 + 36\*x

**mupad** [B] time = 0.03, size = 44, normalized size = 0.81

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^2\*(3\*x + 5\*x^2 + 2)^2,x)

[Out] 36\*x + 42\*x^2 + (241\*x^3)/3 + 59\*x^4 + 78\*x^5 + (86\*x^6)/3 + (321\*x^7)/7 + (5\*x^8)/2 + (100\*x^9)/9

**sympy** [A] time = 0.08, size = 51, normalized size = 0.94

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*2\*(5\*x\*\*2+3\*x+2)\*\*2,x)

[Out] 100\*x\*\*9/9 + 5\*x\*\*8/2 + 321\*x\*\*7/7 + 86\*x\*\*6/3 + 78\*x\*\*5 + 59\*x\*\*4 + 241\*x\*\*3/3 + 42\*x\*\*2 + 36\*x

$$3.22 \quad \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2) dx$$

Optimal. Leaf size=46

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

**Rubi [A]** time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {1657}

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2), x]

[Out] 18\*x + (15\*x^2)/2 + (53\*x^3)/3 + x^4/4 + (61\*x^5)/5 - (4\*x^6)/3 + (20\*x^7)/7

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2) dx &= \int (18 + 15x + 53x^2 + x^3 + 61x^4 - 8x^5 + 20x^6) dx \\ &= 18x + \frac{15x^2}{2} + \frac{53x^3}{3} + \frac{x^4}{4} + \frac{61x^5}{5} - \frac{4x^6}{3} + \frac{20x^7}{7} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 46, normalized size = 1.00

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2), x]

[Out] 18\*x + (15\*x^2)/2 + (53\*x^3)/3 + x^4/4 + (61\*x^5)/5 - (4\*x^6)/3 + (20\*x^7)/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - x + 2x^2)^2 (2 + 3x + 5x^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2), x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2), x]

**fricas [A]** time = 0.34, size = 34, normalized size = 0.74

$$\frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] 20/7\*x^7 - 4/3\*x^6 + 61/5\*x^5 + 1/4\*x^4 + 53/3\*x^3 + 15/2\*x^2 + 18\*x

giac [A] time = 0.19, size = 34, normalized size = 0.74

$$\frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] 20/7\*x^7 - 4/3\*x^6 + 61/5\*x^5 + 1/4\*x^4 + 53/3\*x^3 + 15/2\*x^2 + 18\*x

maple [A] time = 0.00, size = 35, normalized size = 0.76

$$\frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2),x)

[Out] 18\*x+15/2\*x^2+53/3\*x^3+1/4\*x^4+61/5\*x^5-4/3\*x^6+20/7\*x^7

maxima [A] time = 0.44, size = 34, normalized size = 0.74

$$\frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2\*(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] 20/7\*x^7 - 4/3\*x^6 + 61/5\*x^5 + 1/4\*x^4 + 53/3\*x^3 + 15/2\*x^2 + 18\*x

mupad [B] time = 0.03, size = 34, normalized size = 0.74

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^2\*(3\*x + 5\*x^2 + 2),x)

[Out] 18\*x + (15\*x^2)/2 + (53\*x^3)/3 + x^4/4 + (61\*x^5)/5 - (4\*x^6)/3 + (20\*x^7)/7

sympy [A] time = 0.07, size = 41, normalized size = 0.89

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*2\*(5\*x\*\*2+3\*x+2),x)

[Out] 20\*x\*\*7/7 - 4\*x\*\*6/3 + 61\*x\*\*5/5 + x\*\*4/4 + 53\*x\*\*3/3 + 15\*x\*\*2/2 + 18\*x

$$3.23 \quad \int \frac{(3-x+2x^2)^2}{2+3x+5x^2} dx$$

Optimal. Leaf size=56

$$\frac{4x^3}{15} - \frac{16x^2}{25} - \frac{1573 \log(5x^2 + 3x + 2)}{1250} + \frac{381x}{125} + \frac{8349 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1657, 634, 618, 204, 628}

$$\frac{4x^3}{15} - \frac{16x^2}{25} - \frac{1573 \log(5x^2 + 3x + 2)}{1250} + \frac{381x}{125} + \frac{8349 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2),x]

[Out] (381\*x)/125 - (16\*x^2)/25 + (4\*x^3)/15 + (8349\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(625\*Sqrt[31]) - (1573\*Log[2 + 3\*x + 5\*x^2])/1250

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[Expand[Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^2}{2+3x+5x^2} dx &= \int \left( \frac{381}{125} - \frac{32x}{25} + \frac{4x^2}{5} + \frac{121(3-13x)}{125(2+3x+5x^2)} \right) dx \\
&= \frac{381x}{125} - \frac{16x^2}{25} + \frac{4x^3}{15} + \frac{121}{125} \int \frac{3-13x}{2+3x+5x^2} dx \\
&= \frac{381x}{125} - \frac{16x^2}{25} + \frac{4x^3}{15} - \frac{1573 \int \frac{3+10x}{2+3x+5x^2} dx}{1250} + \frac{8349 \int \frac{1}{2+3x+5x^2} dx}{1250} \\
&= \frac{381x}{125} - \frac{16x^2}{25} + \frac{4x^3}{15} - \frac{1573 \log(2+3x+5x^2)}{1250} - \frac{8349}{625} \text{Subst} \left( \int \frac{1}{-31-x^2} dx, x, 3+10x \right) \\
&= \frac{381x}{125} - \frac{16x^2}{25} + \frac{4x^3}{15} + \frac{8349 \tan^{-1} \left( \frac{3+10x}{\sqrt{31}} \right)}{625\sqrt{31}} - \frac{1573 \log(2+3x+5x^2)}{1250}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 53, normalized size = 0.95

$$\frac{10x(100x^2 - 240x + 1143) - 4719 \log(5x^2 + 3x + 2)}{3750} + \frac{8349 \tan^{-1} \left( \frac{10x+3}{\sqrt{31}} \right)}{625\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2), x]

[Out] (8349\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(625\*Sqrt[31]) + (10\*x\*(1143 - 240\*x + 100\*x^2) - 4719\*Log[2 + 3\*x + 5\*x^2])/3750

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2), x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2), x]

**fricas [A]** time = 0.39, size = 43, normalized size = 0.77

$$\frac{4}{15} x^3 - \frac{16}{25} x^2 + \frac{8349}{19375} \sqrt{31} \arctan \left( \frac{1}{31} \sqrt{31} (10x + 3) \right) + \frac{381}{125} x - \frac{1573}{1250} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] 4/15\*x^3 - 16/25\*x^2 + 8349/19375\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 381/125\*x - 1573/1250\*log(5\*x^2 + 3\*x + 2)

**giac [A]** time = 0.21, size = 43, normalized size = 0.77

$$\frac{4}{15} x^3 - \frac{16}{25} x^2 + \frac{8349}{19375} \sqrt{31} \arctan \left( \frac{1}{31} \sqrt{31} (10x + 3) \right) + \frac{381}{125} x - \frac{1573}{1250} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2),x, algorithm="giac")

[Out]  $4/15*x^3 - 16/25*x^2 + 8349/19375*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3))$   
 $+ 381/125*x - 1573/1250*\log(5*x^2 + 3*x + 2)$

**maple** [A] time = 0.00, size = 44, normalized size = 0.79

$$\frac{4x^3}{15} - \frac{16x^2}{25} + \frac{381x}{125} + \frac{8349\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{19375} - \frac{1573 \ln(5x^2 + 3x + 2)}{1250}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^2/(5*x^2+3*x+2), x)`

[Out]  $381/125*x - 16/25*x^2 + 4/15*x^3 - 1573/1250*\ln(5*x^2 + 3*x + 2) + 8349/19375*31^{(1/2)}*$   
 $\arctan(1/31*(10*x+3)*31^{(1/2)})$

**maxima** [A] time = 0.97, size = 43, normalized size = 0.77

$$\frac{4}{15}x^3 - \frac{16}{25}x^2 + \frac{8349}{19375}\sqrt{31} \arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + \frac{381}{125}x - \frac{1573}{1250}\log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2/(5*x^2+3*x+2), x, algorithm="maxima")`

[Out]  $4/15*x^3 - 16/25*x^2 + 8349/19375*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3))$   
 $+ 381/125*x - 1573/1250*\log(5*x^2 + 3*x + 2)$

**mupad** [B] time = 3.45, size = 45, normalized size = 0.80

$$\frac{381x}{125} - \frac{1573 \ln(5x^2 + 3x + 2)}{1250} + \frac{8349\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{19375} - \frac{16x^2}{25} + \frac{4x^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2), x)`

[Out]  $(381*x)/125 - (1573*\log(3*x + 5*x^2 + 2))/1250 + (8349*31^{(1/2)}*\operatorname{atan}((10*31^{(1/2)}*x)/31 + (3*31^{(1/2)})/31))/19375 - (16*x^2)/25 + (4*x^3)/15$

**sympy** [A] time = 0.15, size = 63, normalized size = 1.12

$$\frac{4x^3}{15} - \frac{16x^2}{25} + \frac{381x}{125} - \frac{1573 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{1250} + \frac{8349\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{19375}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**2/(5*x**2+3*x+2), x)`

[Out]  $4*x**3/15 - 16*x**2/25 + 381*x/125 - 1573*\log(x**2 + 3*x/5 + 2/5)/1250 + 8349*\sqrt{31}*\operatorname{atan}(10*\sqrt{31}*x/31 + 3*\sqrt{31}/31)/19375$

$$3.24 \quad \int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=63

$$\frac{121(69x+61)}{3875(5x^2+3x+2)} - \frac{22}{125} \log(5x^2+3x+2) + \frac{4x}{25} + \frac{41932 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{3875\sqrt{31}}$$

**Rubi [A]** time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(69x+61)}{3875(5x^2+3x+2)} - \frac{22}{125} \log(5x^2+3x+2) + \frac{4x}{25} + \frac{41932 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{3875\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^2,x]

[Out] (4\*x)/25 + (121\*(61 + 69\*x))/(3875\*(2 + 3\*x + 5\*x^2)) + (41932\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(3875\*Sqrt[31]) - (22\*Log[2 + 3\*x + 5\*x^2])/125

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[P



```

q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]], Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2} dx &= \frac{121(61+69x)}{3875(2+3x+5x^2)} + \frac{1}{31} \int \frac{\frac{4032}{25} - \frac{992x}{25} + \frac{124x^2}{5}}{2+3x+5x^2} dx \\
&= \frac{121(61+69x)}{3875(2+3x+5x^2)} + \frac{1}{31} \int \left( \frac{124}{25} + \frac{44(86-31x)}{25(2+3x+5x^2)} \right) dx \\
&= \frac{4x}{25} + \frac{121(61+69x)}{3875(2+3x+5x^2)} + \frac{44}{775} \int \frac{86-31x}{2+3x+5x^2} dx \\
&= \frac{4x}{25} + \frac{121(61+69x)}{3875(2+3x+5x^2)} - \frac{22}{125} \int \frac{3+10x}{2+3x+5x^2} dx + \frac{20966}{3875} \int \frac{1}{2+3x+5x^2} dx \\
&= \frac{4x}{25} + \frac{121(61+69x)}{3875(2+3x+5x^2)} - \frac{22}{125} \log(2+3x+5x^2) - \frac{41932 \operatorname{Subst}\left(\int \frac{1}{-31-x^2} dx, x, 3\right)}{3875} \\
&= \frac{4x}{25} + \frac{121(61+69x)}{3875(2+3x+5x^2)} + \frac{41932 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{3875\sqrt{31}} - \frac{22}{125} \log(2+3x+5x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 59, normalized size = 0.94

$$\frac{\frac{3751(69x+61)}{5x^2+3x+2} - 21142 \log(5x^2+3x+2) + 19220x + 41932\sqrt{31} \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{120125}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^2, x]

[Out] (19220\*x + (3751\*(61 + 69\*x))/(2 + 3\*x + 5\*x^2) + 41932\*Sqrt[31]\*ArcTan[(3 + 10\*x)/Sqrt[31]] - 21142\*Log[2 + 3\*x + 5\*x^2])/120125

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^2, x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^2, x]

**fricas [A]** time = 0.40, size = 78, normalized size = 1.24

$$\frac{96100x^3 + 41932\sqrt{31}(5x^2 + 3x + 2)\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 57660x^2 - 21142(5x^2 + 3x + 2)\log(5x^2 + 3x + 2) + 297259x + 228811}{120125(5x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 1/120125\*(96100\*x^3 + 41932\*sqrt(31)\*(5\*x^2 + 3\*x + 2)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 57660\*x^2 - 21142\*(5\*x^2 + 3\*x + 2)\*log(5\*x^2 + 3\*x + 2) + 2\*97259\*x + 228811)/(5\*x^2 + 3\*x + 2)

**giac** [A] time = 0.17, size = 52, normalized size = 0.83

$$\frac{41932}{120125} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{4}{25} x + \frac{121(69x + 61)}{3875(5x^2 + 3x + 2)} - \frac{22}{125} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 41932/120125\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 4/25\*x + 121/3875\*(69\*x + 61)/(5\*x^2 + 3\*x + 2) - 22/125\*log(5\*x^2 + 3\*x + 2)

**maple** [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{4x}{25} + \frac{41932\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{120125} - \frac{22 \ln(5x^2 + 3x + 2)}{125} - \frac{11\left(-\frac{759x}{775} - \frac{671}{775}\right)}{25\left(x^2 + \frac{3}{5}x + \frac{2}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^2,x)

[Out] 4/25\*x-11/25\*(-759/775\*x-671/775)/(x^2+3/5\*x+2/5)-22/125\*ln(5\*x^2+3\*x+2)+41932/120125\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))

**maxima** [A] time = 0.96, size = 52, normalized size = 0.83

$$\frac{41932}{120125} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{4}{25} x + \frac{121(69x + 61)}{3875(5x^2 + 3x + 2)} - \frac{22}{125} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 41932/120125\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 4/25\*x + 121/3875\*(69\*x + 61)/(5\*x^2 + 3\*x + 2) - 22/125\*log(5\*x^2 + 3\*x + 2)

**mupad** [B] time = 0.05, size = 51, normalized size = 0.81

$$\frac{4x}{25} - \frac{22 \ln(5x^2 + 3x + 2)}{125} + \frac{\frac{8349x}{19375} + \frac{7381}{19375}}{x^2 + \frac{3x}{5} + \frac{2}{5}} + \frac{41932 \sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{120125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^2/(3\*x + 5\*x^2 + 2)^2,x)

[Out] (4\*x)/25 - (22\*log(3\*x + 5\*x^2 + 2))/125 + ((8349\*x)/19375 + 7381/19375)/((3\*x)/5 + x^2 + 2/5) + (41932\*31^(1/2)\*atan((10\*31^(1/2)\*x)/31 + (3\*31^(1/2))/31))/120125

**sympy** [A] time = 0.20, size = 65, normalized size = 1.03

$$\frac{4x}{25} + \frac{8349x + 7381}{19375x^2 + 11625x + 7750} - \frac{22 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{125} + \frac{41932\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{120125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)
```

```
[Out] 4*x/25 + (8349*x + 7381)/(19375*x**2 + 11625*x + 7750) - 22*log(x**2 + 3*x/5 + 2/5)/125 + 41932*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/120125
```

$$3.25 \quad \int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3} dx$$

**Optimal.** Leaf size=64

$$\frac{121(69x + 61)}{7750(5x^2 + 3x + 2)^2} + \frac{11(45710x + 17557)}{240250(5x^2 + 3x + 2)} + \frac{4330 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

**Rubi [A]** time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1660, 12, 618, 204}

$$\frac{121(69x + 61)}{7750(5x^2 + 3x + 2)^2} + \frac{11(45710x + 17557)}{240250(5x^2 + 3x + 2)} + \frac{4330 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (121*(61 + 69*x))/(7750*(2 + 3*x + 5*x^2)^2) + (11*(17557 + 45710*x))/(240250*(2 + 3*x + 5*x^2)) + (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3} dx &= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{1}{62} \int \frac{\frac{48669}{125} - \frac{1984x}{25} + \frac{248x^2}{5}}{(2+3x+5x^2)^2} dx \\
&= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} + \frac{\int \frac{4330}{2+3x+5x^2} dx}{1922} \\
&= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} + \frac{2165}{961} \int \frac{1}{2+3x+5x^2} dx \\
&= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} - \frac{4330}{961} \text{Subst}\left(\int \frac{1}{-31-x^2} dx, x, 3x+2\right) \\
&= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} + \frac{4330 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 53, normalized size = 0.83

$$\frac{11(45710x^3 + 44983x^2 + 33524x + 11183)}{48050(5x^2 + 3x + 2)^2} + \frac{4330 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^3, x]

[Out] (11\*(11183 + 33524\*x + 44983\*x^2 + 45710\*x^3))/(48050\*(2 + 3\*x + 5\*x^2)^2) + (4330\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(961\*Sqrt[31])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^3, x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^3, x]

**fricas [A]** time = 0.38, size = 75, normalized size = 1.17

$$\frac{15587110x^3 + 216500\sqrt{31}(25x^4 + 30x^3 + 29x^2 + 12x + 4) \arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 15339203x^2 + 11431684x + 3813403}{1489550(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/1489550\*(15587110\*x^3 + 216500\*sqrt(31)\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 15339203\*x^2 + 11431684\*x + 3813403)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)

**giac [A]** time = 0.18, size = 46, normalized size = 0.72

$$\frac{4330}{29791} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{11(45710x^3 + 44983x^2 + 33524x + 11183)}{48050(5x^2 + 3x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 4330/29791\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 11/48050\*(45710\*x^3 + 44983\*x^2 + 33524\*x + 11183)/(5\*x^2 + 3\*x + 2)^2

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{4330\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{29791} + \frac{\frac{50281}{4805}x^3 + \frac{494813}{48050}x^2 + \frac{184382}{24025}x + \frac{123013}{48050}}{(5x^2 + 3x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x)

[Out] 25\*(50281/120125\*x^3+494813/1201250\*x^2+184382/600625\*x+123013/1201250)/(5\*x^2+3\*x+2)^2+4330/29791\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))

maxima [A] time = 0.97, size = 56, normalized size = 0.88

$$\frac{4330}{29791} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{11(45710x^3 + 44983x^2 + 33524x + 11183)}{48050(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 4330/29791\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 11/48050\*(45710\*x^3 + 44983\*x^2 + 33524\*x + 11183)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)

mupad [B] time = 3.44, size = 55, normalized size = 0.86

$$\frac{4330 \sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{29791} + \frac{\frac{50281x^3}{120125} + \frac{494813x^2}{1201250} + \frac{184382x}{600625} + \frac{123013}{1201250}}{x^4 + \frac{6x^3}{5} + \frac{29x^2}{25} + \frac{12x}{25} + \frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^2/(3\*x + 5\*x^2 + 2)^3,x)

[Out] (4330\*31^(1/2)\*atan((10\*31^(1/2)\*x)/31 + (3\*31^(1/2))/31))/29791 + ((184382\*x)/600625 + (494813\*x^2)/1201250 + (50281\*x^3)/120125 + 123013/1201250)/((12\*x)/25 + (29\*x^2)/25 + (6\*x^3)/5 + x^4 + 4/25)

sympy [A] time = 0.20, size = 63, normalized size = 0.98

$$\frac{502810x^3 + 494813x^2 + 368764x + 123013}{1201250x^4 + 1441500x^3 + 1393450x^2 + 576600x + 192200} + \frac{4330\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{29791}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*2/(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] (502810\*x\*\*3 + 494813\*x\*\*2 + 368764\*x + 123013)/((1201250\*x\*\*4 + 1441500\*x\*\*3 + 1393450\*x\*\*2 + 576600\*x + 192200) + 4330\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/29791

$$3.26 \quad \int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4} dx$$

**Optimal.** Leaf size=85

$$\frac{16688(10x+3)}{148955(5x^2+3x+2)} + \frac{11(12060x+4579)}{120125(5x^2+3x+2)^2} + \frac{121(69x+61)}{11625(5x^2+3x+2)^3} + \frac{66752 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

**Rubi [A]** time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1660, 12, 614, 618, 204}

$$\frac{16688(10x+3)}{148955(5x^2+3x+2)} + \frac{11(12060x+4579)}{120125(5x^2+3x+2)^2} + \frac{121(69x+61)}{11625(5x^2+3x+2)^3} + \frac{66752 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^4,x]

[Out] (121\*(61 + 69\*x))/(11625\*(2 + 3\*x + 5\*x^2)^3) + (11\*(4579 + 12060\*x))/(120125\*(2 + 3\*x + 5\*x^2)^2) + (16688\*(3 + 10\*x))/(148955\*(2 + 3\*x + 5\*x^2)) + (66752\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(29791\*Sqrt[31])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 614

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] - Dist[(2\*c\*(2\*p + 3))/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4\*p]

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2

- 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4} dx &= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{1}{93} \int \frac{\frac{77178}{125} - \frac{2976x}{25} + \frac{372x^2}{5}}{(2+3x+5x^2)^3} dx \\
 &= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{\int \frac{100128}{5(2+3x+5x^2)^2} dx}{5766} \\
 &= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{16688 \int \frac{1}{(2+3x+5x^2)^2} dx}{4805} \\
 &= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{16688(3+10x)}{148955(2+3x+5x^2)} + \frac{33376 \int \frac{1}{2+3x+5x^2} dx}{29791\sqrt{31}} \\
 &= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{16688(3+10x)}{148955(2+3x+5x^2)} - \frac{66752 \operatorname{ArcTan}\left[\frac{10x+3}{\sqrt{31}}\right]}{29791\sqrt{31}} \\
 &= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{16688(3+10x)}{148955(2+3x+5x^2)} + \frac{66752 \operatorname{ArcTan}\left[\frac{10x+3}{\sqrt{31}}\right]}{29791\sqrt{31}}
 \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 63, normalized size = 0.74

$$\frac{12516000x^5 + 18774000x^4 + 21491796x^3 + 12780597x^2 + 5674908x + 1259239}{446865(5x^2 + 3x + 2)^3} + \frac{66752 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^4, x]

[Out] (1259239 + 5674908\*x + 12780597\*x^2 + 21491796\*x^3 + 18774000\*x^4 + 12516000\*x^5)/(446865\*(2 + 3\*x + 5\*x^2)^3) + (66752\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(29791\*Sqrt[31])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^4, x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^2/(2 + 3\*x + 5\*x^2)^4, x]

**fricas [A]** time = 0.38, size = 105, normalized size = 1.24

$$\frac{387996000x^5 + 581994000x^4 + 666245676x^3 + 1001280\sqrt{31}(125x^6 + 225x^5 + 285x^4 + 207x^3 + 114x^2 + 36x + 8) \arctan\left(\frac{1}{\sqrt{31}}\sqrt{31}(10x+3)\right) + 396198507x^2 + 175922148x + 39036409}{13852815(125x^6 + 225x^5 + 285x^4 + 207x^3 + 114x^2 + 36x + 8)}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^4,x, algorithm="fricas")

[Out] 1/13852815\*(387996000\*x^5 + 581994000\*x^4 + 666245676\*x^3 + 1001280\*sqrt(31)\*(125\*x^6 + 225\*x^5 + 285\*x^4 + 207\*x^3 + 114\*x^2 + 36\*x + 8)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 396198507\*x^2 + 175922148\*x + 39036409)/(125\*x^6 + 225\*x^5 + 285\*x^4 + 207\*x^3 + 114\*x^2 + 36\*x + 8)

**giac** [A] time = 0.18, size = 56, normalized size = 0.66

$$\frac{66752}{923521} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x+3)\right) + \frac{12516000x^5 + 18774000x^4 + 21491796x^3 + 12780597x^2 + 5674908x + 1259239}{446865(5x^2 + 3x + 2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^4,x, algorithm="giac")

[Out] 66752/923521\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 1/446865\*(12516000\*x^5 + 18774000\*x^4 + 21491796\*x^3 + 12780597\*x^2 + 5674908\*x + 1259239)/(5\*x^2 + 3\*x + 2)^3

**maple** [A] time = 0.01, size = 57, normalized size = 0.67

$$\frac{66752\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{923521} + \frac{\frac{834400}{29791}x^5 + \frac{1251600}{29791}x^4 + \frac{7163932}{148955}x^3 + \frac{4260199}{148955}x^2 + \frac{1891636}{148955}x + \frac{1259239}{446865}}{(5x^2 + 3x + 2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^4,x)

[Out] 125\*(33376/148955\*x^5+50064/148955\*x^4+7163932/18619375\*x^3+4260199/18619375\*x^2+1891636/18619375\*x+1259239/55858125)/(5\*x^2+3\*x+2)^3+66752/923521\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))

**maxima** [A] time = 0.96, size = 76, normalized size = 0.89

$$\frac{66752}{923521} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x+3)\right) + \frac{12516000x^5 + 18774000x^4 + 21491796x^3 + 12780597x^2 + 5674908x + 1259239}{446865(125x^6 + 225x^5 + 285x^4 + 207x^3 + 114x^2 + 36x + 8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^4,x, algorithm="maxima")

[Out] 66752/923521\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 1/446865\*(12516000\*x^5 + 18774000\*x^4 + 21491796\*x^3 + 12780597\*x^2 + 5674908\*x + 1259239)/(125\*x^6 + 225\*x^5 + 285\*x^4 + 207\*x^3 + 114\*x^2 + 36\*x + 8)

**mupad** [B] time = 3.47, size = 75, normalized size = 0.88

$$\frac{66752 \sqrt{31} \operatorname{atan}\left(\frac{10 \sqrt{31} x}{31} + \frac{3 \sqrt{31}}{31}\right)}{923521} + \frac{\frac{33376x^5}{148955} + \frac{50064x^4}{148955} + \frac{7163932x^3}{18619375} + \frac{4260199x^2}{18619375} + \frac{1891636x}{18619375} + \frac{1259239}{55858125}}{x^6 + \frac{9x^5}{5} + \frac{57x^4}{25} + \frac{207x^3}{125} + \frac{114x^2}{125} + \frac{36x}{125} + \frac{8}{125}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^2/(3\*x + 5\*x^2 + 2)^4,x)

[Out] (66752\*31^(1/2)\*atan((10\*31^(1/2)\*x)/31 + (3\*31^(1/2))/31))/923521 + ((1891636\*x)/18619375 + (4260199\*x^2)/18619375 + (7163932\*x^3)/18619375 + (50064\*x^4)/148955 + (33376\*x^5)/148955 + 1259239/55858125)/((36\*x)/125 + (114\*x^2)/125 + (207\*x^3)/125 + (57\*x^4)/25 + (9\*x^5)/5 + x^6 + 8/125)

sympy [A] time = 0.23, size = 83, normalized size = 0.98

$$\frac{12516000x^5 + 18774000x^4 + 21491796x^3 + 12780597x^2 + 5674908x + 1259239}{55858125x^6 + 100544625x^5 + 127356525x^4 + 92501055x^3 + 50942610x^2 + 16087140x + 3574920} + \frac{66752\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{923521}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*2/(5\*x\*\*2+3\*x+2)\*\*4,x)

[Out] (12516000\*x\*\*5 + 18774000\*x\*\*4 + 21491796\*x\*\*3 + 12780597\*x\*\*2 + 5674908\*x + 1259239)/(55858125\*x\*\*6 + 100544625\*x\*\*5 + 127356525\*x\*\*4 + 92501055\*x\*\*3 + 50942610\*x\*\*2 + 16087140\*x + 3574920) + 66752\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/923521

$$3.27 \quad \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^4 dx$$

**Optimal.** Leaf size=96

$$\frac{1000x^{15}}{3} + \frac{2250x^{14}}{7} + \frac{27050x^{13}}{13} + \frac{30395x^{12}}{12} + \frac{68583x^{11}}{11} + \frac{75311x^{10}}{10} + \frac{103583x^9}{9} + \frac{94881x^8}{8} + \frac{91349x^7}{7} + \frac{64529x^6}{6} +$$

**Rubi [A]** time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1657}

$$\frac{1000x^{15}}{3} + \frac{2250x^{14}}{7} + \frac{27050x^{13}}{13} + \frac{30395x^{12}}{12} + \frac{68583x^{11}}{11} + \frac{75311x^{10}}{10} + \frac{103583x^9}{9} + \frac{94881x^8}{8} + \frac{91349x^7}{7} + \frac{64529x^6}{6} + \frac{43083x^5}{5} + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^4, x]

[Out] 432\*x + 1080\*x^2 + 2856\*x^3 + 5144\*x^4 + (43083\*x^5)/5 + (64529\*x^6)/6 + (91349\*x^7)/7 + (94881\*x^8)/8 + (103583\*x^9)/9 + (75311\*x^10)/10 + (68583\*x^11)/11 + (30395\*x^12)/12 + (27050\*x^13)/13 + (2250\*x^14)/7 + (1000\*x^15)/3

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^4 dx &= \int (432 + 2160x + 8568x^2 + 20576x^3 + 43083x^4 + 64529x^5 + 91349x^6 \\ &+ 94881x^7 + 103583x^8 + 75311x^9 + 68583x^{10} + 30395x^{11} + 27050x^{12} + 2250x^{13} + 1000x^{14} + 1000x^{15}) dx \\ &= 432x + 1080x^2 + 2856x^3 + 5144x^4 + \frac{43083x^5}{5} + \frac{64529x^6}{6} + \frac{91349x^7}{7} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 96, normalized size = 1.00

$$\frac{1000x^{15}}{3} + \frac{2250x^{14}}{7} + \frac{27050x^{13}}{13} + \frac{30395x^{12}}{12} + \frac{68583x^{11}}{11} + \frac{75311x^{10}}{10} + \frac{103583x^9}{9} + \frac{94881x^8}{8} + \frac{91349x^7}{7} + \frac{64529x^6}{6} + \frac{43083x^5}{5} + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^4, x]

[Out] 432\*x + 1080\*x^2 + 2856\*x^3 + 5144\*x^4 + (43083\*x^5)/5 + (64529\*x^6)/6 + (91349\*x^7)/7 + (94881\*x^8)/8 + (103583\*x^9)/9 + (75311\*x^10)/10 + (68583\*x^11)/11 + (30395\*x^12)/12 + (27050\*x^13)/13 + (2250\*x^14)/7 + (1000\*x^15)/3

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^4 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^4, x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^4, x]

**fricas [A]** time = 0.34, size = 74, normalized size = 0.77

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + \frac{43083}{5}x^5 + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^4,x, algorithm="fricas")

[Out] 1000/3\*x^15 + 2250/7\*x^14 + 27050/13\*x^13 + 30395/12\*x^12 + 68583/11\*x^11 + 75311/10\*x^10 + 103583/9\*x^9 + 94881/8\*x^8 + 91349/7\*x^7 + 64529/6\*x^6 + 43083/5\*x^5 + 5144\*x^4 + 2856\*x^3 + 1080\*x^2 + 432\*x

**giac** [A] time = 0.18, size = 74, normalized size = 0.77

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + \frac{43083}{5}x^5 + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^4,x, algorithm="giac")

[Out] 1000/3\*x^15 + 2250/7\*x^14 + 27050/13\*x^13 + 30395/12\*x^12 + 68583/11\*x^11 + 75311/10\*x^10 + 103583/9\*x^9 + 94881/8\*x^8 + 91349/7\*x^7 + 64529/6\*x^6 + 43083/5\*x^5 + 5144\*x^4 + 2856\*x^3 + 1080\*x^2 + 432\*x

**maple** [A] time = 0.00, size = 75, normalized size = 0.78

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + \frac{43083}{5}x^5 + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^4,x)

[Out] 432\*x+1080\*x^2+2856\*x^3+5144\*x^4+43083/5\*x^5+64529/6\*x^6+91349/7\*x^7+94881/8\*x^8+103583/9\*x^9+75311/10\*x^10+68583/11\*x^11+30395/12\*x^12+27050/13\*x^13+2250/7\*x^14+1000/3\*x^15

**maxima** [A] time = 0.43, size = 74, normalized size = 0.77

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + \frac{43083}{5}x^5 + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^4,x, algorithm="maxima")

[Out] 1000/3\*x^15 + 2250/7\*x^14 + 27050/13\*x^13 + 30395/12\*x^12 + 68583/11\*x^11 + 75311/10\*x^10 + 103583/9\*x^9 + 94881/8\*x^8 + 91349/7\*x^7 + 64529/6\*x^6 + 43083/5\*x^5 + 5144\*x^4 + 2856\*x^3 + 1080\*x^2 + 432\*x

**mupad** [B] time = 0.12, size = 74, normalized size = 0.77

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + \frac{43083}{5}x^5 + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^3\*(3\*x + 5\*x^2 + 2)^4,x)

[Out] 432\*x + 1080\*x^2 + 2856\*x^3 + 5144\*x^4 + (43083\*x^5)/5 + (64529\*x^6)/6 + (91349\*x^7)/7 + (94881\*x^8)/8 + (103583\*x^9)/9 + (75311\*x^10)/10 + (68583\*x^11)/11 + (30395\*x^12)/12 + (27050\*x^13)/13 + (2250\*x^14)/7 + (1000\*x^15)/3

**sympy** [A] time = 0.10, size = 92, normalized size = 0.96

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + \frac{43083}{5}x^5 + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*3\*(5\*x\*\*2+3\*x+2)\*\*4,x)

[Out] 1000\*x\*\*15/3 + 2250\*x\*\*14/7 + 27050\*x\*\*13/13 + 30395\*x\*\*12/12 + 68583\*x\*\*11/11 + 75311\*x\*\*10/10 + 103583\*x\*\*9/9 + 94881\*x\*\*8/8 + 91349\*x\*\*7/7 + 64529\*x\*\*6/6 + 43083\*x\*\*5/5 + 5144\*x\*\*4 + 2856\*x\*\*3 + 1080\*x\*\*2 + 432\*x

$$3.28 \quad \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^3 dx$$

**Optimal.** Leaf size=82

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2 + 216x$$

**Rubi [A]** time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1657}

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2 + 216x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] 216\*x + 378\*x^2 + 870\*x^3 + (4483\*x^4)/4 + (8292\*x^5)/5 + (2873\*x^6)/2 + (12016\*x^7)/7 + (7869\*x^8)/8 + (3316\*x^9)/3 + (3061\*x^10)/10 + (4830\*x^11)/11 + 25\*x^12 + (1000\*x^13)/13

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^3 dx &= \int (216 + 756x + 2610x^2 + 4483x^3 + 8292x^4 + 8619x^5 + 12016x^6 + 7560x^7 + 2160x^8 + 216x^9) dx \\ &= 216x + 378x^2 + 870x^3 + \frac{4483x^4}{4} + \frac{8292x^5}{5} + \frac{2873x^6}{2} + \frac{12016x^7}{7} + \frac{7560x^8}{8} + \frac{2160x^9}{9} + C \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 82, normalized size = 1.00

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2 + 216x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] 216\*x + 378\*x^2 + 870\*x^3 + (4483\*x^4)/4 + (8292\*x^5)/5 + (2873\*x^6)/2 + (12016\*x^7)/7 + (7869\*x^8)/8 + (3316\*x^9)/3 + (3061\*x^10)/10 + (4830\*x^11)/11 + 25\*x^12 + (1000\*x^13)/13

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^3, x]

**fricas** [A] time = 0.33, size = 64, normalized size = 0.78

$$\frac{1000}{13}x^{13} + 25x^{12} + \frac{4830}{11}x^{11} + \frac{3061}{10}x^{10} + \frac{3316}{3}x^9 + \frac{7869}{8}x^8 + \frac{12016}{7}x^7 + \frac{2873}{2}x^6 + \frac{8292}{5}x^5 + \frac{4483}{4}x^4 + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1000/13\*x^13 + 25\*x^12 + 4830/11\*x^11 + 3061/10\*x^10 + 3316/3\*x^9 + 7869/8\*x^8 + 12016/7\*x^7 + 2873/2\*x^6 + 8292/5\*x^5 + 4483/4\*x^4 + 870\*x^3 + 378\*x^2 + 216\*x

**giac** [A] time = 0.21, size = 64, normalized size = 0.78

$$\frac{1000}{13}x^{13} + 25x^{12} + \frac{4830}{11}x^{11} + \frac{3061}{10}x^{10} + \frac{3316}{3}x^9 + \frac{7869}{8}x^8 + \frac{12016}{7}x^7 + \frac{2873}{2}x^6 + \frac{8292}{5}x^5 + \frac{4483}{4}x^4 + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 1000/13\*x^13 + 25\*x^12 + 4830/11\*x^11 + 3061/10\*x^10 + 3316/3\*x^9 + 7869/8\*x^8 + 12016/7\*x^7 + 2873/2\*x^6 + 8292/5\*x^5 + 4483/4\*x^4 + 870\*x^3 + 378\*x^2 + 216\*x

**maple** [A] time = 0.00, size = 65, normalized size = 0.79

$$\frac{1000}{13}x^{13} + 25x^{12} + \frac{4830}{11}x^{11} + \frac{3061}{10}x^{10} + \frac{3316}{3}x^9 + \frac{7869}{8}x^8 + \frac{12016}{7}x^7 + \frac{2873}{2}x^6 + \frac{8292}{5}x^5 + \frac{4483}{4}x^4 + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^3,x)

[Out] 216\*x+378\*x^2+870\*x^3+4483/4\*x^4+8292/5\*x^5+2873/2\*x^6+12016/7\*x^7+7869/8\*x^8+3316/3\*x^9+3061/10\*x^10+4830/11\*x^11+25\*x^12+1000/13\*x^13

**maxima** [A] time = 0.43, size = 64, normalized size = 0.78

$$\frac{1000}{13}x^{13} + 25x^{12} + \frac{4830}{11}x^{11} + \frac{3061}{10}x^{10} + \frac{3316}{3}x^9 + \frac{7869}{8}x^8 + \frac{12016}{7}x^7 + \frac{2873}{2}x^6 + \frac{8292}{5}x^5 + \frac{4483}{4}x^4 + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 1000/13\*x^13 + 25\*x^12 + 4830/11\*x^11 + 3061/10\*x^10 + 3316/3\*x^9 + 7869/8\*x^8 + 12016/7\*x^7 + 2873/2\*x^6 + 8292/5\*x^5 + 4483/4\*x^4 + 870\*x^3 + 378\*x^2 + 216\*x

**mupad** [B] time = 0.08, size = 64, normalized size = 0.78

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^3\*(3\*x + 5\*x^2 + 2)^3,x)

[Out] 216\*x + 378\*x^2 + 870\*x^3 + (4483\*x^4)/4 + (8292\*x^5)/5 + (2873\*x^6)/2 + (12016\*x^7)/7 + (7869\*x^8)/8 + (3316\*x^9)/3 + (3061\*x^10)/10 + (4830\*x^11)/11 + 25\*x^12 + (1000\*x^13)/13

**sympy** [A] time = 0.09, size = 78, normalized size = 0.95

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**3,x)
```

```
[Out] 1000*x**13/13 + 25*x**12 + 4830*x**11/11 + 3061*x**10/10 + 3316*x**9/3 + 78  
69*x**8/8 + 12016*x**7/7 + 2873*x**6/2 + 8292*x**5/5 + 4483*x**4/4 + 870*x*  
*3 + 378*x**2 + 216*x
```

$$3.29 \quad \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^2 dx$$

**Optimal.** Leaf size=68

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

**Rubi [A]** time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1657}

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] 108\*x + 108\*x^2 + 237\*x^3 + (635\*x^4)/4 + (1416\*x^5)/5 + (299\*x^6)/3 + (1571\*x^7)/7 + (83\*x^8)/8 + (922\*x^9)/9 - 6\*x^10 + (200\*x^11)/11

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^2 dx &= \int (108 + 216x + 711x^2 + 635x^3 + 1416x^4 + 598x^5 + 1571x^6 + 83x^7 + 922x^8 + 200x^9 - 6x^{10} + 200x^{11}) dx \\ &= 108x + 108x^2 + 237x^3 + \frac{635x^4}{4} + \frac{1416x^5}{5} + \frac{299x^6}{3} + \frac{1571x^7}{7} + \frac{83x^8}{8} + \frac{200x^9}{9} - 6x^{10} + \frac{200x^{11}}{11} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 68, normalized size = 1.00

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] 108\*x + 108\*x^2 + 237\*x^3 + (635\*x^4)/4 + (1416\*x^5)/5 + (299\*x^6)/3 + (1571\*x^7)/7 + (83\*x^8)/8 + (922\*x^9)/9 - 6\*x^10 + (200\*x^11)/11

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^2, x]

**fricas [A]** time = 0.34, size = 54, normalized size = 0.79

$$\frac{200}{11}x^{11} - 6x^{10} + \frac{922}{9}x^9 + \frac{83}{8}x^8 + \frac{1571}{7}x^7 + \frac{299}{3}x^6 + \frac{1416}{5}x^5 + \frac{635}{4}x^4 + 237x^3 + 108x^2 + 108x$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 200/11\*x^11 - 6\*x^10 + 922/9\*x^9 + 83/8\*x^8 + 1571/7\*x^7 + 299/3\*x^6 + 1416/5\*x^5 + 635/4\*x^4 + 237\*x^3 + 108\*x^2 + 108\*x

**giac** [A] time = 0.20, size = 54, normalized size = 0.79

$$\frac{200}{11}x^{11} - 6x^{10} + \frac{922}{9}x^9 + \frac{83}{8}x^8 + \frac{1571}{7}x^7 + \frac{299}{3}x^6 + \frac{1416}{5}x^5 + \frac{635}{4}x^4 + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 200/11\*x^11 - 6\*x^10 + 922/9\*x^9 + 83/8\*x^8 + 1571/7\*x^7 + 299/3\*x^6 + 1416/5\*x^5 + 635/4\*x^4 + 237\*x^3 + 108\*x^2 + 108\*x

**maple** [A] time = 0.00, size = 55, normalized size = 0.81

$$\frac{200}{11}x^{11} - 6x^{10} + \frac{922}{9}x^9 + \frac{83}{8}x^8 + \frac{1571}{7}x^7 + \frac{299}{3}x^6 + \frac{1416}{5}x^5 + \frac{635}{4}x^4 + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^2,x)

[Out] 108\*x+108\*x^2+237\*x^3+635/4\*x^4+1416/5\*x^5+299/3\*x^6+1571/7\*x^7+83/8\*x^8+922/9\*x^9-6\*x^10+200/11\*x^11

**maxima** [A] time = 0.43, size = 54, normalized size = 0.79

$$\frac{200}{11}x^{11} - 6x^{10} + \frac{922}{9}x^9 + \frac{83}{8}x^8 + \frac{1571}{7}x^7 + \frac{299}{3}x^6 + \frac{1416}{5}x^5 + \frac{635}{4}x^4 + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 200/11\*x^11 - 6\*x^10 + 922/9\*x^9 + 83/8\*x^8 + 1571/7\*x^7 + 299/3\*x^6 + 1416/5\*x^5 + 635/4\*x^4 + 237\*x^3 + 108\*x^2 + 108\*x

**mupad** [B] time = 0.05, size = 54, normalized size = 0.79

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^3\*(3\*x + 5\*x^2 + 2)^2,x)

[Out] 108\*x + 108\*x^2 + 237\*x^3 + (635\*x^4)/4 + (1416\*x^5)/5 + (299\*x^6)/3 + (1571\*x^7)/7 + (83\*x^8)/8 + (922\*x^9)/9 - 6\*x^10 + (200\*x^11)/11

**sympy** [A] time = 0.08, size = 65, normalized size = 0.96

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*3\*(5\*x\*\*2+3\*x+2)\*\*2,x)

[Out] 200\*x\*\*11/11 - 6\*x\*\*10 + 922\*x\*\*9/9 + 83\*x\*\*8/8 + 1571\*x\*\*7/7 + 299\*x\*\*6/3 + 1416\*x\*\*5/5 + 635\*x\*\*4/4 + 237\*x\*\*3 + 108\*x\*\*2 + 108\*x

$$3.30 \quad \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2) dx$$

**Optimal.** Leaf size=56

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

**Rubi [A]** time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {1657}

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2), x]

[Out] 54\*x + (27\*x^2)/2 + 60\*x^3 - 5\*x^4 + (288\*x^5)/5 - (83\*x^6)/6 + (190\*x^7)/7 - (9\*x^8)/2 + (40\*x^9)/9

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2) dx &= \int (54 + 27x + 180x^2 - 20x^3 + 288x^4 - 83x^5 + 190x^6 - 36x^7 + 40x^8) dx \\ &= 54x + \frac{27x^2}{2} + 60x^3 - 5x^4 + \frac{288x^5}{5} - \frac{83x^6}{6} + \frac{190x^7}{7} - \frac{9x^8}{2} + \frac{40x^9}{9} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 56, normalized size = 1.00

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2), x]

[Out] 54\*x + (27\*x^2)/2 + 60\*x^3 - 5\*x^4 + (288\*x^5)/5 - (83\*x^6)/6 + (190\*x^7)/7 - (9\*x^8)/2 + (40\*x^9)/9

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - x + 2x^2)^3 (2 + 3x + 5x^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2), x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2), x]

**fricas [A]** time = 0.33, size = 44, normalized size = 0.79

$$\frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out]  $40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27/2*x^2 + 54*x$

**giac** [A] time = 0.20, size = 44, normalized size = 0.79

$$\frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2),x, algorithm="giac")

[Out]  $40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27/2*x^2 + 54*x$

**maple** [A] time = 0.00, size = 45, normalized size = 0.80

$$\frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2),x)

[Out]  $54*x+27/2*x^2+60*x^3-5*x^4+288/5*x^5-83/6*x^6+190/7*x^7-9/2*x^8+40/9*x^9$

**maxima** [A] time = 0.43, size = 44, normalized size = 0.79

$$\frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3\*(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out]  $40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27/2*x^2 + 54*x$

**mupad** [B] time = 0.03, size = 44, normalized size = 0.79

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^3\*(3\*x + 5\*x^2 + 2),x)

[Out]  $54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7 - (9*x^8)/2 + (40*x^9)/9$

**sympy** [A] time = 0.08, size = 53, normalized size = 0.95

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*3\*(5\*x\*\*2+3\*x+2),x)

[Out]  $40*x**9/9 - 9*x**8/2 + 190*x**7/7 - 83*x**6/6 + 288*x**5/5 - 5*x**4 + 60*x**3 + 27*x**2/2 + 54*x$

$$3.31 \quad \int \frac{(3-x+2x^2)^3}{2+3x+5x^2} dx$$

**Optimal.** Leaf size=70

$$\frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} - \frac{158389 \log(5x^2 + 3x + 2)}{31250} + \frac{49508x}{3125} + \frac{328757 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}$$

**Rubi [A]** time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1657, 634, 618, 204, 628}

$$\frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} - \frac{158389 \log(5x^2 + 3x + 2)}{31250} + \frac{49508x}{3125} + \frac{328757 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2), x]
```

```
[Out] (49508*x)/3125 - (7451*x^2)/1250 + (1222*x^3)/375 - (21*x^4)/25 + (8*x^5)/25 + (328757*ArcTan[(3 + 10*x)/Sqrt[31]])/(15625*Sqrt[31]) - (158389*Log[2 + 3*x + 5*x^2])/31250
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

#### Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^3}{2+3x+5x^2} dx &= \int \left( \frac{49508}{3125} - \frac{7451x}{625} + \frac{1222x^2}{125} - \frac{84x^3}{25} + \frac{8x^4}{5} - \frac{1331(11+119x)}{3125(2+3x+5x^2)} \right) dx \\
&= \frac{49508x}{3125} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25} - \frac{1331 \int \frac{11+119x}{2+3x+5x^2} dx}{3125} \\
&= \frac{49508x}{3125} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25} - \frac{158389 \int \frac{3+10x}{2+3x+5x^2} dx}{31250} + \frac{328757 \int \frac{1}{2+3x+5x^2} dx}{31250} \\
&= \frac{49508x}{3125} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25} - \frac{158389 \log(2+3x+5x^2)}{31250} - \frac{328757 \operatorname{arctan}\left(\frac{10x+3}{\sqrt{31}}\right)}{31250} \\
&= \frac{49508x}{3125} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25} + \frac{328757 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15625\sqrt{31}} - \frac{158389 \log(2+3x+5x^2)}{31250}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 63, normalized size = 0.90

$$\frac{31(5x(6000x^4 - 15750x^3 + 61100x^2 - 111765x + 297048) - 475167 \log(5x^2 + 3x + 2)) + 1972542\sqrt{31} \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{2906250}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2), x]

[Out] (1972542\*sqrt(31)\*ArcTan[(3 + 10\*x)/sqrt(31)] + 31\*(5\*x\*(297048 - 111765\*x + 61100\*x^2 - 15750\*x^3 + 6000\*x^4) - 475167\*Log[2 + 3\*x + 5\*x^2]))/2906250

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2), x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2), x]

**fricas [A]** time = 0.41, size = 53, normalized size = 0.76

$$\frac{8}{25}x^5 - \frac{21}{25}x^4 + \frac{1222}{375}x^3 - \frac{7451}{1250}x^2 + \frac{328757}{484375}\sqrt{31} \arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{49508}{3125}x - \frac{158389}{31250} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2), x, algorithm="fricas")

[Out] 8/25\*x^5 - 21/25\*x^4 + 1222/375\*x^3 - 7451/1250\*x^2 + 328757/484375\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 49508/3125\*x - 158389/31250\*log(5\*x^2 + 3\*x + 2)

**giac [A]** time = 0.21, size = 53, normalized size = 0.76

$$\frac{8}{25}x^5 - \frac{21}{25}x^4 + \frac{1222}{375}x^3 - \frac{7451}{1250}x^2 + \frac{328757}{484375}\sqrt{31} \arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{49508}{3125}x - \frac{158389}{31250} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2), x, algorithm="giac")

[Out]  $8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 49508/3125*x - 158389/31250*\log(5*x^2 + 3*x + 2)$

**maple [A]** time = 0.00, size = 54, normalized size = 0.77

$$\frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} + \frac{49508x}{3125} + \frac{328757\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{484375} - \frac{158389 \ln(5x^2 + 3x + 2)}{31250}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^3/(5*x^2+3*x+2),x)`

[Out]  $49508/3125*x - 7451/1250*x^2 + 1222/375*x^3 - 21/25*x^4 + 8/25*x^5 - 158389/31250*\ln(5*x^2 + 3*x + 2) + 328757/484375*31^{(1/2)}*\arctan(1/31*(10*x+3)*31^{(1/2)})$

**maxima [A]** time = 0.96, size = 53, normalized size = 0.76

$$\frac{8}{25}x^5 - \frac{21}{25}x^4 + \frac{1222}{375}x^3 - \frac{7451}{1250}x^2 + \frac{328757}{484375}\sqrt{31} \arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{49508}{3125}x - \frac{158389}{31250} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="maxima")`

[Out]  $8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 49508/3125*x - 158389/31250*\log(5*x^2 + 3*x + 2)$

**mupad [B]** time = 0.04, size = 55, normalized size = 0.79

$$\frac{49508x}{3125} - \frac{158389 \ln(5x^2 + 3x + 2)}{31250} + \frac{328757\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{484375} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2),x)`

[Out]  $(49508*x)/3125 - (158389*\log(3*x + 5*x^2 + 2))/31250 + (328757*31^{(1/2)}*\operatorname{atan}((10*31^{(1/2)}*x)/31 + (3*31^{(1/2)})/31))/484375 - (7451*x^2)/1250 + (1222*x^3)/375 - (21*x^4)/25 + (8*x^5)/25$

**sympy [A]** time = 0.16, size = 76, normalized size = 1.09

$$\frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} + \frac{49508x}{3125} - \frac{158389 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{31250} + \frac{328757\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{484375}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**3/(5*x**2+3*x+2),x)`

[Out]  $8*x**5/25 - 21*x**4/25 + 1222*x**3/375 - 7451*x**2/1250 + 49508*x/3125 - 158389*\log(x**2 + 3*x/5 + 2/5)/31250 + 328757*\sqrt{31}*\operatorname{atan}(10*\sqrt{31}*x/31 + 3*\sqrt{31}/31)/484375$

$$3.32 \quad \int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=77

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1331(247x+443)}{96875(5x^2+3x+2)} - \frac{10769 \log(5x^2+3x+2)}{6250} + \frac{1466x}{625} + \frac{3819607 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

**Rubi [A]** time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1331(247x+443)}{96875(5x^2+3x+2)} - \frac{10769 \log(5x^2+3x+2)}{6250} + \frac{1466x}{625} + \frac{3819607 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2)^2,x]

[Out] (1466\*x)/625 - (54\*x^2)/125 + (8\*x^3)/75 + (1331\*(443 + 247\*x))/(96875\*(2 + 3\*x + 5\*x^2)) + (3819607\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(96875\*Sqrt[31]) - (10769\*Log[2 + 3\*x + 5\*x^2])/6250

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[P

```

q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2} dx &= \frac{1331(443+247x)}{96875(2+3x+5x^2)} + \frac{1}{31} \int \frac{\frac{372701}{625} - \frac{230981x}{625} + \frac{37882x^2}{125} - \frac{2604x^3}{25} + \frac{248x^4}{5}}{2+3x+5x^2} dx \\
&= \frac{1331(443+247x)}{96875(2+3x+5x^2)} + \frac{1}{31} \int \left( \frac{45446}{625} - \frac{3348x}{125} + \frac{248x^2}{25} + \frac{121(2329-2759x)}{625(2+3x+5x^2)} \right) dx \\
&= \frac{1466x}{625} - \frac{54x^2}{125} + \frac{8x^3}{75} + \frac{1331(443+247x)}{96875(2+3x+5x^2)} + \frac{121 \int \frac{2329-2759x}{2+3x+5x^2} dx}{19375} \\
&= \frac{1466x}{625} - \frac{54x^2}{125} + \frac{8x^3}{75} + \frac{1331(443+247x)}{96875(2+3x+5x^2)} - \frac{10769 \int \frac{3+10x}{2+3x+5x^2} dx}{6250} + \frac{3819607 \int \frac{1}{2+3x+5x^2} dx}{193750} \\
&= \frac{1466x}{625} - \frac{54x^2}{125} + \frac{8x^3}{75} + \frac{1331(443+247x)}{96875(2+3x+5x^2)} - \frac{10769 \log(2+3x+5x^2)}{6250} - \frac{3819607 \operatorname{Sqrt}[2+3x+5x^2]}{193750} \\
&= \frac{1466x}{625} - \frac{54x^2}{125} + \frac{8x^3}{75} + \frac{1331(443+247x)}{96875(2+3x+5x^2)} + \frac{3819607 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{96875\sqrt{31}} - \frac{10769 \log(2+3x+5x^2)}{6250}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 77, normalized size = 1.00

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1331(247x+443)}{96875(5x^2+3x+2)} - \frac{10769 \log(5x^2+3x+2)}{6250} + \frac{1466x}{625} + \frac{3819607 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2)^2, x]

[Out] (1466\*x)/625 - (54\*x^2)/125 + (8\*x^3)/75 + (1331\*(443 + 247\*x))/(96875\*(2 + 3\*x + 5\*x^2)) + (3819607\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(96875\*Sqrt[31]) - (10769\*Log[2 + 3\*x + 5\*x^2])/6250

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2)^2, x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2)^2, x]

**fricas [A]** time = 0.41, size = 88, normalized size = 1.14

$$\frac{9610000x^5 - 33154500x^4 + 191815600x^3 + 22917642\sqrt{31}(5x^2+3x+2)\arctan\left(\frac{1}{\sqrt{31}}(10x+3)\right) + 111226140x^2 - 31047027(5x^2+3x+2)\log(5x^2+3x+2) + 145678362x + 109671738}{18018750(5x^2+3x+2)}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 1/18018750\*(9610000\*x^5 - 33154500\*x^4 + 191815600\*x^3 + 22917642\*sqrt(31)\*(5\*x^2 + 3\*x + 2)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 111226140\*x^2 - 31047027\*(5\*x^2 + 3\*x + 2)\*log(5\*x^2 + 3\*x + 2) + 145678362\*x + 109671738)/(5\*x^2 + 3\*x + 2)

**giac** [A] time = 0.21, size = 62, normalized size = 0.81

$$\frac{8}{75}x^3 - \frac{54}{125}x^2 + \frac{3819607}{3003125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{1466}{625}x + \frac{1331(247x+443)}{96875(5x^2+3x+2)} - \frac{10769}{6250}\log(5x^2+3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 8/75\*x^3 - 54/125\*x^2 + 3819607/3003125\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 1466/625\*x + 1331/96875\*(247\*x + 443)/(5\*x^2 + 3\*x + 2) - 10769/6250\*log(5\*x^2 + 3\*x + 2)

**maple** [A] time = 0.01, size = 61, normalized size = 0.79

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1466x}{625} + \frac{3819607\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{3003125} - \frac{10769\ln(5x^2+3x+2)}{6250} - \frac{121\left(-\frac{2717x}{775} - \frac{4873}{775}\right)}{625\left(x^2 + \frac{3}{5}x + \frac{2}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^2,x)

[Out] 8/75\*x^3-54/125\*x^2+1466/625\*x-121/625\*(-2717/775\*x-4873/775)/(x^2+3/5\*x+2/5)-10769/6250\*ln(5\*x^2+3\*x+2)+3819607/3003125\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))

**maxima** [A] time = 0.96, size = 62, normalized size = 0.81

$$\frac{8}{75}x^3 - \frac{54}{125}x^2 + \frac{3819607}{3003125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{1466}{625}x + \frac{1331(247x+443)}{96875(5x^2+3x+2)} - \frac{10769}{6250}\log(5x^2+3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 8/75\*x^3 - 54/125\*x^2 + 3819607/3003125\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 1466/625\*x + 1331/96875\*(247\*x + 443)/(5\*x^2 + 3\*x + 2) - 10769/6250\*log(5\*x^2 + 3\*x + 2)

**mupad** [B] time = 3.43, size = 61, normalized size = 0.79

$$\frac{1466x}{625} - \frac{10769\ln(5x^2+3x+2)}{6250} + \frac{\frac{328757x}{484375} + \frac{589633}{484375}}{x^2 + \frac{3x}{5} + \frac{2}{5}} + \frac{3819607\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{3003125} - \frac{54x^2}{125} + \frac{8x^3}{75}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^3/(3\*x + 5\*x^2 + 2)^2,x)

[Out] (1466\*x)/625 - (10769\*log(3\*x + 5\*x^2 + 2))/6250 + ((328757\*x)/484375 + 589633/484375)/((3\*x)/5 + x^2 + 2/5) + (3819607\*31^(1/2)\*atan((10\*31^(1/2)\*x)/31 + (3\*31^(1/2))/31))/3003125 - (54\*x^2)/125 + (8\*x^3)/75

**sympy [A]** time = 0.19, size = 78, normalized size = 1.01

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1466x}{625} + \frac{328757x + 589633}{484375x^2 + 290625x + 193750} - \frac{10769 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{6250} + \frac{3819607\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{3003125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*3/(5\*x\*\*2+3\*x+2)\*\*2,x)

[Out] 8\*x\*\*3/75 - 54\*x\*\*2/125 + 1466\*x/625 + (328757\*x + 589633)/(484375\*x\*\*2 + 290625\*x + 193750) - 10769\*log(x\*\*2 + 3\*x/5 + 2/5)/6250 + 3819607\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/3003125

$$3.33 \quad \int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3} dx$$

**Optimal.** Leaf size=84

$$\frac{121(342840x + 188381)}{6006250(5x^2 + 3x + 2)} + \frac{1331(247x + 443)}{193750(5x^2 + 3x + 2)^2} - \frac{66}{625} \log(5x^2 + 3x + 2) + \frac{8x}{125} + \frac{11341176 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{600625\sqrt{31}}$$

**Rubi [A]** time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(342840x + 188381)}{6006250(5x^2 + 3x + 2)} + \frac{1331(247x + 443)}{193750(5x^2 + 3x + 2)^2} - \frac{66}{625} \log(5x^2 + 3x + 2) + \frac{8x}{125} + \frac{11341176 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{600625\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2)^3,x]

[Out] (8\*x)/125 + (1331\*(443 + 247\*x))/(193750\*(2 + 3\*x + 5\*x^2)^2) + (121\*(188381 + 342840\*x))/(6006250\*(2 + 3\*x + 5\*x^2)) + (11341176\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(600625\*Sqrt[31]) - (66\*Log[2 + 3\*x + 5\*x^2])/625

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[P

```

q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]], Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3} dx &= \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{1}{62} \int \frac{\frac{4055767}{3125} - \frac{461962x}{625} + \frac{75764x^2}{125} - \frac{5208x^3}{25} + \frac{496x^4}{5}}{(2+3x+5x^2)^2} dx \\
&= \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \int \frac{\frac{2222876}{125} - \frac{207576x}{125} + \frac{15376x^2}{25}}{2+3x+5x^2} dx \\
&= \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{\int \left( \frac{15376}{125} + \frac{132(16607-1922x)}{125(2+3x+5x^2)} \right) dx}{1922} \\
&= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{66 \int \frac{16607-1922x}{2+3x+5x^2} dx}{120125} \\
&= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} - \frac{66}{625} \int \frac{3+10x}{2+3x+5x^2} dx + \frac{1}{1} \\
&= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} - \frac{66}{625} \log(2+3x+5x^2) - \frac{1}{1} \\
&= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{11341176 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{600625\sqrt{31}} - \frac{1}{1}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 78, normalized size = 0.93

$$\frac{\frac{3751(342840x+188381)}{5x^2+3x+2} + \frac{1279091(247x+443)}{(5x^2+3x+2)^2} - 19662060 \log(5x^2+3x+2) + 11916400x + 113411760\sqrt{31} \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{186193750}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2)^3, x]

[Out] (11916400\*x + (1279091\*(443 + 247\*x))/(2 + 3\*x + 5\*x^2)^2 + (3751\*(188381 + 342840\*x))/(2 + 3\*x + 5\*x^2) + 113411760\*sqrt[31]\*ArcTan[(3 + 10\*x)/sqrt[31]]) - 19662060\*Log[2 + 3\*x + 5\*x^2])/186193750

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2)^3, x]

[Out] IntegrateAlgebraic[(3 - x + 2\*x^2)^3/(2 + 3\*x + 5\*x^2)^3, x]

**fricas** [A] time = 0.39, size = 118, normalized size = 1.40

$$\frac{59582000x^5 + 71498400x^4 + 1355107960x^3 + 22682352\sqrt{31}(25x^4 + 30x^3 + 29x^2 + 12x + 4)\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 1506812195x^2 - 3932412(25x^4 + 30x^3 + 29x^2 + 12x + 4)\log(5x^2 + 3x + 2) + 1011087630x + 395974315}{37238750(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/37238750\*(59582000\*x^5 + 71498400\*x^4 + 1355107960\*x^3 + 22682352\*sqrt(31)\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 1506812195\*x^2 - 3932412\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*log(5\*x^2 + 3\*x + 2) + 1011087630\*x + 395974315)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)

**giac** [A] time = 0.19, size = 62, normalized size = 0.74

$$\frac{11341176}{18619375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{8}{125}x + \frac{121(68568x^3 + 78817x^2 + 53402x + 21113)}{240250(5x^2 + 3x + 2)^2} - \frac{66}{625}\log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 11341176/18619375\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 8/125\*x + 121/240250\*(68568\*x^3 + 78817\*x^2 + 53402\*x + 21113)/(5\*x^2 + 3\*x + 2)^2 - 66/625\*log(5\*x^2 + 3\*x + 2)

**maple** [A] time = 0.01, size = 63, normalized size = 0.75

$$\frac{8x}{125} + \frac{11341176\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{18619375} - \frac{66\ln(5x^2 + 3x + 2)}{625} - \frac{11\left(-\frac{377124}{24025}x^3 - \frac{866987}{48050}x^2 - \frac{293711}{24025}x - \frac{232243}{48050}\right)}{5(5x^2 + 3x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^3,x)

[Out] 8/125\*x-11/5\*(-377124/24025\*x^3-866987/48050\*x^2-293711/24025\*x-232243/48050)/(5\*x^2+3\*x+2)^2-66/625\*ln(5\*x^2+3\*x+2)+11341176/18619375\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))

**maxima** [A] time = 0.97, size = 72, normalized size = 0.86

$$\frac{11341176}{18619375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{8}{125}x + \frac{121(68568x^3 + 78817x^2 + 53402x + 21113)}{240250(25x^4 + 30x^3 + 29x^2 + 12x + 4)} - \frac{66}{625}\log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 11341176/18619375\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 8/125\*x + 121/240250\*(68568\*x^3 + 78817\*x^2 + 53402\*x + 21113)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4) - 66/625\*log(5\*x^2 + 3\*x + 2)

**mupad** [B] time = 3.43, size = 71, normalized size = 0.85

$$\frac{8x}{125} - \frac{66\ln(5x^2 + 3x + 2)}{625} + \frac{11341176\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{18619375} + \frac{\frac{4148364x^3}{3003125} + \frac{9536857x^2}{6006250} + \frac{3230821x}{3003125} + \frac{2554673}{6006250}}{x^4 + \frac{6x^3}{5} + \frac{29x^2}{25} + \frac{12x}{25} + \frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^3/(3\*x + 5\*x^2 + 2)^3,x)

[Out]  $(8x)/125 - (66 \log(3x + 5x^2 + 2))/625 + (11341176 \cdot 31^{1/2} \operatorname{atan}((10 \cdot 31^{1/2} x)/31 + (3 \cdot 31^{1/2})/31))/18619375 + ((3230821x)/3003125 + (9536857x^2)/6006250 + (4148364x^3)/3003125 + 2554673/6006250)/((12x)/25 + (29x^2)/25 + (6x^3)/5 + x^4 + 4/25)$

**sympy** [A] time = 0.23, size = 85, normalized size = 1.01

$$\frac{8x}{125} + \frac{8296728x^3 + 9536857x^2 + 6461642x + 2554673}{6006250x^4 + 7207500x^3 + 6967250x^2 + 2883000x + 961000} - \frac{66 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{625} + \frac{11341176\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{18619375}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)`

[Out]  $8x/125 + (8296728x^3 + 9536857x^2 + 6461642x + 2554673)/(6006250x^4 + 7207500x^3 + 6967250x^2 + 2883000x + 961000) - 66 \log(x^2 + 3x/5 + 2/5)/625 + 11341176 \sqrt{31} \operatorname{atan}(10 \sqrt{31} x/31 + 3 \sqrt{31}/31)/18619375$

$$3.34 \quad \int \frac{(2+3x+5x^2)^4}{3-x+2x^2} dx$$

**Optimal.** Leaf size=84

$$\frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{307461}{512} \log(2x^2 - x + 3) + \frac{122691x}{128} + \frac{1156639 \tan^{-1}}{256\sqrt{23}}$$

**Rubi [A]** time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1657, 634, 618, 204, 628}

$$\frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{307461}{512} \log(2x^2 - x + 3) + \frac{122691x}{128} + \frac{1156639 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2), x]

[Out] (122691\*x)/128 - (28747\*x^2)/128 - (21229\*x^3)/96 + (6245\*x^4)/64 + (1855\*x^5)/8 + (3625\*x^6)/24 + (625\*x^7)/14 + (1156639\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(256\*Sqrt[23]) + (307461\*Log[3 - x + 2\*x^2])/512

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[Expand[Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

#### Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^4}{3 - x + 2x^2} dx &= \int \left( \frac{122691}{128} - \frac{28747x}{64} - \frac{21229x^2}{32} + \frac{6245x^3}{16} + \frac{9275x^4}{8} + \frac{3625x^5}{4} + \frac{625x^6}{2} - \frac{14641(25 - x)}{128(3 - x + 2x^2)} \right) dx \\
&= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} - \frac{14641}{128} \int \frac{25 - x}{3 - x + 2x^2} dx \\
&= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{307461}{512} \int \frac{25 - x}{3 - x + 2x^2} dx \\
&= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{307461}{512} \log \left| \frac{2x - 1}{3 - x + 2x^2} \right| \\
&= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{1156639}{256\sqrt{23}} \arctan \left( \frac{4x - 1}{\sqrt{23}} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 72, normalized size = 0.86

$$\frac{307461}{512} \log(2x^2 - x + 3) + \frac{x(120000x^6 + 406000x^5 + 623280x^4 + 262290x^3 - 594412x^2 - 603687x + 2576511)}{2688} - \frac{1156639 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{256\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2), x]

[Out] (x\*(2576511 - 603687\*x - 594412\*x^2 + 262290\*x^3 + 623280\*x^4 + 406000\*x^5 + 120000\*x^6))/2688 - (1156639\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(256\*Sqrt[23]) + (307461\*Log[3 - x + 2\*x^2])/512

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2 + 3x + 5x^2)^4}{3 - x + 2x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2), x]

[Out] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2), x]

**fricas [A]** time = 0.40, size = 63, normalized size = 0.75

$$\frac{625}{14}x^7 + \frac{3625}{24}x^6 + \frac{1855}{8}x^5 + \frac{6245}{64}x^4 - \frac{21229}{96}x^3 - \frac{28747}{128}x^2 - \frac{1156639}{5888}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{122691}{128}x + \frac{307461}{512} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3), x, algorithm="fricas")

[Out] 625/14\*x^7 + 3625/24\*x^6 + 1855/8\*x^5 + 6245/64\*x^4 - 21229/96\*x^3 - 28747/128\*x^2 - 1156639/5888\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 122691/128\*x + 307461/512\*log(2\*x^2 - x + 3)

**giac [A]** time = 0.21, size = 63, normalized size = 0.75

$$\frac{625}{14}x^7 + \frac{3625}{24}x^6 + \frac{1855}{8}x^5 + \frac{6245}{64}x^4 - \frac{21229}{96}x^3 - \frac{28747}{128}x^2 - \frac{1156639}{5888}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{122691}{128}x + \frac{307461}{512} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3), x, algorithm="giac")



[Out]  $625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/128*x^2 - 1156639/5888*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 122691/128*x + 307461/512*\log(2*x^2 - x + 3)$

**maple [A]** time = 0.01, size = 64, normalized size = 0.76

$$\frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{122691x}{128} - \frac{1156639\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{5888} + \frac{307461 \ln(2x^2 - x + 3)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^4/(2*x^2-x+3), x)`

[Out]  $625/14*x^7+3625/24*x^6+1855/8*x^5+6245/64*x^4-21229/96*x^3-28747/128*x^2+122691/128*x+307461/512*\ln(2*x^2-x+3)-1156639/5888*23^{(1/2)}*\arctan(1/23*(4*x-1)*23^{(1/2)})$

**maxima [A]** time = 0.98, size = 63, normalized size = 0.75

$$\frac{625}{14}x^7 + \frac{3625}{24}x^6 + \frac{1855}{8}x^5 + \frac{6245}{64}x^4 - \frac{21229}{96}x^3 - \frac{28747}{128}x^2 - \frac{1156639}{5888}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{122691}{128}x + \frac{307461}{512} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^4/(2*x^2-x+3), x, algorithm="maxima")`

[Out]  $625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/128*x^2 - 1156639/5888*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 122691/128*x + 307461/512*\log(2*x^2 - x + 3)$

**mupad [B]** time = 3.44, size = 65, normalized size = 0.77

$$\frac{122691x}{128} + \frac{307461 \ln(2x^2 - x + 3)}{512} - \frac{1156639\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x - \sqrt{23}}{23}\right)}{5888} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3), x)`

[Out]  $(122691*x)/128 + (307461*\log(2*x^2 - x + 3))/512 - (1156639*23^{(1/2)}*\operatorname{atan}((4*23^{(1/2)}*x)/23 - 23^{(1/2)}/23))/5888 - (28747*x^2)/128 - (21229*x^3)/96 + (6245*x^4)/64 + (1855*x^5)/8 + (3625*x^6)/24 + (625*x^7)/14$

**sympy [A]** time = 0.17, size = 87, normalized size = 1.04

$$\frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{122691x}{128} + \frac{307461 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{512} - \frac{1156639\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x - \sqrt{23}}{23}\right)}{5888}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**4/(2*x**2-x+3), x)`

[Out]  $625*x**7/14 + 3625*x**6/24 + 1855*x**5/8 + 6245*x**4/64 - 21229*x**3/96 - 28747*x**2/128 + 122691*x/128 + 307461*\log(x**2 - x/2 + 3/2)/512 - 1156639*\sqrt{23}*\operatorname{atan}(4*\sqrt{23}*x/23 - \sqrt{23}/23)/5888$

$$3.35 \quad \int \frac{(2+3x+5x^2)^3}{3-x+2x^2} dx$$

**Optimal.** Leaf size=70

$$\frac{25x^5}{2} + \frac{575x^4}{16} + \frac{965x^3}{24} - \frac{829x^2}{32} + \frac{1331}{128} \log(2x^2 - x + 3) - \frac{4795x}{32} - \frac{59895 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}$$

**Rubi [A]** time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1657, 634, 618, 204, 628}

$$\frac{25x^5}{2} + \frac{575x^4}{16} + \frac{965x^3}{24} - \frac{829x^2}{32} + \frac{1331}{128} \log(2x^2 - x + 3) - \frac{4795x}{32} - \frac{59895 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2), x]
```

```
[Out] (-4795*x)/32 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16 + (25*x^5)/2 - (59895*ArcTan[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (1331*Log[3 - x + 2*x^2])/128
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

#### Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^3}{3-x+2x^2} dx &= \int \left( -\frac{4795}{32} - \frac{829x}{16} + \frac{965x^2}{8} + \frac{575x^3}{4} + \frac{125x^4}{2} + \frac{1331(11+x)}{32(3-x+2x^2)} \right) dx \\
&= -\frac{4795x}{32} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2} + \frac{1331}{32} \int \frac{11+x}{3-x+2x^2} dx \\
&= -\frac{4795x}{32} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2} + \frac{1331}{128} \int \frac{-1+4x}{3-x+2x^2} dx + \frac{59895}{128} \int \frac{1}{3-x+2x^2} dx \\
&= -\frac{4795x}{32} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2} + \frac{1331}{128} \log(3-x+2x^2) - \frac{59895}{64} \operatorname{Subst} \int \frac{1}{3-x+2x^2} dx \\
&= -\frac{4795x}{32} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2} - \frac{59895 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}} + \frac{1331}{128} \log(3-x+2x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 63, normalized size = 0.90

$$\frac{1}{384} \left( 3993 \log(2x^2 - x + 3) + 4x(1200x^4 + 3450x^3 + 3860x^2 - 2487x - 14385) \right) + \frac{59895 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{64\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2), x]

[Out] (59895\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(64\*Sqrt[23]) + (4\*x\*(-14385 - 2487\*x + 3860\*x^2 + 3450\*x^3 + 1200\*x^4) + 3993\*Log[3 - x + 2\*x^2])/384

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2), x]

[Out] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2), x]

**fricas [A]** time = 0.41, size = 53, normalized size = 0.76

$$\frac{25}{2}x^5 + \frac{575}{16}x^4 + \frac{965}{24}x^3 - \frac{829}{32}x^2 + \frac{59895}{1472}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{4795}{32}x + \frac{1331}{128} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3), x, algorithm="fricas")

[Out] 25/2\*x^5 + 575/16\*x^4 + 965/24\*x^3 - 829/32\*x^2 + 59895/1472\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 4795/32\*x + 1331/128\*log(2\*x^2 - x + 3)

**giac [A]** time = 0.21, size = 53, normalized size = 0.76

$$\frac{25}{2}x^5 + \frac{575}{16}x^4 + \frac{965}{24}x^3 - \frac{829}{32}x^2 + \frac{59895}{1472}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{4795}{32}x + \frac{1331}{128} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3), x, algorithm="giac")

[Out] 25/2\*x^5 + 575/16\*x^4 + 965/24\*x^3 - 829/32\*x^2 + 59895/1472\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 4795/32\*x + 1331/128\*log(2\*x^2 - x + 3)

**maple [A]** time = 0.00, size = 54, normalized size = 0.77

$$\frac{25x^5}{2} + \frac{575x^4}{16} + \frac{965x^3}{24} - \frac{829x^2}{32} - \frac{4795x}{32} + \frac{59895\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{1472} + \frac{1331 \ln(2x^2 - x + 3)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^3/(2\*x^2-x+3),x)

[Out] 25/2\*x^5+575/16\*x^4+965/24\*x^3-829/32\*x^2-4795/32\*x+1331/128\*ln(2\*x^2-x+3)+59895/1472\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima [A]** time = 0.97, size = 53, normalized size = 0.76

$$\frac{25}{2}x^5 + \frac{575}{16}x^4 + \frac{965}{24}x^3 - \frac{829}{32}x^2 + \frac{59895}{1472}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{4795}{32}x + \frac{1331}{128} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3),x, algorithm="maxima")

[Out] 25/2\*x^5 + 575/16\*x^4 + 965/24\*x^3 - 829/32\*x^2 + 59895/1472\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 4795/32\*x + 1331/128\*log(2\*x^2 - x + 3)

**mupad [B]** time = 0.04, size = 55, normalized size = 0.79

$$\frac{1331 \ln(2x^2 - x + 3)}{128} - \frac{4795x}{32} + \frac{59895\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{1472} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^3/(2\*x^2 - x + 3),x)

[Out] (1331\*log(2\*x^2 - x + 3))/128 - (4795\*x)/32 + (59895\*23^(1/2)\*atan((4\*23^(1/2)\*x)/23 - 23^(1/2)/23))/1472 - (829\*x^2)/32 + (965\*x^3)/24 + (575\*x^4)/16 + (25\*x^5)/2

**sympy [A]** time = 0.15, size = 73, normalized size = 1.04

$$\frac{25x^5}{2} + \frac{575x^4}{16} + \frac{965x^3}{24} - \frac{829x^2}{32} - \frac{4795x}{32} + \frac{1331 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{128} + \frac{59895\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{1472}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*3/(2\*x\*\*2-x+3),x)

[Out] 25\*x\*\*5/2 + 575\*x\*\*4/16 + 965\*x\*\*3/24 - 829\*x\*\*2/32 - 4795\*x/32 + 1331\*log(x\*\*2 - x/2 + 3/2)/128 + 59895\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/1472

$$3.36 \quad \int \frac{(2+3x+5x^2)^2}{3-x+2x^2} dx$$

Optimal. Leaf size=56

$$\frac{25x^3}{6} + \frac{85x^2}{8} - \frac{363}{32} \log(2x^2 - x + 3) + \frac{51x}{8} + \frac{847 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}$$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1657, 634, 618, 204, 628}

$$\frac{25x^3}{6} + \frac{85x^2}{8} - \frac{363}{32} \log(2x^2 - x + 3) + \frac{51x}{8} + \frac{847 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2), x]

[Out] (51\*x)/8 + (85\*x^2)/8 + (25\*x^3)/6 + (847\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(16\*Sqrt[23]) - (363\*Log[3 - x + 2\*x^2])/32

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[Expand[Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^2}{3-x+2x^2} dx &= \int \left( \frac{51}{8} + \frac{85x}{4} + \frac{25x^2}{2} - \frac{121(1+3x)}{8(3-x+2x^2)} \right) dx \\
&= \frac{51x}{8} + \frac{85x^2}{8} + \frac{25x^3}{6} - \frac{121}{8} \int \frac{1+3x}{3-x+2x^2} dx \\
&= \frac{51x}{8} + \frac{85x^2}{8} + \frac{25x^3}{6} - \frac{363}{32} \int \frac{-1+4x}{3-x+2x^2} dx - \frac{847}{32} \int \frac{1}{3-x+2x^2} dx \\
&= \frac{51x}{8} + \frac{85x^2}{8} + \frac{25x^3}{6} - \frac{363}{32} \log(3-x+2x^2) + \frac{847}{16} \operatorname{Subst} \left( \int \frac{1}{-23-x^2} dx, x, -1+4x \right) \\
&= \frac{51x}{8} + \frac{85x^2}{8} + \frac{25x^3}{6} + \frac{847 \tan^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{16\sqrt{23}} - \frac{363}{32} \log(3-x+2x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 52, normalized size = 0.93

$$\frac{1}{24}x(100x^2 + 255x + 153) - \frac{363}{32} \log(2x^2 - x + 3) - \frac{847 \tan^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{16\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2), x]

[Out] (x\*(153 + 255\*x + 100\*x^2))/24 - (847\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(16\*Sqrt[23]) - (363\*Log[3 - x + 2\*x^2])/32

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2), x]

[Out] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2), x]

**fricas [A]** time = 0.43, size = 43, normalized size = 0.77

$$\frac{25}{6}x^3 + \frac{85}{8}x^2 - \frac{847}{368}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{51}{8}x - \frac{363}{32} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3),x, algorithm="fricas")

[Out] 25/6\*x^3 + 85/8\*x^2 - 847/368\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 51/8\*x - 363/32\*log(2\*x^2 - x + 3)

**giac [A]** time = 0.18, size = 43, normalized size = 0.77

$$\frac{25}{6}x^3 + \frac{85}{8}x^2 - \frac{847}{368}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{51}{8}x - \frac{363}{32} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3),x, algorithm="giac")

[Out]  $25/6*x^3 + 85/8*x^2 - 847/368*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 51/8*x - 363/32*\log(2*x^2 - x + 3)$

**maple** [A] time = 0.00, size = 44, normalized size = 0.79

$$\frac{25x^3}{6} + \frac{85x^2}{8} + \frac{51x}{8} - \frac{847\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{368} - \frac{363 \ln(2x^2 - x + 3)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^2/(2*x^2-x+3), x)`

[Out]  $25/6*x^3+85/8*x^2+51/8*x-363/32*\ln(2*x^2-x+3)-847/368*23^{(1/2)}*\arctan(1/23*(4*x-1)*23^{(1/2)})$

**maxima** [A] time = 0.96, size = 43, normalized size = 0.77

$$\frac{25}{6}x^3 + \frac{85}{8}x^2 - \frac{847}{368}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{51}{8}x - \frac{363}{32}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2/(2*x^2-x+3), x, algorithm="maxima")`

[Out]  $25/6*x^3 + 85/8*x^2 - 847/368*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 51/8*x - 363/32*\log(2*x^2 - x + 3)$

**mupad** [B] time = 3.44, size = 45, normalized size = 0.80

$$\frac{51x}{8} - \frac{363 \ln(2x^2 - x + 3)}{32} - \frac{847\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{368} + \frac{85x^2}{8} + \frac{25x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3), x)`

[Out]  $(51*x)/8 - (363*\log(2*x^2 - x + 3))/32 - (847*23^{(1/2)}*\operatorname{atan}((4*23^{(1/2)}*x)/23 - 23^{(1/2)}/23))/368 + (85*x^2)/8 + (25*x^3)/6$

**sympy** [A] time = 0.15, size = 60, normalized size = 1.07

$$\frac{25x^3}{6} + \frac{85x^2}{8} + \frac{51x}{8} - \frac{363 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{32} - \frac{847\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{368}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**2/(2*x**2-x+3), x)`

[Out]  $25*x**3/6 + 85*x**2/8 + 51*x/8 - 363*\log(x**2 - x/2 + 3/2)/32 - 847*\sqrt{23}*\operatorname{atan}(4*\sqrt{23}*x/23 - \sqrt{23}/23)/368$

$$3.37 \quad \int \frac{2+3x+5x^2}{3-x+2x^2} dx$$

**Optimal.** Leaf size=42

$$\frac{11}{8} \log(2x^2 - x + 3) + \frac{5x}{2} + \frac{33 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}$$

**Rubi [A]** time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {1657, 634, 618, 204, 628}

$$\frac{11}{8} \log(2x^2 - x + 3) + \frac{5x}{2} + \frac{33 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2),x]

[Out] (5\*x)/2 + (33\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(4\*Sqrt[23]) + (11\*Log[3 - x + 2\*x^2])/8

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Int[Expand[Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

#### Rubi steps



$$\begin{aligned}
\int \frac{2+3x+5x^2}{3-x+2x^2} dx &= \int \left( \frac{5}{2} - \frac{11(1-x)}{2(3-x+2x^2)} \right) dx \\
&= \frac{5x}{2} - \frac{11}{2} \int \frac{1-x}{3-x+2x^2} dx \\
&= \frac{5x}{2} + \frac{11}{8} \int \frac{-1+4x}{3-x+2x^2} dx - \frac{33}{8} \int \frac{1}{3-x+2x^2} dx \\
&= \frac{5x}{2} + \frac{11}{8} \log(3-x+2x^2) + \frac{33}{4} \text{Subst} \left( \int \frac{1}{-23-x^2} dx, x, -1+4x \right) \\
&= \frac{5x}{2} + \frac{33 \tan^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{4\sqrt{23}} + \frac{11}{8} \log(3-x+2x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 42, normalized size = 1.00

$$\frac{11}{8} \log(2x^2 - x + 3) + \frac{5x}{2} - \frac{33 \tan^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{4\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2), x]

[Out] (5\*x)/2 - (33\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(4\*Sqrt[23]) + (11\*Log[3 - x + 2\*x^2])/8

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+3x+5x^2}{3-x+2x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2), x]

[Out] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2), x]

**fricas [A]** time = 0.39, size = 33, normalized size = 0.79

$$-\frac{33}{92} \sqrt{23} \arctan \left( \frac{1}{23} \sqrt{23} (4x-1) \right) + \frac{5}{2} x + \frac{11}{8} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3), x, algorithm="fricas")

[Out] -33/92\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 5/2\*x + 11/8\*log(2\*x^2 - x + 3)

**giac [A]** time = 0.21, size = 33, normalized size = 0.79

$$-\frac{33}{92} \sqrt{23} \arctan \left( \frac{1}{23} \sqrt{23} (4x-1) \right) + \frac{5}{2} x + \frac{11}{8} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3), x, algorithm="giac")

[Out] -33/92\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 5/2\*x + 11/8\*log(2\*x^2 - x + 3)

**maple [A]** time = 0.00, size = 34, normalized size = 0.81

$$\frac{5x}{2} - \frac{33\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{92} + \frac{11 \ln(2x^2 - x + 3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)/(2\*x^2-x+3),x)

[Out] 5/2\*x+11/8\*ln(2\*x^2-x+3)-33/92\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima [A]** time = 0.95, size = 33, normalized size = 0.79

$$-\frac{33}{92}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{5}{2}x + \frac{11}{8}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3),x, algorithm="maxima")

[Out] -33/92\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 5/2\*x + 11/8\*log(2\*x^2 - x + 3)

**mupad [B]** time = 0.04, size = 35, normalized size = 0.83

$$\frac{5x}{2} + \frac{11 \ln(2x^2 - x + 3)}{8} - \frac{33\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{92}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)/(2\*x^2 - x + 3),x)

[Out] (5\*x)/2 + (11\*log(2\*x^2 - x + 3))/8 - (33\*23^(1/2)\*atan((4\*23^(1/2)\*x)/23 - 23^(1/2)/23))/92

**sympy [A]** time = 0.14, size = 46, normalized size = 1.10

$$\frac{5x}{2} + \frac{11 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{8} - \frac{33\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{92}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)/(2\*x\*\*2-x+3),x)

[Out] 5\*x/2 + 11\*log(x\*\*2 - x/2 + 3/2)/8 - 33\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/92

$$3.38 \quad \int \frac{1}{(3-x+2x^2)(2+3x+5x^2)} dx$$

Optimal. Leaf size=73

$$-\frac{1}{44} \log(2x^2 - x + 3) + \frac{1}{44} \log(5x^2 + 3x + 2) + \frac{3 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {980, 634, 618, 204, 628}

$$-\frac{1}{44} \log(2x^2 - x + 3) + \frac{1}{44} \log(5x^2 + 3x + 2) + \frac{3 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)),x]

[Out] (3\*ArcTan[(1 - 4\*x)/Sqrt[23]]/(22\*Sqrt[23]) + (13\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(22\*Sqrt[31]) - Log[3 - x + 2\*x^2]/44 + Log[2 + 3\*x + 5\*x^2]/44

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

Rule 980

Int[1/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)), x\_Symbol] := With[{q = c^2\*d^2 - b\*c\*d\*e + a\*c\*e^2 + b^2\*d\*f - 2\*a\*c\*d\*f - a\*b\*e\*f + a^2\*f^2}, Dist[1/q, Int[(c^2\*d - b\*c\*e + b^2\*f - a\*c\*f - (c^2\*e - b\*c\*f)\*x)/(a + b\*x + c\*x^2), x], x] + Dist[1/q, Int[(c\*e^2 - c\*d\*f - b\*e\*f + a\*f^2 + (c\*e\*f - b\*f^2)\*x)/(d + e\*x + f\*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)} dx &= \frac{1}{242} \int \frac{-11-22x}{3-x+2x^2} dx + \frac{1}{242} \int \frac{88+55x}{2+3x+5x^2} dx \\
&= -\left(\frac{1}{44} \int \frac{-1+4x}{3-x+2x^2} dx\right) + \frac{1}{44} \int \frac{3+10x}{2+3x+5x^2} dx - \frac{3}{44} \int \frac{1}{3-x+2x^2} dx \\
&= -\frac{1}{44} \log(3-x+2x^2) + \frac{1}{44} \log(2+3x+5x^2) + \frac{3}{22} \text{Subst}\left(\int \frac{1}{-23-x^2} dx\right) \\
&= \frac{3 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{22\sqrt{31}} - \frac{1}{44} \log(3-x+2x^2) + \frac{1}{44} \log(2+3x+5x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 73, normalized size = 1.00

$$-\frac{1}{44} \log(2x^2 - x + 3) + \frac{1}{44} \log(5x^2 + 3x + 2) - \frac{3 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)), x]

[Out] (-3\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(22\*Sqrt[23]) + (13\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(22\*Sqrt[31]) - Log[3 - x + 2\*x^2]/44 + Log[2 + 3\*x + 5\*x^2]/44

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)), x]

[Out] IntegrateAlgebraic[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)), x]

**fricas [A]** time = 0.41, size = 59, normalized size = 0.81

$$\frac{13}{682} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x+3)\right) - \frac{3}{506} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x-1)\right) + \frac{1}{44} \log(5x^2+3x+2) - \frac{1}{44} \log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2), x, algorithm="fricas")

[Out] 13/682\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 3/506\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/44\*log(5\*x^2 + 3\*x + 2) - 1/44\*log(2\*x^2 - x + 3)

**giac [A]** time = 0.19, size = 59, normalized size = 0.81

$$\frac{13}{682} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x+3)\right) - \frac{3}{506} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x-1)\right) + \frac{1}{44} \log(5x^2+3x+2) - \frac{1}{44} \log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2), x, algorithm="giac")

[Out] 13/682\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 3/506\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/44\*log(5\*x^2 + 3\*x + 2) - 1/44\*log(2\*x^2 - x + 3)

**maple [A]** time = 0.01, size = 60, normalized size = 0.82

$$\frac{13\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{682} - \frac{3\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{506} - \frac{\ln(2x^2 - x + 3)}{44} + \frac{\ln(5x^2 + 3x + 2)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2), x)

[Out] 1/44\*ln(5\*x^2+3\*x+2)+13/682\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))-1/44\*ln(2\*x^2-x+3)-3/506\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima [A]** time = 0.96, size = 59, normalized size = 0.81

$$\frac{13}{682} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) - \frac{3}{506} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{1}{44} \log(5x^2 + 3x + 2) - \frac{1}{44} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2), x, algorithm="maxima")

[Out] 13/682\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 3/506\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/44\*log(5\*x^2 + 3\*x + 2) - 1/44\*log(2\*x^2 - x + 3)

**mupad [B]** time = 0.19, size = 79, normalized size = 1.08

$$\ln\left(x - \frac{1}{4} - \frac{\sqrt{23} 1i}{4}\right) \left(-\frac{1}{44} + \frac{\sqrt{23} 3i}{1012}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{23} 1i}{4}\right) \left(\frac{1}{44} + \frac{\sqrt{23} 3i}{1012}\right) - \ln\left(x + \frac{3}{10} - \frac{\sqrt{31} 1i}{10}\right) \left(-\frac{1}{44} + \frac{\sqrt{31} 13i}{1364}\right) + \ln\left(x + \frac{3}{10} + \frac{\sqrt{31} 1i}{10}\right) \left(\frac{1}{44} + \frac{\sqrt{31} 13i}{1364}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)\*(3\*x + 5\*x^2 + 2)), x)

[Out] log(x - (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*3i)/1012 - 1/44) - log(x + (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*3i)/1012 + 1/44) - log(x - (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*13i)/1364 - 1/44) + log(x + (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*13i)/1364 + 1/44)

**sympy [A]** time = 0.24, size = 83, normalized size = 1.14

$$-\frac{\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{44} + \frac{\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{44} - \frac{3\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{506} + \frac{13\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{682}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)/(5\*x\*\*2+3\*x+2), x)

[Out] -log(x\*\*2 - x/2 + 3/2)/44 + log(x\*\*2 + 3\*x/5 + 2/5)/44 - 3\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/506 + 13\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/682

$$3.39 \quad \int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=94

$$\frac{65x+4}{682(5x^2+3x+2)} + \frac{3}{968} \log(2x^2-x+3) - \frac{3}{968} \log(5x^2+3x+2) + \frac{7 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

**Rubi [A]** time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {974, 1072, 634, 618, 204, 628}

$$\frac{65x+4}{682(5x^2+3x+2)} + \frac{3}{968} \log(2x^2-x+3) - \frac{3}{968} \log(5x^2+3x+2) + \frac{7 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] (4 + 65\*x)/(682\*(2 + 3\*x + 5\*x^2)) + (7\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(484\*Sqrt[23]) + (2891\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(15004\*Sqrt[31]) + (3\*Log[3 - x + 2\*x^2])/968 - (3\*Log[2 + 3\*x + 5\*x^2])/968

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)))\*x\*(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^(q+1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), Int[(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p+1) - c\*d\*(p+2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p+q+2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f +

```

b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

### Rule 1072

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2} dx &= \frac{4+65x}{682(2+3x+5x^2)} - \frac{\int \frac{-1804+1397x-1430x^2}{(3-x+2x^2)(2+3x+5x^2)} dx}{7502} \\
&= \frac{4+65x}{682(2+3x+5x^2)} - \frac{\int \frac{18755-22506x}{3-x+2x^2} dx}{1815484} - \frac{\int \frac{-158026+56265x}{2+3x+5x^2} dx}{1815484} \\
&= \frac{4+65x}{682(2+3x+5x^2)} + \frac{3}{968} \int \frac{-1+4x}{3-x+2x^2} dx - \frac{3}{968} \int \frac{3+10x}{2+3x+5x^2} dx \\
&= \frac{4+65x}{682(2+3x+5x^2)} + \frac{3}{968} \log(3-x+2x^2) - \frac{3}{968} \log(2+3x+5x^2) \\
&= \frac{4+65x}{682(2+3x+5x^2)} + \frac{7 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15004\sqrt{31}} + \frac{3}{968} \log(
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 94, normalized size = 1.00

$$\frac{65x+4}{682(5x^2+3x+2)} + \frac{3}{968} \log(2x^2-x+3) - \frac{3}{968} \log(5x^2+3x+2) - \frac{7 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] (4 + 65\*x)/(682\*(2 + 3\*x + 5\*x^2)) - (7\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(484\*Sqrt[23]) + (2891\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(15004\*Sqrt[31]) + (3\*Log[3 - x + 2\*x^2])/968 - (3\*Log[2 + 3\*x + 5\*x^2])/968

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] IntegrateAlgebraic[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^2), x]

**fricas** [A] time = 0.41, size = 117, normalized size = 1.24

$$\frac{132986\sqrt{31}(5x^2+3x+2)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - 13454\sqrt{23}(5x^2+3x+2)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - 66309(5x^2+3x+2)\log(5x^2+3x+2) + 66309(5x^2+3x+2)\log(2x^2-x+3) + 2039180x + 125488}{21395704(5x^2+3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 1/21395704\*(132986\*sqrt(31)\*(5\*x^2 + 3\*x + 2)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 13454\*sqrt(23)\*(5\*x^2 + 3\*x + 2)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 66309\*(5\*x^2 + 3\*x + 2)\*log(5\*x^2 + 3\*x + 2) + 66309\*(5\*x^2 + 3\*x + 2)\*log(2\*x^2 - x + 3) + 2039180\*x + 125488)/(5\*x^2 + 3\*x + 2)

**giac** [A] time = 0.20, size = 78, normalized size = 0.83

$$\frac{2891}{465124}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{7}{11132}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{65x+4}{682(5x^2+3x+2)} - \frac{3}{968}\log(5x^2+3x+2) + \frac{3}{968}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 2891/465124\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 7/11132\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/682\*(65\*x + 4)/(5\*x^2 + 3\*x + 2) - 3/968\*log(5\*x^2 + 3\*x + 2) + 3/968\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 77, normalized size = 0.82

$$\frac{2891\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right) - 7\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right) + 3\ln(2x^2-x+3) - \frac{3\ln(5x^2+3x+2)}{968} - \frac{\frac{286x}{31} - \frac{88}{155}}{484\left(x^2 + \frac{3}{5}x + \frac{2}{5}\right)}}{465124} - \frac{7\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{11132} + \frac{3\ln(2x^2-x+3)}{968} - \frac{3\ln(5x^2+3x+2)}{968} - \frac{\frac{286x}{31} - \frac{88}{155}}{484\left(x^2 + \frac{3}{5}x + \frac{2}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^2,x)

[Out] -1/484\*(-286/31\*x-88/155)/(x^2+3/5\*x+2/5)-3/968\*ln(5\*x^2+3\*x+2)+2891/465124\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))+3/968\*ln(2\*x^2-x+3)-7/11132\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.96, size = 78, normalized size = 0.83

$$\frac{2891}{465124}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{7}{11132}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{65x+4}{682(5x^2+3x+2)} - \frac{3}{968}\log(5x^2+3x+2) + \frac{3}{968}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 2891/465124\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 7/11132\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/682\*(65\*x + 4)/(5\*x^2 + 3\*x + 2) - 3/968\*log(5\*x^2 + 3\*x + 2) + 3/968\*log(2\*x^2 - x + 3)

**mupad** [B] time = 3.57, size = 95, normalized size = 1.01

$$\frac{\frac{13x}{682} + \frac{2}{1705} + \ln\left(x - \frac{1}{4} - \frac{\sqrt{23}1i}{4}\right)\left(\frac{3}{968} + \frac{\sqrt{23}7i}{22264}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{23}1i}{4}\right)\left(-\frac{3}{968} + \frac{\sqrt{23}7i}{22264}\right) - \ln\left(x + \frac{3}{10} - \frac{\sqrt{31}1i}{10}\right)\left(\frac{3}{968} + \frac{\sqrt{31}2891i}{930248}\right) + \ln\left(x + \frac{3}{10} + \frac{\sqrt{31}1i}{10}\right)\left(-\frac{3}{968} + \frac{\sqrt{31}2891i}{930248}\right)}{x^2 + \frac{3x}{5} + \frac{2}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] int(1/((2\*x^2 - x + 3)\*(3\*x + 5\*x^2 + 2)^2),x)

[Out] ((13\*x)/682 + 2/1705)/((3\*x)/5 + x^2 + 2/5) + log(x - (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*7i)/22264 + 3/968) - log(x + (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*7i)/22264 - 3/968) - log(x - (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*2891i)/930248 + 3/968) + log(x + (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*2891i)/930248 - 3/968)

**sympy [A]** time = 0.32, size = 102, normalized size = 1.09

$$\frac{65x + 4}{3410x^2 + 2046x + 1364} + \frac{3 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{968} - \frac{3 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{968} - \frac{7\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{11132} + \frac{2891\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{465124}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)/(5\*x\*\*2+3\*x+2)\*\*2,x)

[Out] (65\*x + 4)/(3410\*x\*\*2 + 2046\*x + 1364) + 3\*log(x\*\*2 - x/2 + 3/2)/968 - 3\*log(x\*\*2 + 3\*x/5 + 2/5)/968 - 7\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/11132 + 2891\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/465124

$$3.40 \quad \int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3} dx$$

**Optimal.** Leaf size=115

$$\frac{65x+4}{1364(5x^2+3x+2)^2} + \frac{21605x+7923}{465124(5x^2+3x+2)} - \frac{\log(2x^2-x+3)}{21296} + \frac{\log(5x^2+3x+2)}{21296} - \frac{45 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{847793}{10232728\sqrt{31}}$$

**Rubi [A]** time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{65x+4}{1364(5x^2+3x+2)^2} + \frac{21605x+7923}{465124(5x^2+3x+2)} - \frac{\log(2x^2-x+3)}{21296} + \frac{\log(5x^2+3x+2)}{21296} - \frac{45 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{847793 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{10232728\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] (4 + 65\*x)/(1364\*(2 + 3\*x + 5\*x^2)^2) + (7923 + 21605\*x)/(465124\*(2 + 3\*x + 5\*x^2)) - (45\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(10648\*Sqrt[23]) + (847793\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(10232728\*Sqrt[31]) - Log[3 - x + 2\*x^2]/21296 + Log[2 + 3\*x + 5\*x^2]/21296

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^(q+1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), Int[(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p+1) - c\*d\*(p+2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b

```

^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

### Rule 1060

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_
)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

### Rule 1072

```

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*((d_) + (e_)*(x_) + (f_)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3} dx &= \frac{4+65x}{1364(2+3x+5x^2)^2} - \frac{\int \frac{-5753+3509x-4290x^2}{(3-x+2x^2)(2+3x+5x^2)^2} dx}{15004} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-14522420+3833038x-1045}{(3-x+2x^2)(2+3x+5x^2)} dx}{112560008} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-58838186+5116364x}{3-x+2x^2} dx}{27239521936} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-1+4x}{3-x+2x^2} dx}{21296} + \frac{\int \frac{3+10}{2+3x+5x^2} dx}{21296} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\log(3-x+2x^2)}{21296} + \frac{\log(2+3x+5x^2)}{21296} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{45 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{8477}{10648\sqrt{23}}
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 104, normalized size = 0.90

$$\frac{31 \left( -961 \log(2x^2 - x + 3) + 961 \log(5x^2 + 3x + 2) + \frac{44(108025x^3 + 104430x^2 + 89144x + 17210)}{(5x^2 + 3x + 2)^2} \right) + 1695586\sqrt{31} \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right) + \frac{45 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{10648\sqrt{23}}}{634429136}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] (45\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(10648\*Sqrt[23]) + (1695586\*Sqrt[31]\*ArcTan[(3 + 10\*x)/Sqrt[31]] + 31\*((44\*(17210 + 89144\*x + 104430\*x^2 + 108025\*x^3))/(2 + 3\*x + 5\*x^2)^2 - 961\*Log[3 - x + 2\*x^2] + 961\*Log[2 + 3\*x + 5\*x^2]))/634429136

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] IntegrateAlgebraic[1/((3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^3), x]

**fricas [A]** time = 0.40, size = 177, normalized size = 1.54

$$\frac{3388960300x^3 + 38998478\sqrt{31}(25x^4 + 30x^3 + 29x^2 + 12x + 4)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + 2681190\sqrt{23}(25x^4 + 30x^3 + 29x^2 + 12x + 4)\arctan\left(\frac{1}{\sqrt{23}}(4x-1)\right) + 327617960x^2 + 685193(25x^4 + 30x^3 + 29x^2 + 12x + 4)\log(5x^2 + 3x + 2) - 685193(25x^4 + 30x^3 + 29x^2 + 12x + 4)\log(2x^2 - x + 3) + 2796625568x + 539912120}{14591870128(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/14591870128\*(3388960300\*x^3 + 38998478\*sqrt(31)\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 2681190\*sqrt(23)\*(25\*x^4 +

$$30*x^3 + 29*x^2 + 12*x + 4)*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 3276177960*x^2 + 685193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*\log(5*x^2 + 3*x + 2) - 685193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*\log(2*x^2 - x + 3) + 2796625568*x + 539912120)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)$$

**giac [A]** time = 0.18, size = 88, normalized size = 0.77

$$\frac{847793}{317214568} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{45}{244904} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{108025x^3 + 104430x^2 + 89144x + 17210}{465124(5x^2 + 3x + 2)^2} + \frac{1}{21296} \log(5x^2 + 3x + 2) - \frac{1}{21296} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 847793/317214568\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 45/244904\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/465124\*(108025\*x^3 + 104430\*x^2 + 89144\*x + 17210)/(5\*x^2 + 3\*x + 2)^2 + 1/21296\*log(5\*x^2 + 3\*x + 2) - 1/21296\*log(2\*x^2 - x + 3)

**maple [A]** time = 0.01, size = 89, normalized size = 0.77

$$\frac{847793\sqrt{31} \arctan\left(\frac{10x+3}{31}\sqrt{31}\right)}{317214568} + \frac{45\sqrt{23} \arctan\left(\frac{4x-1}{23}\sqrt{23}\right)}{244904} - \frac{\ln(2x^2 - x + 3)}{21296} + \frac{\ln(5x^2 + 3x + 2)}{21296} + \frac{108025x^3 + 52215x^2 + 2026x + 8605}{465124(5x^2 + 3x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x)

[Out] 25/10648\*(95062/961\*x^3+459492/4805\*x^2+1961168/24025\*x+75724/4805)/(5\*x^2+3\*x+2)^2+1/21296\*ln(5\*x^2+3\*x+2)+847793/317214568\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))-1/21296\*ln(2\*x^2-x+3)+45/244904\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima [A]** time = 0.96, size = 98, normalized size = 0.85

$$\frac{847793}{317214568} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{45}{244904} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{108025x^3 + 104430x^2 + 89144x + 17210}{465124(25x^4 + 30x^3 + 29x^2 + 12x + 4)} + \frac{1}{21296} \log(5x^2 + 3x + 2) - \frac{1}{21296} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 847793/317214568\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 45/244904\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/465124\*(108025\*x^3 + 104430\*x^2 + 89144\*x + 17210)/(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4) + 1/21296\*log(5\*x^2 + 3\*x + 2) - 1/21296\*log(2\*x^2 - x + 3)

**mupad [B]** time = 0.18, size = 115, normalized size = 1.00

$$\frac{4321x^3 + 10443x^2 + 2026x + 1721}{465124x^4 + 1162810x^3 + 264275x^2 + 1162810x + 1721} + \ln\left(x - \frac{1}{4} + \frac{\sqrt{23}}{4}i\right) \left(-\frac{1}{21296} + \frac{\sqrt{23}}{489808}i\right) - \ln\left(x + \frac{3}{10} - \frac{\sqrt{31}}{10}i\right) \left(-\frac{1}{21296} + \frac{\sqrt{31}}{634429136}i\right) + \ln\left(x + \frac{3}{10} + \frac{\sqrt{31}}{10}i\right) \left(\frac{1}{21296} + \frac{\sqrt{31}}{634429136}i\right) - \ln\left(x - \frac{1}{4} - \frac{\sqrt{23}}{4}i\right) \left(\frac{1}{21296} + \frac{\sqrt{23}}{489808}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)\*(3\*x + 5\*x^2 + 2)^3),x)

[Out] log(x + (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*45i)/489808 - 1/21296) - log(x - (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*45i)/489808 + 1/21296) - log(x - (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*847793i)/634429136 - 1/21296) + log(x + (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*847793i)/634429136 + 1/21296) + ((2026\*x)/264275 + (10443\*x^2)/1162810 + (4321\*x^3)/465124 + 1721/1162810)/((12\*x)/25 + (29\*x^2)/25 + (6\*x^3)/5 + x^4 + 4/25)

**sympy [A]** time = 0.36, size = 119, normalized size = 1.03

$$\frac{108025x^3 + 104430x^2 + 89144x + 17210}{11628100x^4 + 13953720x^3 + 13488596x^2 + 5581488x + 1860496} - \frac{\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{21296} + \frac{\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{21296} + \frac{45\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x - \sqrt{23}}{23}\right)}{244904} + \frac{847793\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x + 3\sqrt{31}}{31}\right)}{317214568}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**3,x)
```

```
[Out] (108025*x**3 + 104430*x**2 + 89144*x + 17210)/(11628100*x**4 + 13953720*x**  
3 + 13488596*x**2 + 5581488*x + 1860496) - log(x**2 - x/2 + 3/2)/21296 + lo  
g(x**2 + 3*x/5 + 2/5)/21296 + 45*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/2  
3)/244904 + 847793*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/31721456  
8
```

$$3.41 \quad \int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2} dx$$

**Optimal.** Leaf size=91

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128} \log(2x^2-x+3) - \frac{89359x}{64} - \frac{13292697 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1472\sqrt{23}}$$

**Rubi [A]** time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128} \log(2x^2-x+3) - \frac{89359x}{64} - \frac{13292697 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1472\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^2, x]

[Out] (-89359\*x)/64 - (1185\*x^2)/8 + (9775\*x^3)/48 + (2125\*x^4)/16 + (125\*x^5)/4 - (14641\*(101 + 79\*x))/(2944\*(3 - x + 2\*x^2)) - (13292697\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(1472\*Sqrt[23]) - (30613\*Log[3 - x + 2\*x^2])/128

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[P

```

q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]], Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^4}{(3 - x + 2x^2)^2} dx &= -\frac{14641(101 + 79x)}{2944(3 - x + 2x^2)} + \frac{1}{23} \int \frac{\frac{832627}{64} - \frac{661181x}{64} - \frac{488267x^2}{32} + \frac{143635x^3}{16} + \frac{213325x^4}{8} + \frac{83375x^5}{4}}{3 - x + 2x^2} \\
&= -\frac{14641(101 + 79x)}{2944(3 - x + 2x^2)} + \frac{1}{23} \int \left( -\frac{2055257}{64} - \frac{27255x}{4} + \frac{224825x^2}{16} + \frac{48875x^3}{4} + \frac{14375x^4}{4} \right) \\
&= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101 + 79x)}{2944(3 - x + 2x^2)} + \frac{1331}{736} \int \frac{2629}{3 - x + 2x^2} \\
&= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101 + 79x)}{2944(3 - x + 2x^2)} - \frac{30613}{128} \int \frac{-1}{3 - x + 2x^2} \\
&= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101 + 79x)}{2944(3 - x + 2x^2)} - \frac{30613}{128} \log(3 - x + 2x^2) \\
&= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101 + 79x)}{2944(3 - x + 2x^2)} - \frac{13292697 \arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{1472\sqrt{23}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 91, normalized size = 1.00

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{14641(79x + 101)}{2944(2x^2 - x + 3)} - \frac{30613}{128} \log(2x^2 - x + 3) - \frac{89359x}{64} + \frac{13292697 \arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{1472\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^2, x]

[Out] (-89359\*x)/64 - (1185\*x^2)/8 + (9775\*x^3)/48 + (2125\*x^4)/16 + (125\*x^5)/4 - (14641\*(101 + 79\*x))/(2944\*(3 - x + 2\*x^2)) + (13292697\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(1472\*Sqrt[23]) - (30613\*Log[3 - x + 2\*x^2])/128

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2 + 3x + 5x^2)^4}{(3 - x + 2x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^2, x]

[Out] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^2, x]

**fricas [A]** time = 0.39, size = 98, normalized size = 1.08



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^2,x, algorithm="fricas")

[Out] 1/203136\*(12696000\*x^7 + 47610000\*x^6 + 74800600\*x^5 - 20609840\*x^4 - 413058012\*x^3 + 79756182\*sqrt(23)\*(2\*x^2 - x + 3)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 193356906\*x^2 - 48582831\*(2\*x^2 - x + 3)\*log(2\*x^2 - x + 3) - 930684489\*x - 102033129)/(2\*x^2 - x + 3)

**giac** [A] time = 0.18, size = 72, normalized size = 0.79

$$\frac{125}{4}x^5 + \frac{2125}{16}x^4 + \frac{9775}{48}x^3 - \frac{1185}{8}x^2 + \frac{13292697}{33856}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{89359}{64}x - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^2,x, algorithm="giac")

[Out] 125/4\*x^5 + 2125/16\*x^4 + 9775/48\*x^3 - 1185/8\*x^2 + 13292697/33856\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 89359/64\*x - 14641/2944\*(79\*x + 101)/(2\*x^2 - x + 3) - 30613/128\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 71, normalized size = 0.78

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{89359x}{64} + \frac{13292697\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{33856} - \frac{30613\ln(2x^2-x+3)}{128} - \frac{1331\left(\frac{869x}{92} + \frac{1111}{92}\right)}{64\left(x^2 - \frac{1}{2}x + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^2,x)

[Out] 125/4\*x^5+2125/16\*x^4+9775/48\*x^3-1185/8\*x^2-89359/64\*x-1331/64\*(869/92\*x+1111/92)/(x^2-1/2\*x+3/2)-30613/128\*ln(2\*x^2-x+3)+13292697/33856\*23^(1/2)\*arc tan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.96, size = 72, normalized size = 0.79

$$\frac{125}{4}x^5 + \frac{2125}{16}x^4 + \frac{9775}{48}x^3 - \frac{1185}{8}x^2 + \frac{13292697}{33856}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{89359}{64}x - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^2,x, algorithm="maxima")

[Out] 125/4\*x^5 + 2125/16\*x^4 + 9775/48\*x^3 - 1185/8\*x^2 + 13292697/33856\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 89359/64\*x - 14641/2944\*(79\*x + 101)/(2\*x^2 - x + 3) - 30613/128\*log(2\*x^2 - x + 3)

**mupad** [B] time = 3.46, size = 72, normalized size = 0.79

$$\frac{13292697\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x-\sqrt{23}}{23}\right)}{33856} - \frac{30613\ln(2x^2-x+3)}{128} - \frac{\frac{1156639x}{5888} + \frac{1478741}{5888}}{x^2 - \frac{x}{2} + \frac{3}{2}} - \frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^4/(2\*x^2 - x + 3)^2,x)

[Out] (13292697\*23^(1/2)\*atan((4\*23^(1/2)\*x)/23 - 23^(1/2)/23))/33856 - (30613\*log(2\*x^2 - x + 3))/128 - ((1156639\*x)/5888 + 1478741/5888)/(x^2 - x/2 + 3/2) - (89359\*x)/64 - (1185\*x^2)/8 + (9775\*x^3)/48 + (2125\*x^4)/16 + (125\*x^5)/4

sympy [A] time = 0.20, size = 90, normalized size = 0.99

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{89359x}{64} + \frac{-1156639x - 1478741}{5888x^2 - 2944x + 8832} - \frac{30613 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{128} + \frac{13292697\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{33856}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*4/(2\*x\*\*2-x+3)\*\*2,x)

[Out] 125\*x\*\*5/4 + 2125\*x\*\*4/16 + 9775\*x\*\*3/48 - 1185\*x\*\*2/8 - 89359\*x/64 + (-1156639\*x - 1478741)/(5888\*x\*\*2 - 2944\*x + 8832) - 30613\*log(x\*\*2 - x/2 + 3/2)/128 + 13292697\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/33856

$$3.42 \quad \int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2} dx$$

**Optimal.** Leaf size=77

$$\frac{125x^3}{12} + \frac{175x^2}{4} - \frac{1331(17-45x)}{736(2x^2-x+3)} - \frac{2057}{32} \log(2x^2-x+3) + \frac{915x}{16} + \frac{223971 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}$$

**Rubi [A]** time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{125x^3}{12} + \frac{175x^2}{4} - \frac{1331(17-45x)}{736(2x^2-x+3)} - \frac{2057}{32} \log(2x^2-x+3) + \frac{915x}{16} + \frac{223971 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^2,x]

[Out] (915\*x)/16 + (175\*x^2)/4 + (125\*x^3)/12 - (1331\*(17 - 45\*x))/(736\*(3 - x + 2\*x^2)) + (223971\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(368\*Sqrt[23]) - (2057\*Log[3 - x + 2\*x^2])/32

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[Expand[Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[P

```

q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]], Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^3}{(3 - x + 2x^2)^2} dx &= -\frac{1331(17 - 45x)}{736(3 - x + 2x^2)} + \frac{1}{23} \int \frac{-\frac{25195}{16} - \frac{19067x}{16} + \frac{22195x^2}{8} + \frac{13225x^3}{4} + \frac{2875x^4}{2}}{3 - x + 2x^2} dx \\
&= -\frac{1331(17 - 45x)}{736(3 - x + 2x^2)} + \frac{1}{23} \int \left( \frac{21045}{16} + \frac{4025x}{2} + \frac{2875x^2}{4} - \frac{121(365 + 391x)}{8(3 - x + 2x^2)} \right) dx \\
&= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17 - 45x)}{736(3 - x + 2x^2)} - \frac{121}{184} \int \frac{365 + 391x}{3 - x + 2x^2} dx \\
&= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17 - 45x)}{736(3 - x + 2x^2)} - \frac{2057}{32} \int \frac{-1 + 4x}{3 - x + 2x^2} dx - \frac{223971}{736} \int \frac{1}{3 - x + 2x^2} dx \\
&= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17 - 45x)}{736(3 - x + 2x^2)} - \frac{2057}{32} \log(3 - x + 2x^2) + \frac{223971}{368} \operatorname{Subst} \int \frac{1}{3 - x + 2x^2} dx \\
&= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17 - 45x)}{736(3 - x + 2x^2)} + \frac{223971 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}} - \frac{2057}{32} \log(3 - x + 2x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 77, normalized size = 1.00

$$\frac{125x^3}{12} + \frac{175x^2}{4} + \frac{1331(45x - 17)}{736(2x^2 - x + 3)} - \frac{2057}{32} \log(2x^2 - x + 3) + \frac{915x}{16} - \frac{223971 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{368\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^2,x]

[Out] (915\*x)/16 + (175\*x^2)/4 + (125\*x^3)/12 + (1331\*(-17 + 45\*x))/(736\*(3 - x + 2\*x^2)) - (223971\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(368\*Sqrt[23]) - (2057\*Log[3 - x + 2\*x^2])/32

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2 + 3x + 5x^2)^3}{(3 - x + 2x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^2,x]

[Out] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^2, x]

**fricas [A]** time = 0.43, size = 88, normalized size = 1.14

$$\frac{1058000x^5 + 3914600x^4 + 5173620x^3 - 1343826\sqrt{23}(2x^2 - x + 3)\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + 3761190x^2 - 3264459(2x^2 - x + 3)\log(2x^2 - x + 3) + 12845385x - 1561263}{50784(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^2,x, algorithm="fricas")

[Out] 1/50784\*(1058000\*x^5 + 3914600\*x^4 + 5173620\*x^3 - 1343826\*sqrt(23)\*(2\*x^2 - x + 3)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 3761190\*x^2 - 3264459\*(2\*x^2 - x + 3)\*log(2\*x^2 - x + 3) + 12845385\*x - 1561263)/(2\*x^2 - x + 3)

**giac** [A] time = 0.19, size = 62, normalized size = 0.81

$$\frac{125}{12}x^3 + \frac{175}{4}x^2 - \frac{223971}{8464}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{915}{16}x + \frac{1331(45x-17)}{736(2x^2-x+3)} - \frac{2057}{32}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^2,x, algorithm="giac")

[Out] 125/12\*x^3 + 175/4\*x^2 - 223971/8464\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 915/16\*x + 1331/736\*(45\*x - 17)/(2\*x^2 - x + 3) - 2057/32\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 61, normalized size = 0.79

$$\frac{125x^3}{12} + \frac{175x^2}{4} + \frac{915x}{16} - \frac{223971\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{8464} - \frac{2057\ln(2x^2-x+3)}{32} - \frac{121\left(-\frac{495x}{92} + \frac{187}{92}\right)}{16\left(x^2 - \frac{1}{2}x + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^2,x)

[Out] 125/12\*x^3+175/4\*x^2+915/16\*x-121/16\*(-495/92\*x+187/92)/(x^2-1/2\*x+3/2)-2057/32\*ln(2\*x^2-x+3)-223971/8464\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.96, size = 62, normalized size = 0.81

$$\frac{125}{12}x^3 + \frac{175}{4}x^2 - \frac{223971}{8464}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{915}{16}x + \frac{1331(45x-17)}{736(2x^2-x+3)} - \frac{2057}{32}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^2,x, algorithm="maxima")

[Out] 125/12\*x^3 + 175/4\*x^2 - 223971/8464\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 915/16\*x + 1331/736\*(45\*x - 17)/(2\*x^2 - x + 3) - 2057/32\*log(2\*x^2 - x + 3)

**mupad** [B] time = 3.42, size = 61, normalized size = 0.79

$$\frac{915x}{16} - \frac{2057\ln(2x^2-x+3)}{32} + \frac{\frac{59895x}{1472} - \frac{22627}{1472}}{x^2 - \frac{x}{2} + \frac{3}{2}} - \frac{223971\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{8464} + \frac{175x^2}{4} + \frac{125x^3}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^3/(2\*x^2 - x + 3)^2,x)

[Out] (915\*x)/16 - (2057\*log(2\*x^2 - x + 3))/32 + ((59895\*x)/1472 - 22627/1472)/(x^2 - x/2 + 3/2) - (223971\*23^(1/2)\*atan((4\*23^(1/2)\*x)/23 - 23^(1/2)/23))/8464 + (175\*x^2)/4 + (125\*x^3)/12

**sympy** [A] time = 0.19, size = 75, normalized size = 0.97

$$\frac{125x^3}{12} + \frac{175x^2}{4} + \frac{915x}{16} + \frac{59895x - 22627}{1472x^2 - 736x + 2208} - \frac{2057\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{32} - \frac{223971\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{8464}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**2,x)
```

```
[Out] 125*x**3/12 + 175*x**2/4 + 915*x/16 + (59895*x - 22627)/(1472*x**2 - 736*x  
+ 2208) - 2057*log(x**2 - x/2 + 3/2)/32 - 223971*sqrt(23)*atan(4*sqrt(23)*x  
/23 - sqrt(23)/23)/8464
```

$$3.43 \quad \int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2} dx$$

**Optimal.** Leaf size=63

$$\frac{121(19-7x)}{184(2x^2-x+3)} + \frac{55}{8} \log(2x^2-x+3) + \frac{25x}{4} + \frac{1859 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}$$

**Rubi [A]** time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(19-7x)}{184(2x^2-x+3)} + \frac{55}{8} \log(2x^2-x+3) + \frac{25x}{4} + \frac{1859 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^2,x]

[Out] (25\*x)/4 + (121\*(19 - 7\*x))/(184\*(3 - x + 2\*x^2)) + (1859\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(92\*Sqrt[23]) + (55\*Log[3 - x + 2\*x^2])/8

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Int[Expand[Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[P

q, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]], Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(2 + 3x + 5x^2)^2}{(3 - x + 2x^2)^2} dx &= \frac{121(19 - 7x)}{184(3 - x + 2x^2)} + \frac{1}{23} \int \frac{\frac{163}{4} + \frac{1955x}{4} + \frac{575x^2}{2}}{3 - x + 2x^2} dx \\
 &= \frac{121(19 - 7x)}{184(3 - x + 2x^2)} + \frac{1}{23} \int \left( \frac{575}{4} - \frac{11(71 - 115x)}{2(3 - x + 2x^2)} \right) dx \\
 &= \frac{25x}{4} + \frac{121(19 - 7x)}{184(3 - x + 2x^2)} - \frac{11}{46} \int \frac{71 - 115x}{3 - x + 2x^2} dx \\
 &= \frac{25x}{4} + \frac{121(19 - 7x)}{184(3 - x + 2x^2)} + \frac{55}{8} \int \frac{-1 + 4x}{3 - x + 2x^2} dx - \frac{1859}{184} \int \frac{1}{3 - x + 2x^2} dx \\
 &= \frac{25x}{4} + \frac{121(19 - 7x)}{184(3 - x + 2x^2)} + \frac{55}{8} \log(3 - x + 2x^2) + \frac{1859}{92} \text{Subst} \left( \int \frac{1}{-23 - x^2} dx, x, -1 + \right. \\
 &= \frac{25x}{4} + \frac{121(19 - 7x)}{184(3 - x + 2x^2)} + \frac{1859 \tan^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{92\sqrt{23}} + \frac{55}{8} \log(3 - x + 2x^2)
 \end{aligned}$$

**Mathematica** [A] time = 0.03, size = 63, normalized size = 1.00

$$-\frac{121(7x - 19)}{184(2x^2 - x + 3)} + \frac{55}{8} \log(2x^2 - x + 3) + \frac{25x}{4} - \frac{1859 \tan^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{92\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^2,x]

[Out] (25\*x)/4 - (121\*(-19 + 7\*x))/(184\*(3 - x + 2\*x^2)) - (1859\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(92\*Sqrt[23]) + (55\*Log[3 - x + 2\*x^2])/8

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2 + 3x + 5x^2)^2}{(3 - x + 2x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^2,x]

[Out] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^2, x]

**fricas** [A] time = 0.41, size = 78, normalized size = 1.24

$$\frac{52900x^3 - 3718\sqrt{23}(2x^2 - x + 3)\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - 26450x^2 + 29095(2x^2 - x + 3)\log(2x^2 - x + 3) + 59869x + 52877}{4232(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^2,x, algorithm="fricas")

[Out] 1/4232\*(52900\*x^3 - 3718\*sqrt(23)\*(2\*x^2 - x + 3)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 26450\*x^2 + 29095\*(2\*x^2 - x + 3)\*log(2\*x^2 - x + 3) + 59869\*x + 52877)/(2\*x^2 - x + 3)

**giac** [A] time = 0.17, size = 52, normalized size = 0.83

$$-\frac{1859}{2116} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{25}{4} x - \frac{121(7x - 19)}{184(2x^2 - x + 3)} + \frac{55}{8} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^2,x, algorithm="giac")

[Out] -1859/2116\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 25/4\*x - 121/184\*(7\*x - 19)/(2\*x^2 - x + 3) + 55/8\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{25x}{4} - \frac{1859\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2116} + \frac{55 \ln(2x^2 - x + 3)}{8} + \frac{-\frac{847x}{368} + \frac{2299}{368}}{x^2 - \frac{1}{2}x + \frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^2,x)

[Out] 25/4\*x+11/4\*(-77/92\*x+209/92)/(x^2-1/2\*x+3/2)+55/8\*ln(2\*x^2-x+3)-1859/2116\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.97, size = 52, normalized size = 0.83

$$-\frac{1859}{2116} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{25}{4} x - \frac{121(7x - 19)}{184(2x^2 - x + 3)} + \frac{55}{8} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^2,x, algorithm="maxima")

[Out] -1859/2116\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 25/4\*x - 121/184\*(7\*x - 19)/(2\*x^2 - x + 3) + 55/8\*log(2\*x^2 - x + 3)

**mupad** [B] time = 3.40, size = 52, normalized size = 0.83

$$\frac{25x}{4} + \frac{55 \ln(2x^2 - x + 3)}{8} - \frac{\frac{847x}{368} - \frac{2299}{368}}{x^2 - \frac{x}{2} + \frac{3}{2}} - \frac{1859 \sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{2116}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^2/(2\*x^2 - x + 3)^2,x)

[Out] (25\*x)/4 + (55\*log(2\*x^2 - x + 3))/8 - ((847\*x)/368 - 2299/368)/(x^2 - x/2 + 3/2) - (1859\*23^(1/2)\*atan((4\*23^(1/2)\*x)/23 - 23^(1/2)/23))/2116

**sympy** [A] time = 0.19, size = 61, normalized size = 0.97

$$\frac{25x}{4} + \frac{2299 - 847x}{368x^2 - 184x + 552} + \frac{55 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{8} - \frac{1859\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{2116}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**2,x)
```

```
[Out] 25*x/4 + (2299 - 847*x)/(368*x**2 - 184*x + 552) + 55*log(x**2 - x/2 + 3/2)
/8 - 1859*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/2116
```

$$3.44 \quad \int \frac{2+3x+5x^2}{(3-x+2x^2)^2} dx$$

Optimal. Leaf size=43

$$-\frac{11(3x+5)}{46(2x^2-x+3)} - \frac{82 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$ , Rules used = {1660, 12, 618, 204}

$$-\frac{11(3x+5)}{46(2x^2-x+3)} - \frac{82 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) - (82*ArcTan[(1 - 4*x)/Sqrt[23]])/(23*Sqrt[23])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x+5x^2}{(3-x+2x^2)^2} dx &= -\frac{11(5+3x)}{46(3-x+2x^2)} + \frac{1}{23} \int \frac{41}{3-x+2x^2} dx \\
&= -\frac{11(5+3x)}{46(3-x+2x^2)} + \frac{41}{23} \int \frac{1}{3-x+2x^2} dx \\
&= -\frac{11(5+3x)}{46(3-x+2x^2)} - \frac{82}{23} \text{Subst} \left( \int \frac{1}{-23-x^2} dx, x, -1+4x \right) \\
&= -\frac{11(5+3x)}{46(3-x+2x^2)} - \frac{82 \tan^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{23\sqrt{23}}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 43, normalized size = 1.00

$$\frac{82 \tan^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{23\sqrt{23}} - \frac{11(3x+5)}{46(2x^2-x+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^2, x]

[Out] (-11\*(5 + 3\*x))/(46\*(3 - x + 2\*x^2)) + (82\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(23\*Sqrt[23])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^2, x]

[Out] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^2, x]

**fricas [A]** time = 0.40, size = 45, normalized size = 1.05

$$\frac{164 \sqrt{23} (2x^2 - x + 3) \arctan \left( \frac{1}{23} \sqrt{23} (4x - 1) \right) - 759x - 1265}{1058 (2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^2,x, algorithm="fricas")

[Out] 1/1058\*(164\*sqrt(23)\*(2\*x^2 - x + 3)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 759\*x - 1265)/(2\*x^2 - x + 3)

**giac [A]** time = 0.19, size = 36, normalized size = 0.84

$$\frac{82}{529} \sqrt{23} \arctan \left( \frac{1}{23} \sqrt{23} (4x - 1) \right) - \frac{11(3x+5)}{46(2x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^2,x, algorithm="giac")

[Out]  $82/529*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 - x + 3)$

**maple** [A] time = 0.00, size = 34, normalized size = 0.79

$$\frac{82\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{529} + \frac{-\frac{33x}{92} - \frac{55}{92}}{x^2 - \frac{1}{2}x + \frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)/(2*x^2-x+3)^2,x)`

[Out]  $(-33/92*x-55/92)/(x^2-1/2*x+3/2)+82/529*23^{(1/2)}*\arctan(1/23*(4*x-1)*23^{(1/2)})$

**maxima** [A] time = 0.96, size = 36, normalized size = 0.84

$$\frac{82}{529} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{11(3x + 5)}{46(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="maxima")`

[Out]  $82/529*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 - x + 3)$

**mupad** [B] time = 0.04, size = 36, normalized size = 0.84

$$\frac{82\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{529} - \frac{\frac{33x}{92} + \frac{55}{92}}{x^2 - \frac{x}{2} + \frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^2,x)`

[Out]  $(82*23^{(1/2)}*\operatorname{atan}((4*23^{(1/2)}*x)/23 - 23^{(1/2)}/23))/529 - ((33*x)/92 + 55/92)/(x^2 - x/2 + 3/2)$

**sympy** [A] time = 0.15, size = 42, normalized size = 0.98

$$\frac{-33x - 55}{92x^2 - 46x + 138} + \frac{82\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{529}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)/(2*x**2-x+3)**2,x)`

[Out]  $(-33*x - 55)/(92*x**2 - 46*x + 138) + 82*\sqrt{23}*\operatorname{atan}(4*\sqrt{23}*x/23 - \sqrt{23}/23)/529$

$$3.45 \quad \int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)} dx$$

**Optimal.** Leaf size=94

$$\frac{13-6x}{506(2x^2-x+3)} - \frac{13}{968} \log(2x^2-x+3) + \frac{13}{968} \log(5x^2+3x+2) + \frac{241 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

**Rubi [A]** time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {974, 1072, 634, 618, 204, 628}

$$\frac{13-6x}{506(2x^2-x+3)} - \frac{13}{968} \log(2x^2-x+3) + \frac{13}{968} \log(5x^2+3x+2) + \frac{241 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)), x]

[Out] (13 - 6\*x)/(506\*(3 - x + 2\*x^2)) + (241\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(11132\*Sqrt[23]) + (69\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(484\*Sqrt[31]) - (13\*Log[3 - x + 2\*x^2])/968 + (13\*Log[2 + 3\*x + 5\*x^2])/968

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)))\*x\*(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^(q+1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), Int[(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p+1) - c\*d\*(p+2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p+q+2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f +

```

b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

### Rule 1072

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)} dx &= \frac{13-6x}{506(3-x+2x^2)} - \frac{\int \frac{-1892-1067x+330x^2}{(3-x+2x^2)(2+3x+5x^2)} dx}{5566} \\
&= \frac{13-6x}{506(3-x+2x^2)} - \frac{\int \frac{-3509+72358x}{3-x+2x^2} dx}{1346972} - \frac{\int \frac{-150282-180895x}{2+3x+5x^2} dx}{1346972} \\
&= \frac{13-6x}{506(3-x+2x^2)} - \frac{241 \int \frac{1}{3-x+2x^2} dx}{22264} - \frac{13}{968} \int \frac{-1+4x}{3-x+2x^2} dx + \frac{13}{968} \\
&= \frac{13-6x}{506(3-x+2x^2)} - \frac{13}{968} \log(3-x+2x^2) + \frac{13}{968} \log(2+3x+5x^2) + \\
&= \frac{13-6x}{506(3-x+2x^2)} + \frac{241 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{484\sqrt{31}} - \frac{13}{968} \log(3-x+2x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 94, normalized size = 1.00

$$\frac{13-6x}{506(2x^2-x+3)} - \frac{13}{968} \log(2x^2-x+3) + \frac{13}{968} \log(5x^2+3x+2) - \frac{241 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)), x]

[Out] (13 - 6\*x)/(506\*(3 - x + 2\*x^2)) - (241\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(11132\*Sqrt[23]) + (69\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(484\*Sqrt[31]) - (13\*Log[3 - x + 2\*x^2])/968 + (13\*Log[2 + 3\*x + 5\*x^2])/968

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)), x]

[Out] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)), x]

**fricas** [A] time = 0.41, size = 117, normalized size = 1.24

$$\frac{73002\sqrt{31}(2x^2-x+3)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-14942\sqrt{23}(2x^2-x+3)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+213187(2x^2-x+3)\log(5x^2+3x+2)-213187(2x^2-x+3)\log(2x^2-x+3)-188232x+407836}{15874232(2x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2), x, algorithm="fricas")

[Out] 1/15874232\*(73002\*sqrt(31)\*(2\*x^2 - x + 3)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 14942\*sqrt(23)\*(2\*x^2 - x + 3)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 213187\*(2\*x^2 - x + 3)\*log(5\*x^2 + 3\*x + 2) - 213187\*(2\*x^2 - x + 3)\*log(2\*x^2 - x + 3) - 188232\*x + 407836)/(2\*x^2 - x + 3)

**giac** [A] time = 0.19, size = 78, normalized size = 0.83

$$\frac{69}{15004}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{241}{256036}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{6x-13}{506(2x^2-x+3)}+\frac{13}{968}\log(5x^2+3x+2)-\frac{13}{968}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2), x, algorithm="giac")

[Out] 69/15004\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 241/256036\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 1/506\*(6\*x - 13)/(2\*x^2 - x + 3) + 13/968\*log(5\*x^2 + 3\*x + 2) - 13/968\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 77, normalized size = 0.82

$$\frac{69\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)-241\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)-13\ln(2x^2-x+3)+13\ln(5x^2+3x+2)-\frac{66x-\frac{143}{23}}{484\left(x^2-\frac{1}{2}x+\frac{3}{2}\right)}}{15004-\frac{256036}{256036}-\frac{968}{968}+\frac{13\ln(5x^2+3x+2)}{968}-\frac{66x-\frac{143}{23}}{484\left(x^2-\frac{1}{2}x+\frac{3}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2), x)

[Out] 13/968\*ln(5\*x^2+3\*x+2)+69/15004\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))-1/484\*(66/23\*x-143/23)/(x^2-1/2\*x+3/2)-13/968\*ln(2\*x^2-x+3)-241/256036\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.96, size = 78, normalized size = 0.83

$$\frac{69}{15004}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{241}{256036}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{6x-13}{506(2x^2-x+3)}+\frac{13}{968}\log(5x^2+3x+2)-\frac{13}{968}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2), x, algorithm="maxima")

[Out] 69/15004\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 241/256036\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 1/506\*(6\*x - 13)/(2\*x^2 - x + 3) + 13/968\*log(5\*x^2 + 3\*x + 2) - 13/968\*log(2\*x^2 - x + 3)

**mupad** [B] time = 3.58, size = 96, normalized size = 1.02

$$-\frac{3x-\frac{13}{2}}{x^2-\frac{1}{2}x+\frac{3}{2}}-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}11i}{10}\right)\left(-\frac{13}{968}+\frac{\sqrt{31}69i}{30008}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}11i}{10}\right)\left(\frac{13}{968}+\frac{\sqrt{31}69i}{30008}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}11i}{4}\right)\left(-\frac{13}{968}+\frac{\sqrt{23}241i}{512072}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}11i}{4}\right)\left(\frac{13}{968}+\frac{\sqrt{23}241i}{512072}\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] int(1/((2\*x^2 - x + 3)^2\*(3\*x + 5\*x^2 + 2)),x)

[Out]  $\log(x + (31^{1/2}*1i)/10 + 3/10)*((31^{1/2}*69i)/30008 + 13/968) - \log(x - (31^{1/2}*1i)/10 + 3/10)*((31^{1/2}*69i)/30008 - 13/968) - ((3*x)/506 - 13/1012)/(x^2 - x/2 + 3/2) + \log(x - (23^{1/2}*1i)/4 - 1/4)*((23^{1/2}*241i)/512072 - 13/968) - \log(x + (23^{1/2}*1i)/4 - 1/4)*((23^{1/2}*241i)/512072 + 13/968)$

**sympy [A]** time = 0.32, size = 102, normalized size = 1.09

$$\frac{13 - 6x}{1012x^2 - 506x + 1518} - \frac{13 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{968} + \frac{13 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{968} - \frac{241\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{256036} + \frac{69\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{15004}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*2/(5\*x\*\*2+3\*x+2),x)

[Out]  $(13 - 6*x)/(1012*x**2 - 506*x + 1518) - 13*\log(x**2 - x/2 + 3/2)/968 + 13*\log(x**2 + 3*x/5 + 2/5)/968 - 241*\sqrt{23}*\operatorname{atan}(4*\sqrt{23}*x/23 - \sqrt{23}/23)/256036 + 69*\sqrt{31}*\operatorname{atan}(10*\sqrt{31}*x/31 + 3*\sqrt{31}/31)/15004$

$$3.46 \quad \int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=127

$$-\frac{25(117-137x)}{172546(5x^2+3x+2)} + \frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)} + \frac{19 \log(2x^2-x+3)}{10648} - \frac{19 \log(5x^2+3x+2)}{10648} + \frac{2769}{122452\sqrt{23}} + \frac{12643}{165044\sqrt{31}}$$

**Rubi [A]** time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$-\frac{25(117-137x)}{172546(5x^2+3x+2)} + \frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)} + \frac{19 \log(2x^2-x+3)}{10648} - \frac{19 \log(5x^2+3x+2)}{10648} + \frac{2769 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{122452\sqrt{23}} + \frac{12643 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{165044\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] (-25\*(117 - 137\*x))/(172546\*(2 + 3\*x + 5\*x^2)) + (13 - 6\*x)/(506\*(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)) + (2769\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(122452\*Sqrt[23]) + (12643\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(165044\*Sqrt[31]) + (19\*Log[3 - x + 2\*x^2])/10648 - (19\*Log[2 + 3\*x + 5\*x^2])/10648

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[Rt[-b, 2]\*x]/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^(q+1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), Int[(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p+1) - c\*d\*(p+2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b

```

^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

### Rule 1060

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_
)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

### Rule 1072

```

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*((d_) + (e_)*(x_) + (f_)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx &= \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{-2321-2299x+990x^2}{(3-x+2x^2)(2+3x+5x^2)^2} dx}{5566} \\
&= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{-303419x}{(3-x+2x^2)(2+3x+5x^2)^2} dx}{5566} \\
&= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{13228276}{3-x+2x^2} dx}{1010} \\
&= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} + \frac{19 \int \frac{-1}{3-x} dx}{1064} \\
&= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} + \frac{19 \log(3-x)}{1064} \\
&= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} + \frac{2769 \tan^{-1}\left(\frac{3-x}{\sqrt{31}}\right)}{12245}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 106, normalized size = 0.83

$$\frac{9659011 \log(2x^2 - x + 3) - 9659011 \log(5x^2 + 3x + 2) + \frac{31372(6850x^3 - 9275x^2 + 11154x - 4342)}{10x^4 + x^3 + 16x^2 + 7x + 6} - 5322018\sqrt{23} \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right) + 13376294\sqrt{31} \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{5413113112}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] ((31372\*(-4342 + 11154\*x - 9275\*x^2 + 6850\*x^3))/(6 + 7\*x + 16\*x^2 + x^3 + 10\*x^4) - 5322018\*sqrt(23)\*ArcTan[(-1 + 4\*x)/sqrt(23)] + 13376294\*sqrt(31)\*ArcTan[(3 + 10\*x)/sqrt(31)] + 9659011\*Log[3 - x + 2\*x^2] - 9659011\*Log[2 + 3\*x + 5\*x^2])/5413113112

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^2), x]

**fricas [A]** time = 0.43, size = 167, normalized size = 1.31

$$\frac{214898200x^3 + 13376294\sqrt{31}(10x^4 + x^3 + 16x^2 + 7x + 6)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - 5322018\sqrt{23}(10x^4 + x^3 + 16x^2 + 7x + 6)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - 290975300x^2 - 9659011(10x^4 + x^3 + 16x^2 + 7x + 6)\log(5x^2 + 3x + 2) + 9659011(10x^4 + x^3 + 16x^2 + 7x + 6)\log(2x^2 - x + 3) + 349923288x - 136217224}{5413113112(10x^4 + x^3 + 16x^2 + 7x + 6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 1/5413113112\*(214898200\*x^3 + 13376294\*sqrt(31)\*(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 5322018\*sqrt(23)\*(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 290975300\*x^2 - 9659011\*(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6)\*log(5\*x^2 + 3\*x + 2) + 9659011\*(10\*x^4 +

$$x^3 + 16x^2 + 7x + 6) \cdot \log(2x^2 - x + 3) + 349923288x - 136217224) / (10x^4 + x^3 + 16x^2 + 7x + 6)$$

**giac** [A] time = 0.19, size = 96, normalized size = 0.76

$$\frac{12643}{5116364} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x+3)\right) - \frac{2769}{2816396} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x-1)\right) + \frac{6850x^3 - 9275x^2 + 11154x - 4342}{172546(10x^4 + x^3 + 16x^2 + 7x + 6)} - \frac{19}{10648} \log(5x^2 + 3x + 2) + \frac{19}{10648} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 12643/5116364\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 2769/2816396\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/172546\*(6850\*x^3 - 9275\*x^2 + 11154\*x - 4342)/(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6) - 19/10648\*log(5\*x^2 + 3\*x + 2) + 19/10648\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 94, normalized size = 0.74

$$\frac{12643\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right) - 2769\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right) + 19 \ln(2x^2 - x + 3) - 19 \ln(5x^2 + 3x + 2)}{5116364} - \frac{759x + 1078}{5324\left(x^2 + \frac{3}{5}x + \frac{2}{5}\right)} + \frac{-\frac{77x}{23} - \frac{341}{46}}{5324x^2 - 2662x + 7986}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^2,x)

[Out] -1/5324\*(-759/31\*x+1078/155)/(x^2+3/5\*x+2/5)-19/10648\*ln(5\*x^2+3\*x+2)+12643/5116364\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))+1/5324\*(-77/23\*x-341/46)/(x^2-1/2\*x+3/2)+19/10648\*ln(2\*x^2-x+3)-2769/2816396\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.96, size = 96, normalized size = 0.76

$$\frac{12643}{5116364} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x+3)\right) - \frac{2769}{2816396} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x-1)\right) + \frac{6850x^3 - 9275x^2 + 11154x - 4342}{172546(10x^4 + x^3 + 16x^2 + 7x + 6)} - \frac{19}{10648} \log(5x^2 + 3x + 2) + \frac{19}{10648} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 12643/5116364\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 2769/2816396\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/172546\*(6850\*x^3 - 9275\*x^2 + 11154\*x - 4342)/(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6) - 19/10648\*log(5\*x^2 + 3\*x + 2) + 19/10648\*log(2\*x^2 - x + 3)

**mupad** [B] time = 0.18, size = 115, normalized size = 0.91

$$\ln\left(x - \frac{1}{4} - \frac{\sqrt{23}i}{4}\right) \left(\frac{19}{10648} + \frac{\sqrt{23} 2769i}{5632792}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{23}i}{4}\right) \left(-\frac{19}{10648} + \frac{\sqrt{23} 2769i}{5632792}\right) - \ln\left(x + \frac{3}{10} - \frac{\sqrt{31}i}{10}\right) \left(\frac{19}{10648} + \frac{\sqrt{31} 12643i}{10232728}\right) + \ln\left(x + \frac{3}{10} + \frac{\sqrt{31}i}{10}\right) \left(-\frac{19}{10648} + \frac{\sqrt{31} 12643i}{10232728}\right) + \frac{685x^3 - 1855x^2 + 507x - 2171}{x^4 + \frac{x^3}{10} + \frac{8x^2}{5} + \frac{7x}{10} + \frac{3}{5}}{172546} + \frac{345092}{78430} + \frac{78430}{862730}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^2\*(3\*x + 5\*x^2 + 2)^2),x)

[Out] log(x - (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*2769i)/5632792 + 19/10648) - log(x + (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*2769i)/5632792 - 19/10648) - log(x - (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*12643i)/10232728 + 19/10648) + log(x + (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*12643i)/10232728 - 19/10648) + ((507\*x)/78430 - (1855\*x^2)/345092 + (685\*x^3)/172546 - 2171/862730)/((7\*x)/10 + (8\*x^2)/5 + x^3/10 + x^4 + 3/5)

**sympy** [A] time = 0.36, size = 122, normalized size = 0.96

$$\frac{6850x^3 - 9275x^2 + 11154x - 4342}{1725460x^4 + 172546x^3 + 2760736x^2 + 1207822x + 1035276} + \frac{19 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{10648} - \frac{19 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{10648} - \frac{2769\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x - \sqrt{23}}{23}\right)}{2816396} + \frac{12643\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x + 3\sqrt{31}}{31}\right)}{5116364}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)
```

```
[Out] (6850*x**3 - 9275*x**2 + 11154*x - 4342)/(1725460*x**4 + 172546*x**3 + 2760
736*x**2 + 1207822*x + 1035276) + 19*log(x**2 - x/2 + 3/2)/10648 - 19*log(x
**2 + 3*x/5 + 2/5)/10648 - 2769*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23
)/2816396 + 12643*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/5116364
```

$$3.47 \quad \int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3} dx$$

**Optimal.** Leaf size=148

$$\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)^2} + \frac{3996965x+1765599}{235352744(5x^2+3x+2)} + \frac{5765x-9446}{690184(5x^2+3x+2)^2} + \frac{97 \log(2x^2-x+3)}{468512}$$

**Rubi [A]** time = 0.16, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$-\frac{9446-5765x}{690184(5x^2+3x+2)^2} + \frac{3996965x+1765599}{235352744(5x^2+3x+2)} + \frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)^2} + \frac{97 \log(2x^2-x+3)}{468512} - \frac{97 \log(5x^2+3x+2)}{468512} - \frac{25557 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5387888\sqrt{23}} + \frac{4464079 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{225120016\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] -(9446 - 5765\*x)/(690184\*(2 + 3\*x + 5\*x^2)^2) + (13 - 6\*x)/(506\*(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^2) + (1765599 + 3996965\*x)/(235352744\*(2 + 3\*x + 5\*x^2)) - (25557\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(5387888\*Sqrt[23]) + (4464079\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(225120016\*Sqrt[31]) + (97\*Log[3 - x + 2\*x^2])/468512 - (97\*Log[2 + 3\*x + 5\*x^2])/468512

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^(q+1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), Int[(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1) - (2\*c^2\*d + b^2\*f

```
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

### Rule 1060

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

### Rule 1072

```
Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3} dx &= \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} - \frac{\int \frac{-2750-3531x+1650x^2}{(3-x+2x^2)(2+3x+5x^2)^3} dx}{5566} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} - \frac{\int \frac{-8}{(3-x+2x^2)(2+3x+5x^2)^3} dx}{235} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{17}{235} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{17}{235} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{17}{235} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{17}{235} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{17}{235}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 136, normalized size = 0.92

$$\frac{90x-11}{244904(2x^2-x+3)} + \frac{164380x+67573}{10232728(5x^2+3x+2)} + \frac{345x-98}{30008(5x^2+3x+2)^2} + \frac{97 \log(2x^2-x+3)}{468512} - \frac{97 \log(5x^2+3x+2)}{468512} + \frac{25557 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{5387888\sqrt{23}} + \frac{4464079 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{225120016\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] (-11 + 90\*x)/(244904\*(3 - x + 2\*x^2)) + (-98 + 345\*x)/(30008\*(2 + 3\*x + 5\*x^2)^2) + (67573 + 164380\*x)/(10232728\*(2 + 3\*x + 5\*x^2)) + (25557\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(5387888\*Sqrt[23]) + (4464079\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(225120016\*Sqrt[31]) + (97\*Log[3 - x + 2\*x^2])/468512 - (97\*Log[2 + 3\*x + 5\*x^2])/468512

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^3), x]

**fricas [A]** time = 0.41, size = 237, normalized size = 1.60

$$\frac{1536278988x^7 + 874968428x^6 + 2188211888x^5 + 472198582\sqrt{31}\sqrt{23}(9x^4 + 38x^3 + 85x^2 + 52x + 12)\arctan\left(\frac{4x-1}{\sqrt{23}}\right) + 15270714\sqrt{31}\sqrt{23}(9x^4 + 38x^3 + 85x^2 + 52x + 12)\arctan\left(\frac{10x+3}{\sqrt{31}}\right) - 1802818434x^4 - 173868881(9x^4 + 38x^3 + 85x^2 + 52x + 12)\log(5x^2+3x+2) + 173868881(9x^4 + 38x^3 + 85x^2 + 52x + 12)\log(2x^2-x+3) - 13386895241x + 2188881260}{23848824096(9x^4 + 38x^3 + 85x^2 + 52x + 12)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/7383486284768\*(1253927859800\*x^5 + 679296504260\*x^4 + 2185021181068\*x^3 + 4722995582\*sqrt(31)\*(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^3 + 83\*x^2 + 32\*x + 12)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 1522737174\*sqrt(23)\*(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^3 + 83\*x^2 + 32\*x + 12)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1500218514344\*x^2 - 1528665583\*(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^3 + 83\*x^2 + 32\*x + 12)\*log(5\*x^2 + 3\*x + 2) + 1528665583\*(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^3 + 83\*x^2 + 32\*x + 12)\*log(2\*x^2 - x + 3) + 1338609358240\*x + 218880812656)/(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^3 + 83\*x^2 + 32\*x + 12)

**giac** [A] time = 0.20, size = 110, normalized size = 0.74

$$\frac{4464079}{6978720496} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x+3)\right) + \frac{25557}{123921424} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x-1)\right) + \frac{39969650x^5 + 21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{235352744(5x^2+3x+2)^2(2x^2-x+3)} - \frac{97}{468512} \log(5x^2+3x+2) + \frac{97}{468512} \log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 4464079/6978720496\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 25557/123921424\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/235352744\*(39969650\*x^5 + 21652955\*x^4 + 69648769\*x^3 + 47820302\*x^2 + 42668920\*x + 6976948)/((5\*x^2 + 3\*x + 2)^2\*(2\*x^2 - x + 3)) - 97/468512\*log(5\*x^2 + 3\*x + 2) + 97/468512\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 106, normalized size = 0.72

$$\frac{4464079\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{6978720496} + \frac{25557\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{123921424} + \frac{97 \ln(2x^2-x+3)}{468512} - \frac{97 \ln(5x^2+3x+2)}{468512} - \frac{25 \left( \frac{723272x^3}{961} - \frac{3656422x^2}{4805} - \frac{14280728x}{24025} - \frac{2238016}{24025} \right)}{234256(5x^2+3x+2)^2} + \frac{990x - 121}{234256x^2 - 117128x + 351384}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x)

[Out] -25/234256\*(-723272/961\*x^3-3656422/4805\*x^2-14280728/24025\*x-2238016/24025)/(5\*x^2+3\*x+2)^2-97/468512\*ln(5\*x^2+3\*x+2)+4464079/6978720496\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))+1/234256\*(990/23\*x-121/23)/(x^2-1/2\*x+3/2)+97/468512\*ln(2\*x^2-x+3)+25557/123921424\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.97, size = 118, normalized size = 0.80

$$\frac{4464079}{6978720496} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x+3)\right) + \frac{25557}{123921424} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x-1)\right) + \frac{39969650x^5 + 21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{235352744(50x^6+35x^5+103x^4+85x^3+83x^2+32x+12)} - \frac{97}{468512} \log(5x^2+3x+2) + \frac{97}{468512} \log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^2/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 4464079/6978720496\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 25557/123921424\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/235352744\*(39969650\*x^5 + 21652955\*x^4 + 69648769\*x^3 + 47820302\*x^2 + 42668920\*x + 6976948)/(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^3 + 83\*x^2 + 32\*x + 12) - 97/468512\*log(5\*x^2 + 3\*x + 2) + 97/468512\*log(2\*x^2 - x + 3)

**mupad** [B] time = 3.59, size = 135, normalized size = 0.91

$$\frac{799393x^5}{235352744} + \frac{4330991x^4}{235352744} + \frac{69648769x^3}{1176763720} + \frac{22910151x^2}{8883818600} + \frac{1066723x}{294490900} + \frac{198567}{267446300} + \ln\left(x - \frac{1}{4} + \frac{\sqrt{23} 11}{4}\right) \left(\frac{97}{468512} + \frac{\sqrt{23} 255571}{247842848}\right) - \ln\left(x - \frac{1}{4} - \frac{\sqrt{23} 11}{4}\right) \left(-\frac{97}{468512} + \frac{\sqrt{23} 255571}{247842848}\right) - \ln\left(x + \frac{3}{10} - \frac{\sqrt{31} 11}{10}\right) \left(\frac{97}{468512} + \frac{\sqrt{31} 44640791}{13957440992}\right) + \ln\left(x + \frac{3}{10} + \frac{\sqrt{31} 11}{10}\right) \left(-\frac{97}{468512} + \frac{\sqrt{31} 44640791}{13957440992}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^2\*(3\*x + 5\*x^2 + 2)^3),x)

[Out] log(x + (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*25557i)/247842848 + 97/468512) - log(x - (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*25557i)/247842848 - 97/468512) +

$$\begin{aligned} & ((1066723*x)/294190930 + (23910151*x^2)/5883818600 + (69648769*x^3)/1176763 \\ & 7200 + (4330591*x^4)/2353527440 + (799393*x^5)/235352744 + 158567/267446300 \\ & )/((16*x)/25 + (83*x^2)/50 + (17*x^3)/10 + (103*x^4)/50 + (7*x^5)/10 + x^6 \\ & + 6/25) - \log(x - (31^{(1/2)}*1i)/10 + 3/10)*((31^{(1/2)}*4464079i)/13957440992 \\ & + 97/468512) + \log(x + (31^{(1/2)}*1i)/10 + 3/10)*((31^{(1/2)}*4464079i)/13957 \\ & 440992 - 97/468512) \end{aligned}$$

**sympy [A]** time = 0.40, size = 143, normalized size = 0.97

$$\frac{39969650x^5 + 21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{11767637200x^6 + 8237346040x^5 + 24241332632x^4 + 20004983240x^3 + 19534277752x^2 + 7531287808x + 2824232928} + \frac{97 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{468512} - \frac{97 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{468512} + \frac{25557\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x - \sqrt{23}}{23}\right)}{123921424} + \frac{4464079\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x + 3\sqrt{31}}{31}\right)}{6978720496}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*2/(5\*x\*\*2+3\*x+2)\*\*3, x)

[Out] (39969650\*x\*\*5 + 21652955\*x\*\*4 + 69648769\*x\*\*3 + 47820302\*x\*\*2 + 42668920\*x + 6976948)/(11767637200\*x\*\*6 + 8237346040\*x\*\*5 + 24241332632\*x\*\*4 + 20004983240\*x\*\*3 + 19534277752\*x\*\*2 + 7531287808\*x + 2824232928) + 97\*log(x\*\*2 - x/2 + 3/2)/468512 - 97\*log(x\*\*2 + 3\*x/5 + 2/5)/468512 + 25557\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/123921424 + 4464079\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/6978720496

$$3.48 \quad \int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3} dx$$

**Optimal.** Leaf size=98

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{1331(76420x + 5229)}{135424(2x^2 - x + 3)} - \frac{14641(79x + 101)}{5888(2x^2 - x + 3)^2} - \frac{13915}{64} \log(2x^2 - x + 3) + \frac{2725x}{8} + \frac{63799791 \tan^{-1}}{16928\sqrt{2}}$$

**Rubi [A]** time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{1331(76420x + 5229)}{135424(2x^2 - x + 3)} - \frac{14641(79x + 101)}{5888(2x^2 - x + 3)^2} - \frac{13915}{64} \log(2x^2 - x + 3) + \frac{2725x}{8} + \frac{63799791 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16928\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]
```

```
[Out] (2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3 - x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) + (63799791*ArcTan[(1 - 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*x^2])/64
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

#### Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

#### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(2 + 3x + 5x^2)^4}{(3 - x + 2x^2)^3} dx &= -\frac{14641(101 + 79x)}{5888(3 - x + 2x^2)^2} + \frac{1}{46} \int \frac{\frac{2173869}{128} - \frac{661181x}{32} - \frac{488267x^2}{16} + \frac{143635x^3}{8} + \frac{213325x^4}{4} + \frac{833}{1}}{(3 - x + 2x^2)^2} dx \\ &= -\frac{14641(101 + 79x)}{5888(3 - x + 2x^2)^2} + \frac{1331(5229 + 76420x)}{135424(3 - x + 2x^2)} + \frac{\int \frac{-\frac{5460539}{8} - \frac{626865x}{2} + \frac{5170975x^2}{8} + \frac{1124125x^3}{2} + 3}{3 - x + 2x^2} dx}{1058} \\ &= -\frac{14641(101 + 79x)}{5888(3 - x + 2x^2)^2} + \frac{1331(5229 + 76420x)}{135424(3 - x + 2x^2)} + \frac{\int \left( \frac{1441525}{4} + \frac{2578875x}{8} + \frac{330625x^2}{4} - 1 \right)}{1058} dx \\ &= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101 + 79x)}{5888(3 - x + 2x^2)^2} + \frac{1331(5229 + 76420x)}{135424(3 - x + 2x^2)} - \frac{121 \int \frac{1}{3 - x + 2x^2} dx}{64} \\ &= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101 + 79x)}{5888(3 - x + 2x^2)^2} + \frac{1331(5229 + 76420x)}{135424(3 - x + 2x^2)} - \frac{13915}{64} \int \frac{1}{3 - x + 2x^2} dx \\ &= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101 + 79x)}{5888(3 - x + 2x^2)^2} + \frac{1331(5229 + 76420x)}{135424(3 - x + 2x^2)} - \frac{13915}{64} \ln|3 - x + 2x^2| \\ &= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101 + 79x)}{5888(3 - x + 2x^2)^2} + \frac{1331(5229 + 76420x)}{135424(3 - x + 2x^2)} + \frac{637997}{16928\sqrt{23}} \arctan\left(\frac{4x-1}{\sqrt{23}}\right) \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 98, normalized size = 1.00

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{1331(76420x + 5229)}{135424(2x^2 - x + 3)} - \frac{14641(79x + 101)}{5888(2x^2 - x + 3)^2} - \frac{13915}{64} \log(2x^2 - x + 3) + \frac{2725x}{8} - \frac{63799791 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{16928\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^3, x]

[Out] (2725\*x)/8 + (4875\*x^2)/32 + (625\*x^3)/24 - (14641\*(101 + 79\*x))/(5888\*(3 - x + 2\*x^2)^2) + (1331\*(5229 + 76420\*x))/(135424\*(3 - x + 2\*x^2)) - (63799791\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(16928\*Sqrt[23]) - (13915\*Log[3 - x + 2\*x^2])/64

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2 + 3x + 5x^2)^4}{(3 - x + 2x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^3,x]

[Out] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^3, x]

**fricas** [A] time = 0.41, size = 128, normalized size = 1.31

$$\frac{486680000x^7 + 2360398000x^6 + 5100406400x^5 + 2157209100x^4 + 24531516180x^3 - 765597492\sqrt{23}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - 6171678159x^2 - 1015822830(4x^4 - 4x^3 + 13x^2 - 6x + 9)\log(2x^2 - x + 3) + 23692590858x - 453041787}{4672128(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^3,x, algorithm="fricas")

[Out] 1/4672128\*(486680000\*x^7 + 2360398000\*x^6 + 5100406400\*x^5 + 2157209100\*x^4 + 24531516180\*x^3 - 765597492\*sqrt(23)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 6171678159\*x^2 - 1015822830\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(2\*x^2 - x + 3) + 23692590858\*x - 453041787)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac** [A] time = 0.21, size = 72, normalized size = 0.73

$$\frac{625}{24}x^3 + \frac{4875}{32}x^2 - \frac{63799791}{389344}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{2725}{8}x + \frac{1331(76420x^3 - 32981x^2 + 102022x - 4933)}{67712(2x^2 - x + 3)^2} - \frac{13915}{64}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^3,x, algorithm="giac")

[Out] 625/24\*x^3 + 4875/32\*x^2 - 63799791/389344\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 2725/8\*x + 1331/67712\*(76420\*x^3 - 32981\*x^2 + 102022\*x - 4933)/(2\*x^2 - x + 3)^2 - 13915/64\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 73, normalized size = 0.74

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{2725x}{8} - \frac{63799791\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{389344} - \frac{13915\ln(2x^2 - x + 3)}{64} - \frac{121\left(-\frac{210155}{4232}x^3 + \frac{362791}{16928}x^2 - \frac{561121}{8464}x + \frac{54263}{16928}\right)}{4(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^3,x)

[Out] 625/24\*x^3+4875/32\*x^2+2725/8\*x-121/4\*(-210155/4232\*x^3+362791/16928\*x^2-561121/8464\*x+54263/16928)/(2\*x^2-x+3)^2-13915/64\*ln(2\*x^2-x+3)-63799791/389344\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.95, size = 82, normalized size = 0.84

$$\frac{625}{24}x^3 + \frac{4875}{32}x^2 - \frac{63799791}{389344}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{2725}{8}x + \frac{1331(76420x^3 - 32981x^2 + 102022x - 4933)}{67712(4x^4 - 4x^3 + 13x^2 - 6x + 9)} - \frac{13915}{64}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^3,x, algorithm="maxima")

[Out] 625/24\*x^3 + 4875/32\*x^2 - 63799791/389344\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 2725/8\*x + 1331/67712\*(76420\*x^3 - 32981\*x^2 + 102022\*x - 4933)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9) - 13915/64\*log(2\*x^2 - x + 3)

**mupad** [B] time = 0.05, size = 81, normalized size = 0.83

$$\frac{2725x}{8} - \frac{13915\ln(2x^2 - x + 3)}{64} - \frac{63799791\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x - \sqrt{23}}{23}\right)}{389344} + \frac{4875x^2}{32} + \frac{625x^3}{24} + \frac{25428755x^3}{67712} - \frac{43897711x^2}{270848} + \frac{67895641x}{135424} - \frac{6565823}{270848} - \frac{13x^2}{4} - \frac{3x}{2} + \frac{9}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^3,x)`

[Out]  $(2725*x)/8 - (13915*\log(2*x^2 - x + 3))/64 - (63799791*23^{(1/2)}*\operatorname{atan}((4*23^{(1/2)}*x)/23 - 23^{(1/2)}/23))/389344 + (4875*x^2)/32 + (625*x^3)/24 + ((67895*641*x)/135424 - (43897711*x^2)/270848 + (25428755*x^3)/67712 - 6565823/270848)/((13*x^2)/4 - (3*x)/2 - x^3 + x^4 + 9/4)$

**sympy [A]** time = 0.24, size = 95, normalized size = 0.97

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{2725x}{8} + \frac{101715020x^3 - 43897711x^2 + 135791282x - 6565823}{270848x^4 - 270848x^3 + 880256x^2 - 406272x + 609408} - \frac{13915 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{64} - \frac{63799791\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{389344}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**3,x)`

[Out]  $625*x**3/24 + 4875*x**2/32 + 2725*x/8 + (101715020*x**3 - 43897711*x**2 + 135791282*x - 6565823)/(270848*x**4 - 270848*x**3 + 880256*x**2 - 406272*x + 609408) - 13915*\log(x**2 - x/2 + 3/2)/64 - 63799791*\sqrt{23}*\operatorname{atan}(4*\sqrt{23}*x/23 - \sqrt{23}/23)/389344$

$$3.49 \quad \int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3} dx$$

**Optimal.** Leaf size=84

$$\frac{121(21193 - 12828x)}{33856(2x^2 - x + 3)} - \frac{1331(17 - 45x)}{1472(2x^2 - x + 3)^2} + \frac{825}{32} \log(2x^2 - x + 3) + \frac{125x}{8} + \frac{165099 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

**Rubi [A]** time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(21193 - 12828x)}{33856(2x^2 - x + 3)} - \frac{1331(17 - 45x)}{1472(2x^2 - x + 3)^2} + \frac{825}{32} \log(2x^2 - x + 3) + \frac{125x}{8} + \frac{165099 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]
```

```
[Out] (125*x)/8 - (1331*(17 - 45*x))/(1472*(3 - x + 2*x^2)^2) + (121*(21193 - 12828*x))/(33856*(3 - x + 2*x^2)) + (165099*ArcTan[(1 - 4*x)/Sqrt[23]])/(8464*Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

#### Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

#### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```



```

q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]], Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^3}{(3 - x + 2x^2)^3} dx &= -\frac{1331(17 - 45x)}{1472(3 - x + 2x^2)^2} + \frac{1}{46} \int \frac{-\frac{40885}{32} - \frac{19067x}{8} + \frac{22195x^2}{4} + \frac{13225x^3}{2} + 2875x^4}{(3 - x + 2x^2)^2} dx \\
&= -\frac{1331(17 - 45x)}{1472(3 - x + 2x^2)^2} + \frac{121(21193 - 12828x)}{33856(3 - x + 2x^2)} + \frac{\int \frac{\frac{23997}{2} + 92575x + \frac{66125x^2}{2}}{3 - x + 2x^2} dx}{1058} \\
&= -\frac{1331(17 - 45x)}{1472(3 - x + 2x^2)^2} + \frac{121(21193 - 12828x)}{33856(3 - x + 2x^2)} + \frac{\int \left( \frac{66125}{4} - \frac{33(4557 - 13225x)}{4(3 - x + 2x^2)} \right) dx}{1058} \\
&= \frac{125x}{8} - \frac{1331(17 - 45x)}{1472(3 - x + 2x^2)^2} + \frac{121(21193 - 12828x)}{33856(3 - x + 2x^2)} - \frac{33 \int \frac{4557 - 13225x}{3 - x + 2x^2} dx}{4232} \\
&= \frac{125x}{8} - \frac{1331(17 - 45x)}{1472(3 - x + 2x^2)^2} + \frac{121(21193 - 12828x)}{33856(3 - x + 2x^2)} - \frac{165099 \int \frac{1}{3 - x + 2x^2} dx}{16928} + \frac{825}{32} \int \frac{1}{3 - x + 2x^2} dx \\
&= \frac{125x}{8} - \frac{1331(17 - 45x)}{1472(3 - x + 2x^2)^2} + \frac{121(21193 - 12828x)}{33856(3 - x + 2x^2)} + \frac{825}{32} \log(3 - x + 2x^2) + \frac{165099}{8464\sqrt{23}} \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) + \frac{825}{32} \log(3 - x + 2x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 84, normalized size = 1.00

$$-\frac{121(12828x - 21193)}{33856(2x^2 - x + 3)} + \frac{1331(45x - 17)}{1472(2x^2 - x + 3)^2} + \frac{825}{32} \log(2x^2 - x + 3) + \frac{125x}{8} - \frac{165099 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^3, x]

[Out] (125\*x)/8 + (1331\*(-17 + 45\*x))/((1472\*(3 - x + 2\*x^2)^2) - (121\*(-21193 + 12828\*x))/(33856\*(3 - x + 2\*x^2)) - (165099\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(8464\*Sqrt[23]) + (825\*Log[3 - x + 2\*x^2])/32

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2 + 3x + 5x^2)^3}{(3 - x + 2x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^3,x]

[Out] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^3, x]

**fricas** [A] time = 0.39, size = 118, normalized size = 1.40

$$\frac{24334000x^5 - 24334000x^4 + 43385176x^3 - 330198\sqrt{23}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + 40329281x^2 + 10037775(4x^4 - 4x^3 + 13x^2 - 6x + 9)\log(2x^2 - x + 3) - 12446818x + 82485337}{389344(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^3,x, algorithm="fricas")

[Out] 1/389344\*(24334000\*x^5 - 24334000\*x^4 + 43385176\*x^3 - 330198\*sqrt(23)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 40329281\*x^2 + 10037775\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(2\*x^2 - x + 3) - 12446818\*x + 82485337)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac** [A] time = 0.18, size = 62, normalized size = 0.74

$$-\frac{165099}{194672}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{125}{8}x - \frac{121(12828x^3 - 27607x^2 + 24146x - 29639)}{16928(2x^2 - x + 3)^2} + \frac{825}{32}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^3,x, algorithm="giac")

[Out] -165099/194672\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 125/8\*x - 121/16928\*(12828\*x^3 - 27607\*x^2 + 24146\*x - 29639)/(2\*x^2 - x + 3)^2 + 825/32\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 63, normalized size = 0.75

$$\frac{125x}{8} - \frac{165099\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{194672} + \frac{825\ln(2x^2 - x + 3)}{32} + \frac{-\frac{388047}{4232}x^3 + \frac{3340447}{16928}x^2 - \frac{1460833}{8464}x + \frac{3586319}{16928}}{(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^3,x)

[Out] 125/8\*x+11/2\*(-35277/2116\*x^3+303677/8464\*x^2-132803/4232\*x+326029/8464)/(2\*x^2-x+3)^2+825/32\*ln(2\*x^2-x+3)-165099/194672\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.96, size = 72, normalized size = 0.86

$$-\frac{165099}{194672}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{125}{8}x - \frac{121(12828x^3 - 27607x^2 + 24146x - 29639)}{16928(4x^4 - 4x^3 + 13x^2 - 6x + 9)} + \frac{825}{32}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^3,x, algorithm="maxima")

[Out] -165099/194672\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 125/8\*x - 121/16928\*(12828\*x^3 - 27607\*x^2 + 24146\*x - 29639)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9) + 825/32\*log(2\*x^2 - x + 3)

**mupad** [B] time = 0.05, size = 72, normalized size = 0.86

$$\frac{125x}{8} + \frac{825\ln(2x^2 - x + 3)}{32} - \frac{165099\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{194672} - \frac{\frac{388047x^3}{16928} - \frac{3340447x^2}{67712} + \frac{1460833x}{33856} - \frac{3586319}{67712}}{x^4 - x^3 + \frac{13x^2}{4} - \frac{3x}{2} + \frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^3/(2\*x^2 - x + 3)^3,x)

[Out] (125\*x)/8 + (825\*log(2\*x^2 - x + 3))/32 - (165099\*23^(1/2)\*atan((4\*23^(1/2)\*x)/23 - 23^(1/2)/23))/194672 - ((1460833\*x)/33856 - (3340447\*x^2)/67712 + (388047\*x^3)/16928 - 3586319/67712)/((13\*x^2)/4 - (3\*x)/2 - x^3 + x^4 + 9/4)

**sympy [A]** time = 0.23, size = 82, normalized size = 0.98

$$\frac{125x}{8} + \frac{-1552188x^3 + 3340447x^2 - 2921666x + 3586319}{67712x^4 - 67712x^3 + 220064x^2 - 101568x + 152352} + \frac{825 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{32} - \frac{165099\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{194672}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*3/(2\*x\*\*2-x+3)\*\*3,x)

[Out] 125\*x/8 + (-1552188\*x\*\*3 + 3340447\*x\*\*2 - 2921666\*x + 3586319)/(67712\*x\*\*4 - 67712\*x\*\*3 + 220064\*x\*\*2 - 101568\*x + 152352) + 825\*log(x\*\*2 - x/2 + 3/2)/32 - 165099\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/194672

$$3.50 \quad \int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} dx$$

**Optimal.** Leaf size=64

$$\frac{121(19-7x)}{368(2x^2-x+3)^2} - \frac{55(332x+975)}{8464(2x^2-x+3)} - \frac{4330 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

**Rubi [A]** time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1660, 12, 618, 204}

$$\frac{121(19-7x)}{368(2x^2-x+3)^2} - \frac{55(332x+975)}{8464(2x^2-x+3)} - \frac{4330 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]
```

```
[Out] (121*(19 - 7*x))/(368*(3 - x + 2*x^2)^2) - (55*(975 + 332*x))/(8464*(3 - x + 2*x^2)) - (4330*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} dx &= \frac{121(19-7x)}{368(3-x+2x^2)^2} + \frac{1}{46} \int \frac{-\frac{195}{8} + \frac{1955x}{2} + 575x^2}{(3-x+2x^2)^2} dx \\
&= \frac{121(19-7x)}{368(3-x+2x^2)^2} - \frac{55(975+332x)}{8464(3-x+2x^2)} + \frac{\int \frac{4330}{3-x+2x^2} dx}{1058} \\
&= \frac{121(19-7x)}{368(3-x+2x^2)^2} - \frac{55(975+332x)}{8464(3-x+2x^2)} + \frac{2165}{529} \int \frac{1}{3-x+2x^2} dx \\
&= \frac{121(19-7x)}{368(3-x+2x^2)^2} - \frac{55(975+332x)}{8464(3-x+2x^2)} - \frac{4330}{529} \text{Subst} \left( \int \frac{1}{-23-x^2} dx, x, -1+4x \right) \\
&= \frac{121(19-7x)}{368(3-x+2x^2)^2} - \frac{55(975+332x)}{8464(3-x+2x^2)} - \frac{4330 \tan^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{529\sqrt{23}}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 51, normalized size = 0.80

$$\frac{4330 \tan^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{529\sqrt{23}} - \frac{11(1660x^3 + 4045x^2 + 938x + 4909)}{4232(-2x^2 + x - 3)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^3,x]

[Out] (-11\*(4909 + 938\*x + 4045\*x^2 + 1660\*x^3))/(4232\*(-3 + x - 2\*x^2)^2) + (4330\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(529\*Sqrt[23])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^3,x]

[Out] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^3, x]

**fricas [A]** time = 0.39, size = 75, normalized size = 1.17

$$\frac{419980x^3 - 34640\sqrt{23}(4x^4 - 4x^3 + 13x^2 - 6x + 9) \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + 1023385x^2 + 237314x + 1241977}{97336(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^3,x, algorithm="fricas")

[Out] -1/97336\*(419980\*x^3 - 34640\*sqrt(23)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1023385\*x^2 + 237314\*x + 1241977)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac [A]** time = 0.19, size = 46, normalized size = 0.72

$$\frac{4330}{12167} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x-1)\right) - \frac{11(1660x^3 + 4045x^2 + 938x + 4909)}{4232(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^3,x, algorithm="giac")

[Out] 4330/12167\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 11/4232\*(1660\*x^3 + 4045\*x^2 + 938\*x + 4909)/(2\*x^2 - x + 3)^2

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{4330\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167} + \frac{-\frac{4565}{1058}x^3 - \frac{44495}{4232}x^2 - \frac{5159}{2116}x - \frac{53999}{4232}}{(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^3,x)

[Out] 4\*(-4565/4232\*x^3-44495/16928\*x^2-5159/8464\*x-53999/16928)/(2\*x^2-x+3)^2+4330/12167\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

maxima [A] time = 0.96, size = 56, normalized size = 0.88

$$\frac{4330}{12167} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{11(1660x^3 + 4045x^2 + 938x + 4909)}{4232(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^3,x, algorithm="maxima")

[Out] 4330/12167\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 11/4232\*(1660\*x^3 + 4045\*x^2 + 938\*x + 4909)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

mupad [B] time = 3.47, size = 56, normalized size = 0.88

$$\frac{4330 \sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{12167} - \frac{\frac{4565x^3}{4232} + \frac{44495x^2}{16928} + \frac{5159x}{8464} + \frac{53999}{16928}}{x^4 - x^3 + \frac{13x^2}{4} - \frac{3x}{2} + \frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^2/(2\*x^2 - x + 3)^3,x)

[Out] (4330\*23^(1/2)\*atan((4\*23^(1/2)\*x)/23 - 23^(1/2)/23))/12167 - ((5159\*x)/8464 + (44495\*x^2)/16928 + (4565\*x^3)/4232 + 53999/16928)/((13\*x^2)/4 - (3\*x)/2 - x^3 + x^4 + 9/4)

sympy [A] time = 0.20, size = 63, normalized size = 0.98

$$\frac{-18260x^3 - 44495x^2 - 10318x - 53999}{16928x^4 - 16928x^3 + 55016x^2 - 25392x + 38088} + \frac{4330\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{12167}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*2/(2\*x\*\*2-x+3)\*\*3,x)

[Out] (-18260\*x\*\*3 - 44495\*x\*\*2 - 10318\*x - 53999)/(16928\*x\*\*4 - 16928\*x\*\*3 + 55016\*x\*\*2 - 25392\*x + 38088) + 4330\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/12167

$$3.51 \quad \int \frac{2+3x+5x^2}{(3-x+2x^2)^3} dx$$

Optimal. Leaf size=64

$$-\frac{131(1-4x)}{2116(2x^2-x+3)} - \frac{11(3x+5)}{92(2x^2-x+3)^2} - \frac{262 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {1660, 12, 614, 618, 204}

$$-\frac{131(1-4x)}{2116(2x^2-x+3)} - \frac{11(3x+5)}{92(2x^2-x+3)^2} - \frac{262 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^3,x]

[Out] (-11\*(5 + 3\*x))/(92\*(3 - x + 2\*x^2)^2) - (131\*(1 - 4\*x))/(2116\*(3 - x + 2\*x^2)) - (262\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(529\*Sqrt[23])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] - Dist[(2\*c\*(2\*p + 3))/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4\*p]

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{2+3x+5x^2}{(3-x+2x^2)^3} dx &= -\frac{11(5+3x)}{92(3-x+2x^2)^2} + \frac{1}{46} \int \frac{131}{2(3-x+2x^2)^2} dx \\
&= -\frac{11(5+3x)}{92(3-x+2x^2)^2} + \frac{131}{92} \int \frac{1}{(3-x+2x^2)^2} dx \\
&= -\frac{11(5+3x)}{92(3-x+2x^2)^2} - \frac{131(1-4x)}{2116(3-x+2x^2)} + \frac{131}{529} \int \frac{1}{3-x+2x^2} dx \\
&= -\frac{11(5+3x)}{92(3-x+2x^2)^2} - \frac{131(1-4x)}{2116(3-x+2x^2)} - \frac{262}{529} \text{Subst}\left(\int \frac{1}{-23-x^2} dx, x, -1+4x\right) \\
&= -\frac{11(5+3x)}{92(3-x+2x^2)^2} - \frac{131(1-4x)}{2116(3-x+2x^2)} - \frac{262 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 51, normalized size = 0.80

$$\frac{46(524x^3-393x^2+472x-829)}{(-2x^2+x-3)^2} + 1048\sqrt{23} \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{48668}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^3, x]

[Out] ((46\*(-829 + 472\*x - 393\*x^2 + 524\*x^3))/(-3 + x - 2\*x^2)^2 + 1048\*Sqrt[23]\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/48668

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^3, x]

[Out] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^3, x]

**fricas [A]** time = 0.42, size = 75, normalized size = 1.17

$$\frac{12052x^3 + 524\sqrt{23}(4x^4 - 4x^3 + 13x^2 - 6x + 9) \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - 9039x^2 + 10856x - 19067}{24334(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^3, x, algorithm="fricas")

[Out] 1/24334\*(12052\*x^3 + 524\*sqrt(23)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 9039\*x^2 + 10856\*x - 19067)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac [A]** time = 0.21, size = 46, normalized size = 0.72

$$\frac{262}{12167} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x-1)\right) + \frac{524x^3 - 393x^2 + 472x - 829}{1058(2x^2 - x + 3)^2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^3,x, algorithm="giac")

[Out] 262/12167\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/1058\*(524\*x^3 - 393\*x^2 + 472\*x - 829)/(2\*x^2 - x + 3)^2

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{262\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167} + \frac{\frac{262}{529}x^3 - \frac{393}{1058}x^2 + \frac{236}{529}x - \frac{829}{1058}}{(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)/(2\*x^2-x+3)^3,x)

[Out] 4\*(131/1058\*x^3-393/4232\*x^2+59/529\*x-829/4232)/(2\*x^2-x+3)^2+262/12167\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

maxima [A] time = 0.96, size = 56, normalized size = 0.88

$$\frac{262}{12167} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) + \frac{524x^3 - 393x^2 + 472x - 829}{1058(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^3,x, algorithm="maxima")

[Out] 262/12167\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/1058\*(524\*x^3 - 393\*x^2 + 472\*x - 829)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

mupad [B] time = 0.04, size = 55, normalized size = 0.86

$$\frac{262 \sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{12167} + \frac{\frac{131x^3}{1058} - \frac{393x^2}{4232} + \frac{59x}{529} - \frac{829}{4232}}{x^4 - x^3 + \frac{13x^2}{4} - \frac{3x}{2} + \frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)/(2\*x^2 - x + 3)^3,x)

[Out] (262\*23^(1/2)\*atan((4\*23^(1/2)\*x)/23 - 23^(1/2)/23))/12167 + ((59\*x)/529 - (393\*x^2)/4232 + (131\*x^3)/1058 - 829/4232)/((13\*x^2)/4 - (3\*x)/2 - x^3 + x^4 + 9/4)

sympy [A] time = 0.18, size = 61, normalized size = 0.95

$$\frac{524x^3 - 393x^2 + 472x - 829}{4232x^4 - 4232x^3 + 13754x^2 - 6348x + 9522} + \frac{262\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{12167}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)/(2\*x\*\*2-x+3)\*\*3,x)

[Out] (524\*x\*\*3 - 393\*x\*\*2 + 472\*x - 829)/(4232\*x\*\*4 - 4232\*x\*\*3 + 13754\*x\*\*2 - 6348\*x + 9522) + 262\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/12167

$$3.52 \quad \int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)} dx$$

**Optimal.** Leaf size=115

$$\frac{3625 - 746x}{256036(2x^2 - x + 3)} + \frac{13 - 6x}{1012(2x^2 - x + 3)^2} - \frac{119 \log(2x^2 - x + 3)}{21296} + \frac{119 \log(5x^2 + 3x + 2)}{21296} - \frac{53403 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}}$$

**Rubi [A]** time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{3625 - 746x}{256036(2x^2 - x + 3)} + \frac{13 - 6x}{1012(2x^2 - x + 3)^2} - \frac{119 \log(2x^2 - x + 3)}{21296} + \frac{119 \log(5x^2 + 3x + 2)}{21296} - \frac{53403 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}} + \frac{247 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{10648\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)), x]

[Out] (13 - 6\*x)/(1012\*(3 - x + 2\*x^2)^2) + (3625 - 746\*x)/(256036\*(3 - x + 2\*x^2)) - (53403\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(5632792\*Sqrt[23]) + (247\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(10648\*Sqrt[31]) - (119\*Log[3 - x + 2\*x^2])/21296 + (119\*Log[2 + 3\*x + 5\*x^2])/21296

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b

```

^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

### Rule 1060

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_
)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

### Rule 1072

```

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*((d_) + (e_)*(x_) + (f_)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)} dx &= \frac{13-6x}{1012(3-x+2x^2)^2} - \frac{\int \frac{-3652-1936x+990x^2}{(3-x+2x^2)^2(2+3x+5x^2)} dx}{11132} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2} + \frac{3625-746x}{256036(3-x+2x^2)} - \frac{\int \frac{-6551908-7779574x+902660x^2}{(3-x+2x^2)(2+3x+5x^2)} dx}{61960712} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2} + \frac{3625-746x}{256036(3-x+2x^2)} - \frac{\int \frac{-154867174+335151124x}{3-x+2x^2} dx}{14994492304} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2} + \frac{3625-746x}{256036(3-x+2x^2)} + \frac{53403 \int \frac{1}{3-x+2x^2} dx}{11265584} - \frac{119}{11265584} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2} + \frac{3625-746x}{256036(3-x+2x^2)} - \frac{119 \log(3-x+2x^2)}{21296} + \frac{119}{11265584} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2} + \frac{3625-746x}{256036(3-x+2x^2)} - \frac{53403 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}} + \frac{247}{5632792\sqrt{23}}
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 99, normalized size = 0.86

$$\frac{713 \left( -62951 \log(2x^2 - x + 3) + 62951 \log(5x^2 + 3x + 2) - \frac{44(1492x^3 - 7996x^2 + 7381x - 14164)}{(-2x^2 + x - 3)^2} \right) + 3310986\sqrt{23} \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right) + 6010498\sqrt{31} \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{8032361392}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)), x]

[Out] (3310986\*Sqrt[23]\*ArcTan[(-1 + 4\*x)/Sqrt[23]] + 6010498\*Sqrt[31]\*ArcTan[(3 + 10\*x)/Sqrt[31]] + 713\*((-44\*(-14164 + 7381\*x - 7996\*x^2 + 1492\*x^3))/(-3 + x - 2\*x^2)^2 - 62951\*Log[3 - x + 2\*x^2] + 62951\*Log[2 + 3\*x + 5\*x^2]))/8032361392

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)), x]

[Out] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)), x]

**fricas [A]** time = 0.43, size = 177, normalized size = 1.54

$$\frac{46807024x^3 - 6010498\sqrt{31}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\arctan\left(\frac{1}{\sqrt{31}}(10x + 3)\right) - 3310986\sqrt{23}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\arctan\left(\frac{1}{\sqrt{23}}(4x - 1)\right) - 250850512x^2 - 44884063(4x^4 - 4x^3 + 13x^2 - 6x + 9)\log(5x^2 + 3x + 2) + 44884063(4x^4 - 4x^3 + 13x^2 - 6x + 9)\log(2x^2 - x + 3) + 231556732x - 44435308}{8032361392(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2), x, algorithm="fricas")

[Out] -1/8032361392\*(46807024\*x^3 - 6010498\*sqrt(31)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) - 3310986\*sqrt(23)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 250850512\*x^2 - 44884063\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(5\*x^2 + 3\*x + 2) + 44884063\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(2\*x^2 - x + 3) + 231556732\*x - 44435308)

$$63*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*\log(5*x^2 + 3*x + 2) + 44884063*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*\log(2*x^2 - x + 3) + 231556732*x - 444353008) / (4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)$$

**giac [A]** time = 0.21, size = 88, normalized size = 0.77

$$\frac{247}{330088} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{53403}{129554216} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{1492x^3 - 7996x^2 + 7381x - 14164}{256036(2x^2 - x + 3)^2} + \frac{119}{21296} \log(5x^2 + 3x + 2) - \frac{119}{21296} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] 247/330088\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 53403/129554216\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 1/256036\*(1492\*x^3 - 7996\*x^2 + 7381\*x - 14164)/(2\*x^2 - x + 3)^2 + 119/21296\*log(5\*x^2 + 3\*x + 2) - 119/21296\*log(2\*x^2 - x + 3)

**maple [A]** time = 0.01, size = 89, normalized size = 0.77

$$\frac{247\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{330088} + \frac{53403\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{129554216} - \frac{119 \ln(2x^2 - x + 3)}{21296} + \frac{119 \ln(5x^2 + 3x + 2)}{21296} - \frac{\frac{8206}{529}x^3 - \frac{43978}{529}x^2 + \frac{81191}{1058}x - \frac{77902}{529}}{2662(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2),x)

[Out] 119/21296\*ln(5\*x^2+3\*x+2)+247/330088\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))-1/2662\*(8206/529\*x^3-43978/529\*x^2+81191/1058\*x-77902/529)/(2\*x^2-x+3)^2-119/21296\*ln(2\*x^2-x+3)+53403/129554216\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima [A]** time = 0.97, size = 98, normalized size = 0.85

$$\frac{247}{330088} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x + 3)\right) + \frac{53403}{129554216} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{1492x^3 - 7996x^2 + 7381x - 14164}{256036(4x^4 - 4x^3 + 13x^2 - 6x + 9)} + \frac{119}{21296} \log(5x^2 + 3x + 2) - \frac{119}{21296} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] 247/330088\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 53403/129554216\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) - 1/256036\*(1492\*x^3 - 7996\*x^2 + 7381\*x - 14164)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9) + 119/21296\*log(5\*x^2 + 3\*x + 2) - 119/21296\*log(2\*x^2 - x + 3)

**mupad [B]** time = 3.58, size = 116, normalized size = 1.01

$$-\ln\left(x + \frac{3}{10} - \frac{\sqrt{31} \operatorname{li}}{10}\right) \left(\frac{119}{21296} + \frac{\sqrt{31} 247i}{660176}\right) + \ln\left(x + \frac{3}{10} + \frac{\sqrt{31} \operatorname{li}}{10}\right) \left(\frac{119}{21296} + \frac{\sqrt{31} 247i}{660176}\right) - \ln\left(x - \frac{1}{4} - \frac{\sqrt{23} \operatorname{li}}{4}\right) \left(\frac{119}{21296} + \frac{\sqrt{23} 53403i}{259108432}\right) + \ln\left(x - \frac{1}{4} + \frac{\sqrt{23} \operatorname{li}}{4}\right) \left(\frac{119}{21296} + \frac{\sqrt{23} 53403i}{259108432}\right) - \frac{\frac{373x^3}{256036} - \frac{1999x^2}{256036} + \frac{61x}{8464} - \frac{3541}{256036}}{x^4 - x^3 + \frac{13x^2}{4} - \frac{3x}{2} + \frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^3\*(3\*x + 5\*x^2 + 2)),x)

[Out] log(x + (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*247i)/660176 + 119/21296) - log(x - (31^(1/2)\*1i)/10 + 3/10)\*((31^(1/2)\*247i)/660176 - 119/21296) - log(x - (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*53403i)/259108432 + 119/21296) + log(x + (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*53403i)/259108432 - 119/21296) - ((61\*x)/8464 - (1999\*x^2)/256036 + (373\*x^3)/256036 - 3541/256036)/((13\*x^2)/4 - (3\*x)/2 - x^3 + x^4 + 9/4)

**sympy [A]** time = 0.36, size = 122, normalized size = 1.06

$$\frac{-1492x^3 + 7996x^2 - 7381x + 14164}{1024144x^4 - 1024144x^3 + 3328468x^2 - 1536216x + 2304324} - \frac{119 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{21296} + \frac{119 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{21296} + \frac{53403\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x - \sqrt{23}}{23}\right)}{129554216} + \frac{247\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x + 3\sqrt{31}}{31}\right)}{330088}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2),x)
```

```
[Out] (-1492*x**3 + 7996*x**2 - 7381*x + 14164)/(1024144*x**4 - 1024144*x**3 + 33
28468*x**2 - 1536216*x + 2304324) - 119*log(x**2 - x/2 + 3/2)/21296 + 119*log(x**2 + 3*x/5 + 2/5)/21296 + 53403*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/129554216 + 247*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/330088
```

$$3.53 \quad \int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=160

$$\frac{-252815x - 2328909}{174616552(5x^2 + 3x + 2)} + \frac{9665 - 1446x}{512072(2x^2 - x + 3)(5x^2 + 3x + 2)} + \frac{13 - 6x}{1012(2x^2 - x + 3)^2(5x^2 + 3x + 2)} + \frac{181 \log[3 - x + 2x^2]}{468512}$$

**Rubi [A]** time = 0.16, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{9665 - 1446x}{512072(2x^2 - x + 3)(5x^2 + 3x + 2)} - \frac{252815x + 2328909}{174616552(5x^2 + 3x + 2)} + \frac{13 - 6x}{1012(2x^2 - x + 3)^2(5x^2 + 3x + 2)} + \frac{181 \log(2x^2 - x + 3)}{468512} - \frac{181 \log(5x^2 + 3x + 2)}{468512} + \frac{2038497 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{123921424\sqrt{23}} + \frac{246757 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{7261936\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] -(2328909 + 252815\*x)/(174616552\*(2 + 3\*x + 5\*x^2)) + (13 - 6\*x)/(1012\*(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)) + (9665 - 1446\*x)/(512072\*(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)) + (2038497\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(123921424\*Sqrt[23]) + (246757\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(7261936\*Sqrt[31]) + (181\*Log[3 - x + 2\*x^2])/468512 - (181\*Log[2 + 3\*x + 5\*x^2])/468512

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b

```

^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

### Rule 1060

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_
)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

### Rule 1072

```

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*((d_) + (e_)*(x_) + (f_)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

### Rubi steps



$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2} dx &= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} - \frac{\int \frac{-4081-3168x+1650x^2}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx}{11132} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{9665-1446x}{512072(3-x+2x^2)(2+3x+5x^2)} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{9665-1446x}{512072(3-x+2x^2)(2+3x+5x^2)} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{9665-1446x}{512072(3-x+2x^2)(2+3x+5x^2)} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{9665-1446x}{512072(3-x+2x^2)(2+3x+5x^2)} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{9665-1446x}{512072(3-x+2x^2)(2+3x+5x^2)} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{9665-1446x}{512072(3-x+2x^2)(2+3x+5x^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 136, normalized size = 0.85

$$\frac{-2923x-1782}{1408198(2x^2-x+3)} + \frac{1235x-1474}{330088(5x^2+3x+2)} + \frac{-14x-31}{22264(2x^2-x+3)^2} + \frac{181 \log(2x^2-x+3)}{468512} - \frac{181 \log(5x^2+3x+2)}{468512} - \frac{2038497 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{123921424\sqrt{23}} + \frac{246757 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{7261936\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] (-31 - 14\*x)/(22264\*(3 - x + 2\*x^2)^2) + (-1782 - 2923\*x)/(1408198\*(3 - x + 2\*x^2)) + (-1474 + 1235\*x)/(330088\*(2 + 3\*x + 5\*x^2)) - (2038497\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(123921424\*Sqrt[23]) + (246757\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(7261936\*Sqrt[31]) + (181\*Log[3 - x + 2\*x^2])/468512 - (181\*Log[2 + 3\*x + 5\*x^2])/468512

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^2), x]

**fricas [A]** time = 0.43, size = 227, normalized size = 1.42

372548701\*x^7 + 24652483921\*x^6 - 15024886268\*x^5 - 600494818\*sqrt(23)\*x^4 - 8\*x^3 + 61\*x^2 + 53\*x + 15) arctan(1/sqrt(23)\*(x+3)) + 392799234\*sqrt(23)\*(20\*x^6 + 61\*x^5 + 53\*x^4 + 15\*x + 18) arctan(1/sqrt(23)\*(x-1)) + 47996648492\*x^2 + 210340147\*(20\*x^6 - 8\*x^5 + 61\*x^4 + 53\*x^3 + 15\*x + 18) log(5\*x^2 + 3\*x + 2) - 210340147\*(20\*x^6 + 61\*x^5 + 53\*x^4 + 15\*x + 18) log(2\*x^2 - x + 3) - 184712690401\*x + 27708109136

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out]  $-1/5478070469344*(31725248720*x^5 + 260524883872*x^4 - 158204886268*x^3 - 6004584838*\sqrt{31}*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 3917991234*\sqrt{23}*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 679966484692*x^2 + 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*\log(5*x^2 + 3*x + 2) - 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*\log(2*x^2 - x + 3) - 184712689040*x + 277008109136)/(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)$

**giac** [A] time = 0.22, size = 110, normalized size = 0.69

$$\frac{246757}{225120016} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x+3)\right) - \frac{2038497}{2850192752} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x-1)\right) - \frac{1011260x^5 + 8304376x^4 - 5042869x^3 + 21674311x^2 - 5887820x + 8829788}{174616552(5x^2+3x+2)(2x^2-x+3)^2} - \frac{181}{468512} \log(5x^2+3x+2) + \frac{181}{468512} \log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out]  $246757/225120016*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) - 2038497/2850192752*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 1/174616552*(1011260*x^5 + 8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^2) - 181/468512*\log(5*x^2 + 3*x + 2) + 181/468512*\log(2*x^2 - x + 3)$

**maple** [A] time = 0.01, size = 106, normalized size = 0.66

$$\frac{246757\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right) - 2038497\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right) + 181 \ln(2x^2-x+3) - 181 \ln(5x^2+3x+2) - \frac{5434x}{31} + \frac{32428}{155}}{225120016} + \frac{128612x^3 - 14102x^2 - 173195x - 321497}{529} + \frac{181}{58564(2x^2-x+3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^2,x)

[Out]  $-1/234256*(-5434/31*x+32428/155)/(x^2+3/5*x+2/5)-181/468512*\ln(5*x^2+3*x+2)+246757/225120016*31^{(1/2)}*\arctan(1/31*(10*x+3)*31^{(1/2)})+1/58564*(-128612/529*x^3-14102/529*x^2-173195/529*x-321497/1058)/(2*x^2-x+3)^2+181/468512*\ln(2*x^2-x+3)-2038497/2850192752*23^{(1/2)}*\arctan(1/23*(4*x-1)*23^{(1/2)})$

**maxima** [A] time = 0.97, size = 116, normalized size = 0.72

$$\frac{246757}{225120016} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x+3)\right) - \frac{2038497}{2850192752} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x-1)\right) - \frac{1011260x^5 + 8304376x^4 - 5042869x^3 + 21674311x^2 - 5887820x + 8829788}{174616552(20x^6 - 8x^5 + 61x^4 + x^3 + 53x^2 + 15x + 18)} - \frac{181}{468512} \log(5x^2+3x+2) + \frac{181}{468512} \log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out]  $246757/225120016*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) - 2038497/2850192752*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 1/174616552*(1011260*x^5 + 8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) - 181/468512*\log(5*x^2 + 3*x + 2) + 181/468512*\log(2*x^2 - x + 3)$

**mupad** [B] time = 3.60, size = 136, normalized size = 0.85

$$\frac{5063x^5 + 103887x^4 + 5042869x^3 + 21674311x^2 + 24491x + 20027}{174616552} + \frac{48543180}{38223180} - \frac{34273180}{34273180} + \frac{174616552}{174616552} + \frac{79717160}{79717160} - \ln\left(x + \frac{3}{10} \frac{\sqrt{31} 11}{10}\right) \left(\frac{181}{468512} + \frac{\sqrt{31} 2467571}{450240032}\right) + \ln\left(x + \frac{3}{10} \frac{\sqrt{31} 11}{10}\right) \left(\frac{181}{468512} + \frac{\sqrt{31} 2467571}{450240032}\right) + \ln\left(x - \frac{1}{4} \frac{\sqrt{23} 11}{4}\right) \left(\frac{181}{468512} + \frac{\sqrt{23} 20384971}{5700385504}\right) - \ln\left(x - \frac{1}{4} \frac{\sqrt{23} 11}{4}\right) \left(\frac{181}{468512} + \frac{\sqrt{23} 20384971}{5700385504}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^3\*(3\*x + 5\*x^2 + 2)^2),x)

[Out]  $\log(x + (31^{(1/2)}*11)/10 + 3/10)*((31^{(1/2)}*2467571)/450240032 - 181/468512) - \log(x - (31^{(1/2)}*11)/10 + 3/10)*((31^{(1/2)}*2467571)/450240032 + 181/468512)$

8512) - ((21674311\*x^2)/3492331040 - (294391\*x)/174616552 - (5042869\*x^3)/3492331040 + (1038047\*x^4)/436541380 + (50563\*x^5)/174616552 + 200677/79371160)/((3\*x)/4 + (53\*x^2)/20 + x^3/20 + (61\*x^4)/20 - (2\*x^5)/5 + x^6 + 9/10) + log(x - (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*2038497i)/5700385504 + 181/468512) - log(x + (23^(1/2)\*1i)/4 - 1/4)\*((23^(1/2)\*2038497i)/5700385504 - 181/468512)

**sympy [A]** time = 0.43, size = 143, normalized size = 0.89

$$\frac{-1011260x^5 - 8304376x^4 + 5042869x^3 - 21674311x^2 + 5887820x - 8829788}{3492331040x^6 - 1396932416x^5 + 10651609672x^4 + 174616552x^3 + 9254677256x^2 + 2619248280x + 3143097936} + \frac{181 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{468512} - \frac{181 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{468512} - \frac{2038497\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{2850192752} + \frac{246757\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{225120016}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*3/(5\*x\*\*2+3\*x+2)\*\*2,x)

[Out] (-1011260\*x\*\*5 - 8304376\*x\*\*4 + 5042869\*x\*\*3 - 21674311\*x\*\*2 + 5887820\*x - 8829788)/(3492331040\*x\*\*6 - 1396932416\*x\*\*5 + 10651609672\*x\*\*4 + 174616552\*x\*\*3 + 9254677256\*x\*\*2 + 2619248280\*x + 3143097936) + 181\*log(x\*\*2 - x/2 + 3/2)/468512 - 181\*log(x\*\*2 + 3\*x/5 + 2/5)/468512 - 2038497\*sqrt(23)\*atan(4\*sqrt(23)\*x/23 - sqrt(23)/23)/2850192752 + 246757\*sqrt(31)\*atan(10\*sqrt(31)\*x/31 + 3\*sqrt(31)/31)/225120016

$$3.54 \quad \int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3} dx$$

**Optimal.** Leaf size=181

$$\frac{5(302 - 35x)}{64009(2x^2 - x + 3)(5x^2 + 3x + 2)^2} + \frac{15(7140435x + 2618306)}{14886061058(5x^2 + 3x + 2)} - \frac{5(77020x + 223707)}{87308276(5x^2 + 3x + 2)^2} + \frac{1}{1012(2x^2 - x + 3)}$$

**Rubi [A]** time = 0.20, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{5(302 - 35x)}{64009(2x^2 - x + 3)(5x^2 + 3x + 2)^2} + \frac{15(7140435x + 2618306)}{14886061058(5x^2 + 3x + 2)} - \frac{5(77020x + 223707)}{87308276(5x^2 + 3x + 2)^2} + \frac{13 - 6x}{1012(2x^2 - x + 3)(5x^2 + 3x + 2)^2} + \frac{405 \log(2x^2 - x + 3)}{1288408} - \frac{405 \log(5x^2 + 3x + 2)}{1288408} - \frac{880575 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{340783916\sqrt{23}} + \frac{2768835 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{619080044\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] (-5\*(223707 + 77020\*x))/(87308276\*(2 + 3\*x + 5\*x^2)^2) + (13 - 6\*x)/(1012\*(3 - x + 2\*x^2)^2\*(2 + 3\*x + 5\*x^2)^2) + (5\*(302 - 35\*x))/(64009\*(3 - x + 2\*x^2)\*(2 + 3\*x + 5\*x^2)^2) + (15\*(2618306 + 7140435\*x))/(14886061058\*(2 + 3\*x + 5\*x^2)) - (880575\*ArcTan[(1 - 4\*x)/Sqrt[23]])/(340783916\*Sqrt[23]) + (2768835\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(619080044\*Sqrt[31]) + (405\*Log[3 - x + 2\*x^2])/1288408 - (405\*Log[2 + 3\*x + 5\*x^2])/1288408

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f

```

- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

### Rule 1060

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)
^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

### Rule 1072

```

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*((d_) + (e_)*(x_) + (f_)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3} dx &= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} - \int \frac{-4510-4400x+2310x^2}{(3-x+2x^2)^2(2+3x+5x^2)^3} dx \\
&= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 151, normalized size = 0.83

$$\frac{405 \log(2x^2 - x + 3)}{1288408} - \frac{405 \log(5x^2 + 3x + 2)}{1288408} + \frac{6850x^3 - 9275x^2 + 11154x - 4342}{345092(10x^4 + x^3 + 16x^2 + 7x + 6)^2} + \frac{5(42842610x^3 - 5711469x^2 + 51156233x + 14085977)}{14886061058(10x^4 + x^3 + 16x^2 + 7x + 6)} + \frac{880575 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{340783916\sqrt{23}} + \frac{2768835 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{619080044\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] (-4342 + 11154\*x - 9275\*x^2 + 6850\*x^3)/(345092\*(6 + 7\*x + 16\*x^2 + x^3 + 10\*x^4)^2) + (5\*(14085977 + 51156233\*x - 5711469\*x^2 + 42842610\*x^3))/(14886061058\*(6 + 7\*x + 16\*x^2 + x^3 + 10\*x^4)) + (880575\*ArcTan[(-1 + 4\*x)/Sqrt[23]])/(340783916\*Sqrt[23]) + (2768835\*ArcTan[(3 + 10\*x)/Sqrt[31]])/(619080044\*Sqrt[31]) + (405\*Log[3 - x + 2\*x^2])/1288408 - (405\*Log[2 + 3\*x + 5\*x^2])/1288408

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^3\*(2 + 3\*x + 5\*x^2)^3), x]

**fricas** [A] time = 0.43, size = 297, normalized size = 1.64

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/467005507511576\*(67202918046000\*x^7 - 2238718468800\*x^6 + 186872434930060\*x^5 + 62827256425340\*x^4 + 173919793526820\*x^3 + 67376830890\*sqrt(31)\*(100\*x^8 + 20\*x^7 + 321\*x^6 + 172\*x^5 + 390\*x^4 + 236\*x^3 + 241\*x^2 + 84\*x + 36)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 52466419650\*sqrt(23)\*(100\*x^8 + 20\*x^7 + 321\*x^6 + 172\*x^5 + 390\*x^4 + 236\*x^3 + 241\*x^2 + 84\*x + 36)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 73595926401690\*x^2 - 146799174285\*(100\*x^8 + 20\*x^7 + 321\*x^6 + 172\*x^5 + 390\*x^4 + 236\*x^3 + 241\*x^2 + 84\*x + 36)\*log(5\*x^2 + 3\*x + 2) + 146799174285\*(100\*x^8 + 20\*x^7 + 321\*x^6 + 172\*x^5 + 390\*x^4 + 236\*x^3 + 241\*x^2 + 84\*x + 36)\*log(2\*x^2 - x + 3) + 78707350628632\*x + 7381223830244)/(100\*x^8 + 20\*x^7 + 321\*x^6 + 172\*x^5 + 390\*x^4 + 236\*x^3 + 241\*x^2 + 84\*x + 36)

**giac** [A] time = 0.22, size = 116, normalized size = 0.64

$$\frac{2768835}{19191481364} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x+3)\right) + \frac{880575}{7838030068} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x-1)\right) + \frac{4284261000x^7 - 142720800x^6 + 11913326210x^5 + 4005307690x^4 + 11087580870x^3 + 4691822415x^2 + 5017681412x + 470561254}{29772122116(10x^4 + x^3 + 16x^2 + 7x + 6)^2} - \frac{405}{1288408} \log(5x^2 + 3x + 2) + \frac{405}{1288408} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 2768835/19191481364\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 880575/7838030068\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/29772122116\*(4284261000\*x^7 - 142720800\*x^6 + 11913326210\*x^5 + 4005307690\*x^4 + 11087580870\*x^3 + 4691822415\*x^2 + 5017681412\*x + 470561254)/(10\*x^4 + x^3 + 16\*x^2 + 7\*x + 6)^2 - 405/1288408\*log(5\*x^2 + 3\*x + 2) + 405/1288408\*log(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 118, normalized size = 0.65

$$\frac{2768835\sqrt{31} \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right) + 880575\sqrt{23} \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right) + 405 \ln(2x^2 - x + 3) - 405 \ln(5x^2 + 3x + 2) - 25 \left( \frac{3013197}{961} x^3 - \frac{14516062}{4805} x^2 - \frac{51193868}{24025} x - \frac{5423968}{24025} \right)}{19191481364} + \frac{302907x^3 - 368291x^2 + 2501587x - 665819}{644204(2x^2 - x + 3)^2} + \frac{105}{1288408}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^3,x)

[Out] -25/2576816\*(-3013197/961\*x^3-14516062/4805\*x^2-51193868/24025\*x-5423968/24025)/(5\*x^2+3\*x+2)^2-405/1288408\*ln(5\*x^2+3\*x+2)+2768835/19191481364\*31^(1/2)\*arctan(1/31\*(10\*x+3)\*31^(1/2))+1/644204\*(302907/529\*x^3-368291/529\*x^2+2501587/2116\*x-665819/1058)/(2\*x^2-x+3)^2+405/1288408\*ln(2\*x^2-x+3)+880575/7838030068\*23^(1/2)\*arctan(1/23\*(4\*x-1)\*23^(1/2))

**maxima** [A] time = 0.97, size = 138, normalized size = 0.76

$$\frac{2768835}{19191481364} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10x+3)\right) + \frac{880575}{7838030068} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4x-1)\right) + \frac{4284261000x^7 - 142720800x^6 + 11913326210x^5 + 4005307690x^4 + 11087580870x^3 + 4691822415x^2 + 5017681412x + 470561254}{29772122116(100x^4 + 20x^3 + 321x^2 + 172x + 390x + 236x^3 + 241x^2 + 84x + 36)} - \frac{405}{1288408} \log(5x^2 + 3x + 2) + \frac{405}{1288408} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^3/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 2768835/19191481364\*sqrt(31)\*arctan(1/31\*sqrt(31)\*(10\*x + 3)) + 880575/7838030068\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(4\*x - 1)) + 1/29772122116\*(4284261000\*x^7 - 142720800\*x^6 + 11913326210\*x^5 + 4005307690\*x^4 + 11087580870\*x^3 + 4691822415\*x^2 + 5017681412\*x + 470561254)/(100\*x^8 + 20\*x^7 + 321\*x^6 + 172\*x^5 + 390\*x^4 + 236\*x^3 + 241\*x^2 + 84\*x + 36) - 405/1288408\*log(5\*x^2 + 3\*x + 2) + 405/1288408\*log(2\*x^2 - x + 3)

**mupad [B]** time = 3.59, size = 155, normalized size = 0.86

$$\frac{2141135x^7 - 35480x^6 + 119132621x^5 + 40530769x^4 + 1108758087x^3 + 4691822415x^2 + 5017681412x + 470561254}{1488606108x^8 + 595442442320x^7 + 9556851199236x^6 + 5120805003952x^5 + 1161127625240x^4 + 7026220819376x^3 + 7175081429956x^2 + 2500858257744x + 1071796396176} + \ln\left(x - \frac{1}{4} + \frac{\sqrt{23}i}{4}\right) \left(\frac{405}{1288408} + \frac{\sqrt{23}880575i}{15676060136}\right) - \ln\left(x + \frac{3}{10} + \frac{\sqrt{31}i}{10}\right) \left(\frac{405}{1288408} + \frac{\sqrt{31}2768835i}{38382962728}\right) + \ln\left(x + \frac{3}{10} + \frac{\sqrt{31}i}{10}\right) \left(-\frac{405}{1288408} - \frac{\sqrt{31}2768835i}{38382962728}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{23}i}{4}\right) \left(-\frac{405}{1288408} - \frac{\sqrt{23}880575i}{15676060136}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^3), x)
[Out] log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*880575i)/15676060136 + 405/1288408) - log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*880575i)/15676060136 - 405/1288408) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2768835i)/38382962728 + 405/1288408) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2768835i)/38382962728 - 405/1288408) + ((1254420353*x)/744303052900 + (938364483*x^2)/595442442320 + (1108758087*x^3)/297721221160 + (400530769*x^4)/297721221160 + (1191332621*x^5)/297721221160 - (356802*x^6)/7443030529 + (21421305*x^7)/14886061058 + 235280627/1488606105800)/((21*x)/25 + (241*x^2)/100 + (59*x^3)/25 + (39*x^4)/10 + (43*x^5)/25 + (321*x^6)/100 + x^7/5 + x^8 + 9/25)
```

**sympy [A]** time = 0.47, size = 163, normalized size = 0.90

$$\frac{4284261000x^7 - 142720800x^6 + 11913326210x^5 + 4005307690x^4 + 11087580870x^3 + 4691822415x^2 + 5017681412x + 470561254}{2977212211600x^8 + 595442442320x^7 + 9556851199236x^6 + 5120805003952x^5 + 1161127625240x^4 + 7026220819376x^3 + 7175081429956x^2 + 2500858257744x + 1071796396176} + \frac{405 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{1288408} - \frac{405 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{1288408} + \frac{880575\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x - \sqrt{23}}{23}\right)}{783803068} + \frac{2768835\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x + 3\sqrt{31}}{31}\right)}{19191481364}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**3, x)
[Out] (4284261000*x**7 - 142720800*x**6 + 11913326210*x**5 + 4005307690*x**4 + 11087580870*x**3 + 4691822415*x**2 + 5017681412*x + 470561254)/(2977212211600*x**8 + 595442442320*x**7 + 9556851199236*x**6 + 5120805003952*x**5 + 1161127625240*x**4 + 7026220819376*x**3 + 7175081429956*x**2 + 2500858257744*x + 1071796396176) + 405*log(x**2 - x/2 + 3/2)/1288408 - 405*log(x**2 + 3*x/5 + 2/5)/1288408 + 880575*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/783803068 + 2768835*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/19191481364
```



$$3.55 \quad \int \sqrt{3 - x + 2x^2} (2 + 3x + 5x^2)^4 dx$$

**Optimal.** Leaf size=208

$$\frac{83948353 (2x^2 - x + 3)^{3/2} x^2}{2293760} + \frac{804243809 (2x^2 - x + 3)^{3/2} x}{36700160} + \frac{27185733541 (2x^2 - x + 3)^{3/2}}{440401920} - \frac{359471503(1 - 4x)\sqrt{2x^2 - x + 3}}{67108864} + \frac{8267844569 \operatorname{arsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{134217728\sqrt{2}}$$

**Rubi [A]** time = 0.31, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 27, number of rules / integrand size = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{4} (2x^2 - x + 3)^{3/2} x^2 + \frac{14125}{144} (2x^2 - x + 3)^{3/2} x + \frac{233225(2x^2 - x + 3)^{3/2} x^2}{1536} + \frac{4796405(2x^2 - x + 3)^{3/2} x^3}{43008} + \frac{8325631(2x^2 - x + 3)^{3/2} x^3}{1032192} + \frac{83948353(2x^2 - x + 3)^{3/2} x^2}{2293760} + \frac{804243809(2x^2 - x + 3)^{3/2} x}{36700160} + \frac{27185733541(2x^2 - x + 3)^{3/2}}{440401920} - \frac{359471503(1 - 4x)\sqrt{2x^2 - x + 3}}{67108864} + \frac{8267844569 \operatorname{arsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{134217728\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] (-359471503\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/67108864 + (27185733541\*(3 - x + 2\*x^2)^(3/2))/440401920 + (804243809\*x\*(3 - x + 2\*x^2)^(3/2))/36700160 - (83948353\*x^2\*(3 - x + 2\*x^2)^(3/2))/2293760 + (8325631\*x^3\*(3 - x + 2\*x^2)^(3/2))/1032192 + (4796405\*x^4\*(3 - x + 2\*x^2)^(3/2))/43008 + (233225\*x^5\*(3 - x + 2\*x^2)^(3/2))/1536 + (14125\*x^6\*(3 - x + 2\*x^2)^(3/2))/144 + (125\*x^7\*(3 - x + 2\*x^2)^(3/2))/4 - (8267844569\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(134217728\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \sqrt{3-x+2x^2} (2+3x+5x^2)^4 dx &= \frac{125}{4}x^7(3-x+2x^2)^{3/2} + \frac{1}{20} \int \sqrt{3-x+2x^2} (320+1920x+7520x^2+1 \\
&= \frac{14125}{144}x^6(3-x+2x^2)^{3/2} + \frac{125}{4}x^7(3-x+2x^2)^{3/2} + \frac{1}{360} \int \sqrt{3-x+2x^2} \\
&= \frac{233225x^5(3-x+2x^2)^{3/2}}{1536} + \frac{14125}{144}x^6(3-x+2x^2)^{3/2} + \frac{125}{4}x^7(3-x+2 \\
&= \frac{4796405x^4(3-x+2x^2)^{3/2}}{43008} + \frac{233225x^5(3-x+2x^2)^{3/2}}{1536} + \frac{14125}{144}x^6(3-x \\
&= \frac{8325631x^3(3-x+2x^2)^{3/2}}{1032192} + \frac{4796405x^4(3-x+2x^2)^{3/2}}{43008} + \frac{233225x^5(3-x \\
&= -\frac{83948353x^2(3-x+2x^2)^{3/2}}{2293760} + \frac{8325631x^3(3-x+2x^2)^{3/2}}{1032192} + \frac{4796405x \\
&= \frac{804243809x(3-x+2x^2)^{3/2}}{36700160} - \frac{83948353x^2(3-x+2x^2)^{3/2}}{2293760} + \frac{8325631x \\
&= \frac{27185733541(3-x+2x^2)^{3/2}}{440401920} + \frac{804243809x(3-x+2x^2)^{3/2}}{36700160} - \frac{8394835 \\
&= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864} + \frac{27185733541(3-x+2x^2)^{3/2}}{440401920} + \frac{80 \\
&= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864} + \frac{27185733541(3-x+2x^2)^{3/2}}{440401920} + \frac{80 \\
&= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864} + \frac{27185733541(3-x+2x^2)^{3/2}}{440401920} + \frac{80
\end{aligned}$$

**Mathematica [A]** time = 0.30, size = 85, normalized size = 0.41

$$\frac{4\sqrt{2x^2-x+3} (1321205760000x^9 + 3486515200000x^8 + 6327795712000x^7 + 7725962035200x^6 + 7612808028160x^5 + 5354741991424x^4 + 2211683657856x^3 - 174418077792x^2 + 537752185764x + 3801512106459) - 2604371039235\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{3}}\right)}{84557168640}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^4, x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(3801512106459 + 537752185764\*x - 174418077792\*x^2 + 2211683657856\*x^3 + 5354741991424\*x^4 + 7612808028160\*x^5 + 7725962035200\*x^6 + 6327795712000\*x^7 + 3486515200000\*x^8 + 1321205760000\*x^9) - 2604371039235\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/84557168640

**IntegrateAlgebraic [A]** time = 1.08, size = 100, normalized size = 0.48

$$\frac{\sqrt{2x^2-x+3} (1321205760000x^9 + 3486515200000x^8 + 6327795712000x^7 + 7725962035200x^6 + 7612808028160x^5 + 5354741991424x^4 + 2211683657856x^3 - 174418077792x^2 + 537752185764x + 3801512106459) - 8267844569 \log(2\sqrt{2}\sqrt{2x^2-x+3} - 4x + 1)}{21139292160}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^4, x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(3801512106459 + 537752185764\*x - 174418077792\*x^2 + 2211683657856\*x^3 + 5354741991424\*x^4 + 7612808028160\*x^5 + 7725962035200\*x^6

$6 + 6327795712000*x^7 + 3486515200000*x^8 + 1321205760000*x^9)/21139292160 - (8267844569*\text{Log}[1 - 4*x + 2*\text{Sqrt}[2]*\text{Sqrt}[3 - x + 2*x^2]])/(134217728*\text{Sqrt}[2])$

**fricas** [A] time = 0.44, size = 98, normalized size = 0.47

$\frac{1}{21139292160}(1321205760000x^9 + 3486515200000x^8 + 6327795712000x^7 + 7725962035200x^6 + 7612808028160x^5 + 5354741991424x^4 + 2211683657856x^3 - 174418077792x^2 + 537752185764x + 3801512106459)\sqrt{2x^2 - x + 3} - \frac{8267844569}{536870912}\sqrt{2}\log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4\*(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out]  $1/21139292160*(1321205760000*x^9 + 3486515200000*x^8 + 6327795712000*x^7 + 7725962035200*x^6 + 7612808028160*x^5 + 5354741991424*x^4 + 2211683657856*x^3 - 174418077792*x^2 + 537752185764*x + 3801512106459)*\text{sqrt}(2*x^2 - x + 3) + 8267844569/536870912*\text{sqrt}(2)*\log(-4*\text{sqrt}(2)*\text{sqrt}(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)$

**giac** [A] time = 0.25, size = 93, normalized size = 0.45

$\frac{1}{21139292160}(4(8(4(16(20(40(140(160(36x + 95)x + 27587)x + 4715553)x + 185859571)x + 2614620113)x + 17278778577)x - 5450564931)x + 134438046441)x + 3801512106459)\sqrt{2x^2 - x + 3} - \frac{8267844569}{268435456}\sqrt{2}\log(-2\sqrt{2}(\sqrt{2x^2 - x + 3} + 1))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4\*(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out]  $1/21139292160*(4*(8*(4*(16*(20*(40*(140*(160*(36*x + 95)*x + 27587)*x + 4715553)*x + 185859571)*x + 2614620113)*x + 17278778577)*x - 5450564931)*x + 134438046441)*x + 3801512106459)*\text{sqrt}(2*x^2 - x + 3) - 8267844569/268435456*\text{sqrt}(2)*\log(-2*\text{sqrt}(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)$

**maple** [A] time = 0.03, size = 166, normalized size = 0.80

$\frac{125(2x^2-x+3)^{\frac{3}{2}}}{4} + \frac{14125(2x^2-x+3)^{\frac{3}{2}}}{144} + \frac{233225(2x^2-x+3)^{\frac{3}{2}}}{1536} + \frac{4796405(2x^2-x+3)^{\frac{3}{2}}}{43008} + \frac{8325631(2x^2-x+3)^{\frac{3}{2}}}{1032192} + \frac{83948353(2x^2-x+3)^{\frac{3}{2}}}{2293760} + \frac{804243809(2x^2-x+3)^{\frac{3}{2}}}{36700160} + \frac{8267844569\sqrt{2}\text{arcsinh}\left(\frac{4\sqrt{23}(x-1)}{23}\right)}{268435456} + \frac{27185733541(2x^2-x+3)^{\frac{3}{2}}}{440401920} + \frac{359471503(4x-1)\sqrt{2x^2-x+3}}{67108864}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^4\*(2\*x^2-x+3)^(1/2),x)

[Out]  $125/4*x^7*(2*x^2-x+3)^{(3/2)}+27185733541/440401920*(2*x^2-x+3)^{(3/2)}+14125/144*x^6*(2*x^2-x+3)^{(3/2)}+233225/1536*x^5*(2*x^2-x+3)^{(3/2)}+4796405/43008*x^4*(2*x^2-x+3)^{(3/2)}+8325631/1032192*x^3*(2*x^2-x+3)^{(3/2)}-83948353/2293760*x^2*(2*x^2-x+3)^{(3/2)}+804243809/36700160*x*(2*x^2-x+3)^{(3/2)}+8267844569/268435456*2^{(1/2)}*\text{arcsinh}(4/23*23^{(1/2)}*(x-1/4))+359471503/67108864*(4*x-1)*(2*x^2-x+3)^{(1/2)}$

**maxima** [A] time = 1.01, size = 177, normalized size = 0.85

$\frac{125}{4}(2x^2-x+3)^{\frac{3}{2}} + \frac{14125}{144}(2x^2-x+3)^{\frac{3}{2}} + \frac{233225}{1536}(2x^2-x+3)^{\frac{3}{2}} + \frac{4796405}{43008}(2x^2-x+3)^{\frac{3}{2}} + \frac{8325631}{1032192}(2x^2-x+3)^{\frac{3}{2}} + \frac{83948353}{2293760}(2x^2-x+3)^{\frac{3}{2}} + \frac{804243809}{36700160}(2x^2-x+3)^{\frac{3}{2}} + \frac{27185733541}{440401920}(2x^2-x+3)^{\frac{3}{2}} + \frac{359471503}{16777216}\sqrt{2x^2-x+3} + \frac{8267844569}{268435456}\sqrt{2}\text{arcsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{359471503}{67108864}\sqrt{2x^2-x+3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4\*(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out]  $125/4*(2*x^2 - x + 3)^{(3/2)}*x^7 + 14125/144*(2*x^2 - x + 3)^{(3/2)}*x^6 + 233225/1536*(2*x^2 - x + 3)^{(3/2)}*x^5 + 4796405/43008*(2*x^2 - x + 3)^{(3/2)}*x^4 + 8325631/1032192*(2*x^2 - x + 3)^{(3/2)}*x^3 - 83948353/2293760*(2*x^2 - x + 3)^{(3/2)}*x^2 + 804243809/36700160*(2*x^2 - x + 3)^{(3/2)}*x + 27185733541/440401920*(2*x^2 - x + 3)^{(3/2)} + 359471503/16777216*\text{sqrt}(2*x^2 - x + 3)*x + 8267844569/268435456*\text{sqrt}(2)*\text{arcsinh}(1/23*\text{sqrt}(23)*(4*x - 1)) - 359471503/67108864*\text{sqrt}(2*x^2 - x + 3)$

**mupad [B]** time = 5.03, size = 221, normalized size = 1.06

$$\frac{8325631x^3(2x^2-x+3)^{3/2}}{1032192} - \frac{83948353x^2(2x^2-x+3)^{3/2}}{2293760} + \frac{4796405x^4(2x^2-x+3)^{3/2}}{43008} + \frac{233225x^5(2x^2-x+3)^{3/2}}{1536} + \frac{14125x^6(2x^2-x+3)^{3/2}}{144} + \frac{125x^7(2x^2-x+3)^{3/2}}{4} - \frac{41987163941\sqrt{2}\ln\left(\sqrt{2x^2-x+3} + \frac{x+1}{2}\right)}{1174405120} - \frac{182552867\left(\frac{x}{2}-\frac{1}{8}\right)\sqrt{2x^2-x+3}}{36700160} - \frac{2718573354\sqrt{2x^2-x+3}(32x^2-4x+45)}{7046430720} + \frac{804243809x(2x^2-x+3)^{3/2}}{36700160} + \frac{625271871443x^2(2x^2-x+3)^{3/2}}{9395240960} + \frac{625271871443\sqrt{2}\ln\left(\sqrt{2x^2-x+3} + \frac{x+1}{2}\right)}{9395240960}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] (8325631*x^3*(2*x^2 - x + 3)^(3/2))/1032192 - (83948353*x^2*(2*x^2 - x + 3)^(3/2))/2293760 + (4796405*x^4*(2*x^2 - x + 3)^(3/2))/43008 + (233225*x^5*(2*x^2 - x + 3)^(3/2))/1536 + (14125*x^6*(2*x^2 - x + 3)^(3/2))/144 + (125*x^7*(2*x^2 - x + 3)^(3/2))/4 - (41987163941*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/1174405120 - (182552867*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/36700160 + (27185733541*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x + 45))/7046430720 + (804243809*x*(2*x^2 - x + 3)^(3/2))/36700160 + (625271871443*2^(1/2)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/9395240960
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**4, x)
```

$$3.56 \quad \int \sqrt{3 - x + 2x^2} (2 + 3x + 5x^2)^3 dx$$

Optimal. Leaf size=166

$$\frac{531681(2x^2 - x + 3)^{3/2} x^2}{71680} - \frac{9627393(2x^2 - x + 3)^{3/2} x}{1146880} - \frac{22548119(2x^2 - x + 3)^{3/2}}{4587520} - \frac{6766097(1 - 4x)\sqrt{2x^2 - x + 3}}{2097152}$$

**Rubi [A]** time = 0.18, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{16}(2x^2 - x + 3)^{3/2} x^5 + \frac{8825}{448}(2x^2 - x + 3)^{3/2} x^4 + \frac{247435(2x^2 - x + 3)^{3/2} x^3}{10752} + \frac{531681(2x^2 - x + 3)^{3/2} x^2}{71680} - \frac{9627393(2x^2 - x + 3)^{3/2} x}{1146880} - \frac{22548119(2x^2 - x + 3)^{3/2}}{4587520} - \frac{6766097(1 - 4x)\sqrt{2x^2 - x + 3}}{2097152} - \frac{155620231 \operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4194304\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] (-6766097\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/2097152 - (22548119\*(3 - x + 2\*x^2)^(3/2))/4587520 - (9627393\*x\*(3 - x + 2\*x^2)^(3/2))/1146880 + (531681\*x^2\*(3 - x + 2\*x^2)^(3/2))/71680 + (247435\*x^3\*(3 - x + 2\*x^2)^(3/2))/10752 + (8825\*x^4\*(3 - x + 2\*x^2)^(3/2))/448 + (125\*x^5\*(3 - x + 2\*x^2)^(3/2))/16 - (155620231\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(4194304\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \sqrt{3-x+2x^2} (2+3x+5x^2)^3 dx &= \frac{125}{16} x^5 (3-x+2x^2)^{3/2} + \frac{1}{16} \int \sqrt{3-x+2x^2} (128+576x+1824x^2+33) \\
&= \frac{8825}{448} x^4 (3-x+2x^2)^{3/2} + \frac{125}{16} x^5 (3-x+2x^2)^{3/2} + \frac{1}{224} \int \sqrt{3-x+2x^2} \\
&= \frac{247435x^3 (3-x+2x^2)^{3/2}}{10752} + \frac{8825}{448} x^4 (3-x+2x^2)^{3/2} + \frac{125}{16} x^5 (3-x+2x^2)^{3/2} \\
&= \frac{531681x^2 (3-x+2x^2)^{3/2}}{71680} + \frac{247435x^3 (3-x+2x^2)^{3/2}}{10752} + \frac{8825}{448} x^4 (3-x+2x^2)^{3/2} \\
&= -\frac{9627393x (3-x+2x^2)^{3/2}}{1146880} + \frac{531681x^2 (3-x+2x^2)^{3/2}}{71680} + \frac{247435x^3 (3-x+2x^2)^{3/2}}{10752} \\
&= -\frac{22548119 (3-x+2x^2)^{3/2}}{4587520} - \frac{9627393x (3-x+2x^2)^{3/2}}{1146880} + \frac{531681x^2 (3-x+2x^2)^{3/2}}{71680} \\
&= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119 (3-x+2x^2)^{3/2}}{4587520} - \frac{9627393x (3-x+2x^2)^{3/2}}{1146880} \\
&= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119 (3-x+2x^2)^{3/2}}{4587520} - \frac{9627393x (3-x+2x^2)^{3/2}}{1146880} \\
&= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119 (3-x+2x^2)^{3/2}}{4587520} - \frac{9627393x (3-x+2x^2)^{3/2}}{1146880}
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 75, normalized size = 0.45

$$\frac{4\sqrt{2x^2-x+3} (3440640000x^7 + 6955008000x^6 + 10958233600x^5 + 11212171264x^4 + 9872163456x^3 + 4583812128x^2 - 1621307916x - 3957369321) - 16340124255\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{880803840}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^3, x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(-3957369321 - 1621307916\*x + 4583812128\*x^2 + 9872163456\*x^3 + 11212171264\*x^4 + 10958233600\*x^5 + 6955008000\*x^6 + 3440640000\*x^7) - 16340124255\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/880803840

**IntegrateAlgebraic [A]** time = 0.87, size = 90, normalized size = 0.54

$$\frac{\sqrt{2x^2-x+3} (3440640000x^7 + 6955008000x^6 + 10958233600x^5 + 11212171264x^4 + 9872163456x^3 + 4583812128x^2 - 1621307916x - 3957369321)}{220200960} - \frac{155620231 \log(2\sqrt{2}\sqrt{2x^2-x+3} - 4x + 1)}{4194304\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^3, x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(-3957369321 - 1621307916\*x + 4583812128\*x^2 + 9872163456\*x^3 + 11212171264\*x^4 + 10958233600\*x^5 + 6955008000\*x^6 + 3440640000\*x^7))/220200960 - (155620231\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(4194304\*Sqrt[2])

**fricas [A]** time = 0.43, size = 88, normalized size = 0.53

$$\frac{1}{220200960} (3440640000x^7 + 6955008000x^6 + 10958233600x^5 + 11212171264x^4 + 9872163456x^3 + 4583812128x^2 - 1621307916x - 3957369321)\sqrt{2x^2-x+3} + \frac{155620231}{1677216} \sqrt{2} \log(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3\*(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/220200960\*(3440640000\*x^7 + 6955008000\*x^6 + 10958233600\*x^5 + 11212171264\*x^4 + 9872163456\*x^3 + 4583812128\*x^2 - 1621307916\*x - 3957369321)\*sqrt(2\*x^2 - x + 3) + 155620231/16777216\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.25, size = 83, normalized size = 0.50

$$\frac{1}{220200960} (4(8(4(16(100(120(140x + 283)x + 53507)x + 5474693)x + 77126277)x + 143244129)x - 405326979)x - 3957369321)\sqrt{2x^2 - x + 3} - \frac{155620231}{8388608} \sqrt{2} \log(-2\sqrt{2}(\sqrt{2x - \sqrt{2x^2 - x + 3}}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3\*(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] 1/220200960\*(4\*(8\*(4\*(16\*(100\*(120\*(140\*x + 283)\*x + 53507)\*x + 5474693)\*x + 77126277)\*x + 143244129)\*x - 405326979)\*x - 3957369321)\*sqrt(2\*x^2 - x + 3) - 155620231/8388608\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 132, normalized size = 0.80

$$\frac{125(2x^2 - x + 3)^{\frac{3}{2}}x^5}{16} + \frac{8825(2x^2 - x + 3)^{\frac{3}{2}}x^4}{448} + \frac{247435(2x^2 - x + 3)^{\frac{3}{2}}x^3}{10752} + \frac{531681(2x^2 - x + 3)^{\frac{3}{2}}x^2}{71680} - \frac{9627393(2x^2 - x + 3)^{\frac{3}{2}}x}{1146880} + \frac{155620231\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}(x-1)}{23}\right)}{8388608} - \frac{22548119(2x^2 - x + 3)^{\frac{3}{2}}}{4587520} + \frac{6766097(4x-1)\sqrt{2x^2 - x + 3}}{2097152}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^3\*(2\*x^2-x+3)^(1/2),x)

[Out] -22548119/4587520\*(2\*x^2-x+3)^(3/2)+125/16\*(2\*x^2-x+3)^(3/2)\*x^5+8825/448\*(2\*x^2-x+3)^(3/2)\*x^4+247435/10752\*(2\*x^2-x+3)^(3/2)\*x^3+531681/71680\*(2\*x^2-x+3)^(3/2)\*x^2-9627393/1146880\*(2\*x^2-x+3)^(3/2)\*x+155620231/8388608\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+6766097/2097152\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)

**maxima** [A] time = 0.98, size = 143, normalized size = 0.86

$$\frac{125}{16}(2x^2 - x + 3)^{\frac{3}{2}}x^5 + \frac{8825}{448}(2x^2 - x + 3)^{\frac{3}{2}}x^4 + \frac{247435}{10752}(2x^2 - x + 3)^{\frac{3}{2}}x^3 + \frac{531681}{71680}(2x^2 - x + 3)^{\frac{3}{2}}x^2 - \frac{9627393}{1146880}(2x^2 - x + 3)^{\frac{3}{2}}x - \frac{22548119}{4587520}(2x^2 - x + 3)^{\frac{3}{2}} + \frac{6766097}{524288}\sqrt{2x^2 - x + 3} + \frac{155620231}{8388608}\sqrt{2} \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{6766097}{2097152}\sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3\*(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] 125/16\*(2\*x^2 - x + 3)^(3/2)\*x^5 + 8825/448\*(2\*x^2 - x + 3)^(3/2)\*x^4 + 247435/10752\*(2\*x^2 - x + 3)^(3/2)\*x^3 + 531681/71680\*(2\*x^2 - x + 3)^(3/2)\*x^2 - 9627393/1146880\*(2\*x^2 - x + 3)^(3/2)\*x - 22548119/4587520\*(2\*x^2 - x + 3)^(3/2) + 6766097/524288\*sqrt(2\*x^2 - x + 3)\*x + 155620231/8388608\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 6766097/2097152\*sqrt(2\*x^2 - x + 3)

**mupad** [B] time = 4.69, size = 187, normalized size = 1.13

$$\frac{531681x^2(2x^2 - x + 3)^{\frac{3}{2}}}{71680} + \frac{247435x^3(2x^2 - x + 3)^{\frac{3}{2}}}{10752} + \frac{8825x^4(2x^2 - x + 3)^{\frac{3}{2}}}{448} + \frac{125x^5(2x^2 - x + 3)^{\frac{3}{2}}}{16} + \frac{875316037\sqrt{2} \ln\left(\frac{\sqrt{2x^2 - x + 3} + \sqrt{2x^2 - x + 3}}{2}\right)}{36700160} + \frac{38057219\left(\frac{1}{2} - \frac{1}{8}\right)\sqrt{2x^2 - x + 3}}{1146880} - \frac{22548119\sqrt{2x^2 - x + 3}(32x^2 - 4x + 45)}{73400320} - \frac{9627393x(2x^2 - x + 3)^{\frac{3}{2}}}{1146880} - \frac{1555820211\sqrt{2} \ln\left(2\sqrt{2x^2 - x + 3} + \frac{\sqrt{2x^2 - x + 3}}{2}\right)}{293601280}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(1/2)\*(3\*x + 5\*x^2 + 2)^3,x)

[Out] (531681\*x^2\*(2\*x^2 - x + 3)^(3/2))/71680 + (247435\*x^3\*(2\*x^2 - x + 3)^(3/2))/10752 + (8825\*x^4\*(2\*x^2 - x + 3)^(3/2))/448 + (125\*x^5\*(2\*x^2 - x + 3)^(3/2))/16 + (875316037\*2^(1/2)\*log((2\*x^2 - x + 3)^(1/2) + (2^(1/2)\*(2\*x - 1/2))/2))/36700160 + (38057219\*(x/2 - 1/8)\*(2\*x^2 - x + 3)^(1/2))/1146880 - (22548119\*(2\*x^2 - x + 3)^(1/2)\*(32\*x^2 - 4\*x + 45))/73400320 - (9627393\*x\*(2\*x^2 - x + 3)^(3/2))/1146880 - (1555820211\*2^(1/2)\*log(2\*(2\*x^2 - x + 3)^(1/2) + (2^(1/2)\*(4\*x - 1))/2))/293601280

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*3\*(2\*x\*\*2-x+3)\*\*(1/2),x)

[Out] Integral(sqrt(2\*x\*\*2 - x + 3)\*(5\*x\*\*2 + 3\*x + 2)\*\*3, x)



$$3.57 \quad \int \sqrt{3 - x + 2x^2} (2 + 3x + 5x^2)^2 dx$$

Optimal. Leaf size=124

$$\frac{63}{16} (2x^2 - x + 3)^{3/2} x^2 + \frac{769}{256} (2x^2 - x + 3)^{3/2} x - \frac{2107 (2x^2 - x + 3)^{3/2}}{3072} + \frac{12371(1 - 4x)\sqrt{2x^2 - x + 3}}{16384} + \frac{25}{12} (2x^2 - x + 3)^{3/2}$$

**Rubi [A]** time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{12} (2x^2 - x + 3)^{3/2} x^3 + \frac{63}{16} (2x^2 - x + 3)^{3/2} x^2 + \frac{769}{256} (2x^2 - x + 3)^{3/2} x - \frac{2107 (2x^2 - x + 3)^{3/2}}{3072} + \frac{12371(1 - 4x)\sqrt{2x^2 - x + 3}}{16384} + \frac{284533 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32768\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] (12371\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/16384 - (2107\*(3 - x + 2\*x^2)^(3/2))/3072 + (769\*x\*(3 - x + 2\*x^2)^(3/2))/256 + (63\*x^2\*(3 - x + 2\*x^2)^(3/2))/16 + (25\*x^3\*(3 - x + 2\*x^2)^(3/2))/12 + (284533\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(32768\*Sqrt[2])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \sqrt{3-x+2x^2} (2+3x+5x^2)^2 dx &= \frac{25}{12}x^3(3-x+2x^2)^{3/2} + \frac{1}{12} \int \sqrt{3-x+2x^2} \left(48+144x+123x^2 + \frac{945x^3}{2}\right) dx \\
&= \frac{63}{16}x^2(3-x+2x^2)^{3/2} + \frac{25}{12}x^3(3-x+2x^2)^{3/2} + \frac{1}{120} \int \sqrt{3-x+2x^2} \left(48+144x+123x^2 + \frac{945x^3}{2}\right) dx \\
&= \frac{769}{256}x(3-x+2x^2)^{3/2} + \frac{63}{16}x^2(3-x+2x^2)^{3/2} + \frac{25}{12}x^3(3-x+2x^2)^{3/2} + \frac{1}{120} \int \sqrt{3-x+2x^2} \left(48+144x+123x^2 + \frac{945x^3}{2}\right) dx \\
&= -\frac{2107(3-x+2x^2)^{3/2}}{3072} + \frac{769}{256}x(3-x+2x^2)^{3/2} + \frac{63}{16}x^2(3-x+2x^2)^{3/2} + \frac{1}{120} \int \sqrt{3-x+2x^2} \left(48+144x+123x^2 + \frac{945x^3}{2}\right) dx \\
&= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107(3-x+2x^2)^{3/2}}{3072} + \frac{769}{256}x(3-x+2x^2)^{3/2} + \frac{1}{120} \int \sqrt{3-x+2x^2} \left(48+144x+123x^2 + \frac{945x^3}{2}\right) dx \\
&= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107(3-x+2x^2)^{3/2}}{3072} + \frac{769}{256}x(3-x+2x^2)^{3/2} + \frac{1}{120} \int \sqrt{3-x+2x^2} \left(48+144x+123x^2 + \frac{945x^3}{2}\right) dx \\
&= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107(3-x+2x^2)^{3/2}}{3072} + \frac{769}{256}x(3-x+2x^2)^{3/2} + \frac{1}{120} \int \sqrt{3-x+2x^2} \left(48+144x+123x^2 + \frac{945x^3}{2}\right) dx
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 65, normalized size = 0.52

$$\frac{4\sqrt{2x^2-x+3} (204800x^5 + 284672x^4 + 408960x^3 + 365536x^2 + 328204x - 64023) + 853599\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{196608}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(-64023 + 328204\*x + 365536\*x^2 + 408960\*x^3 + 284672\*x^4 + 204800\*x^5) + 853599\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/196608

**IntegrateAlgebraic [A]** time = 0.56, size = 80, normalized size = 0.65

$$\frac{284533 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{32768\sqrt{2}} + \frac{\sqrt{2x^2-x+3} (204800x^5 + 284672x^4 + 408960x^3 + 365536x^2 + 328204x - 64023)}{49152}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(-64023 + 328204\*x + 365536\*x^2 + 408960\*x^3 + 284672\*x^4 + 204800\*x^5))/49152 + (284533\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(32768\*Sqrt[2])

**fricas [A]** time = 0.42, size = 78, normalized size = 0.63

$$\frac{1}{49152} (204800x^5 + 284672x^4 + 408960x^3 + 365536x^2 + 328204x - 64023)\sqrt{2x^2-x+3} + \frac{284533}{131072} \sqrt{2} \log(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2\*(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/49152\*(204800\*x^5 + 284672\*x^4 + 408960\*x^3 + 365536\*x^2 + 328204\*x - 64023)\*sqrt(2\*x^2 - x + 3) + 284533/131072\*sqrt(2)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.22, size = 73, normalized size = 0.59

$$\frac{1}{49152} (4 (8 (4 (16 (100x + 139)x + 3195)x + 11423)x + 82051)x - 64023) \sqrt{2x^2 - x + 3} + \frac{284533}{65536} \sqrt{2} \log \left( -2 \sqrt{2} \left( \sqrt{2x} - \sqrt{2x^2 - x + 3} \right) + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2\*(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] 1/49152\*(4\*(8\*(4\*(16\*(100\*x + 139)\*x + 3195)\*x + 11423)\*x + 82051)\*x - 64023)\*sqrt(2\*x^2 - x + 3) + 284533/65536\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 98, normalized size = 0.79

$$\frac{25(2x^2 - x + 3)^{\frac{3}{2}}x^3}{12} + \frac{63(2x^2 - x + 3)^{\frac{3}{2}}x^2}{16} + \frac{769(2x^2 - x + 3)^{\frac{3}{2}}x}{256} - \frac{284533\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}(x-1)}{23}\right)}{65536} - \frac{2107(2x^2 - x + 3)^{\frac{3}{2}}}{3072} - \frac{12371(4x-1)\sqrt{2x^2 - x + 3}}{16384}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^2\*(2\*x^2-x+3)^(1/2),x)

[Out] 25/12\*(2\*x^2-x+3)^(3/2)\*x^3+63/16\*(2\*x^2-x+3)^(3/2)\*x^2+769/256\*(2\*x^2-x+3)^(3/2)\*x-2107/3072\*(2\*x^2-x+3)^(3/2)-12371/16384\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)-284533/65536\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))

**maxima** [A] time = 0.98, size = 109, normalized size = 0.88

$$\frac{25}{12} (2x^2 - x + 3)^{\frac{3}{2}} x^3 + \frac{63}{16} (2x^2 - x + 3)^{\frac{3}{2}} x^2 + \frac{769}{256} (2x^2 - x + 3)^{\frac{3}{2}} x - \frac{2107}{3072} (2x^2 - x + 3)^{\frac{3}{2}} - \frac{12371}{4096} \sqrt{2x^2 - x + 3} - \frac{284533}{65536} \sqrt{2} \operatorname{arcsinh}\left(\frac{1}{23} \sqrt{23}(4x-1)\right) + \frac{12371}{16384} \sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2\*(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] 25/12\*(2\*x^2 - x + 3)^(3/2)\*x^3 + 63/16\*(2\*x^2 - x + 3)^(3/2)\*x^2 + 769/256\*(2\*x^2 - x + 3)^(3/2)\*x - 2107/3072\*(2\*x^2 - x + 3)^(3/2) - 12371/4096\*sqrt(2\*x^2 - x + 3)\*x - 284533/65536\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) + 12371/16384\*sqrt(2\*x^2 - x + 3)

**mupad** [B] time = 4.19, size = 153, normalized size = 1.23

$$\frac{63x^2(2x^2 - x + 3)^{\frac{3}{2}}}{16} + \frac{25x^2(2x^2 - x + 3)^{\frac{3}{2}}}{12} - \frac{29509\sqrt{2} \ln\left(\sqrt{2x^2 - x + 3} + \frac{\sqrt{2}(x-1)}{2}\right)}{8192} - \frac{1283\left(\frac{x}{2} - \frac{1}{8}\right)\sqrt{2x^2 - x + 3}}{256} - \frac{2107\sqrt{2x^2 - x + 3}(32x^2 - 4x + 45)}{49152} + \frac{769x(2x^2 - x + 3)^{\frac{3}{2}}}{256} - \frac{48461\sqrt{2} \ln\left(2\sqrt{2x^2 - x + 3} + \frac{\sqrt{2}(4x-1)}{2}\right)}{65536}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(1/2)\*(3\*x + 5\*x^2 + 2)^2,x)

[Out] (63\*x^2\*(2\*x^2 - x + 3)^(3/2))/16 + (25\*x^3\*(2\*x^2 - x + 3)^(3/2))/12 - (29509\*2^(1/2)\*log((2\*x^2 - x + 3)^(1/2) + (2^(1/2)\*(2\*x - 1/2))/2))/8192 - (1283\*(x/2 - 1/8)\*(2\*x^2 - x + 3)^(1/2))/256 - (2107\*(2\*x^2 - x + 3)^(1/2)\*(32\*x^2 - 4\*x + 45))/49152 + (769\*x\*(2\*x^2 - x + 3)^(3/2))/256 - (48461\*2^(1/2)\*log(2\*(2\*x^2 - x + 3)^(1/2) + (2^(1/2)\*(4\*x - 1))/2))/65536

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*2\*(2\*x\*\*2-x+3)\*\*(1/2),x)

[Out] Integral(sqrt(2\*x\*\*2 - x + 3)\*(5\*x\*\*2 + 3\*x + 2)\*\*2, x)

$$3.58 \quad \int \sqrt{3 - x + 2x^2} (2 + 3x + 5x^2) dx$$

**Optimal.** Leaf size=82

$$\frac{5}{8}x(2x^2 - x + 3)^{3/2} + \frac{73}{96}(2x^2 - x + 3)^{3/2} - \frac{81}{512}(1 - 4x)\sqrt{2x^2 - x + 3} - \frac{1863 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

**Rubi [A]** time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{8}x(2x^2 - x + 3)^{3/2} + \frac{73}{96}(2x^2 - x + 3)^{3/2} - \frac{81}{512}(1 - 4x)\sqrt{2x^2 - x + 3} - \frac{1863 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2), x]

[Out] (-81\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/512 + (73\*(3 - x + 2\*x^2)^(3/2))/96 + (5\*x\*(3 - x + 2\*x^2)^(3/2))/8 - (1863\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(1024\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[Rt[b, 2]\*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 612

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \sqrt{3-x+2x^2} (2+3x+5x^2) dx &= \frac{5}{8}x(3-x+2x^2)^{3/2} + \frac{1}{8} \int \left(1 + \frac{73x}{2}\right) \sqrt{3-x+2x^2} dx \\
&= \frac{73}{96}(3-x+2x^2)^{3/2} + \frac{5}{8}x(3-x+2x^2)^{3/2} + \frac{81}{64} \int \sqrt{3-x+2x^2} dx \\
&= -\frac{81}{512}(1-4x)\sqrt{3-x+2x^2} + \frac{73}{96}(3-x+2x^2)^{3/2} + \frac{5}{8}x(3-x+2x^2)^{3/2} \\
&= -\frac{81}{512}(1-4x)\sqrt{3-x+2x^2} + \frac{73}{96}(3-x+2x^2)^{3/2} + \frac{5}{8}x(3-x+2x^2)^{3/2} \\
&= -\frac{81}{512}(1-4x)\sqrt{3-x+2x^2} + \frac{73}{96}(3-x+2x^2)^{3/2} + \frac{5}{8}x(3-x+2x^2)^{3/2}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 55, normalized size = 0.67

$$\frac{4\sqrt{2x^2-x+3} (1920x^3 + 1376x^2 + 2684x + 3261) - 5589\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{6144}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2), x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(3261 + 2684\*x + 1376\*x^2 + 1920\*x^3) - 5589\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/6144

**IntegrateAlgebraic [A]** time = 0.37, size = 70, normalized size = 0.85

$$\frac{\sqrt{2x^2-x+3} (1920x^3 + 1376x^2 + 2684x + 3261)}{1536} - \frac{1863 \log(2\sqrt{2}\sqrt{2x^2-x+3} - 4x + 1)}{1024\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2), x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(3261 + 2684\*x + 1376\*x^2 + 1920\*x^3))/1536 - (1863\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(1024\*Sqrt[2])

**fricas [A]** time = 0.41, size = 68, normalized size = 0.83

$$\frac{1}{1536} (1920x^3 + 1376x^2 + 2684x + 3261)\sqrt{2x^2-x+3} + \frac{1863}{4096} \sqrt{2} \log(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)\*(2\*x^2-x+3)^(1/2), x, algorithm="fricas")

[Out] 1/1536\*(1920\*x^3 + 1376\*x^2 + 2684\*x + 3261)\*sqrt(2\*x^2 - x + 3) + 1863/4096\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac [A]** time = 0.24, size = 63, normalized size = 0.77

$$\frac{1}{1536} (4(8(60x+43)x+671)x+3261)\sqrt{2x^2-x+3} - \frac{1863}{2048} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2-x+3}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)\*(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] 1/1536\*(4\*(8\*(60\*x + 43)\*x + 671)\*x + 3261)\*sqrt(2\*x^2 - x + 3) - 1863/2048\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 64, normalized size = 0.78

$$\frac{5(2x^2 - x + 3)^{\frac{3}{2}}x}{8} + \frac{1863\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2048} + \frac{73(2x^2 - x + 3)^{\frac{3}{2}}}{96} + \frac{81(4x - 1)\sqrt{2x^2 - x + 3}}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)\*(2\*x^2-x+3)^(1/2),x)

[Out] 5/8\*(2\*x^2-x+3)^(3/2)\*x+73/96\*(2\*x^2-x+3)^(3/2)+81/512\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)+1863/2048\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))

**maxima** [A] time = 0.96, size = 75, normalized size = 0.91

$$\frac{5}{8}(2x^2 - x + 3)^{\frac{3}{2}}x + \frac{73}{96}(2x^2 - x + 3)^{\frac{3}{2}} + \frac{81}{128}\sqrt{2x^2 - x + 3}x + \frac{1863}{2048}\sqrt{2} \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - \frac{81}{512}\sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)\*(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] 5/8\*(2\*x^2 - x + 3)^(3/2)\*x + 73/96\*(2\*x^2 - x + 3)^(3/2) + 81/128\*sqrt(2\*x^2 - x + 3)\*x + 1863/2048\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 81/512\*sqrt(2\*x^2 - x + 3)

**mupad** [B] time = 3.84, size = 119, normalized size = 1.45

$$\frac{23\sqrt{2} \ln\left(\sqrt{2x^2 - x + 3} + \frac{\sqrt{2}\left(2x-\frac{1}{2}\right)}{2}\right)}{256} + \frac{\left(\frac{x}{2} - \frac{1}{8}\right)\sqrt{2x^2 - x + 3}}{8} + \frac{73\sqrt{2x^2 - x + 3}(32x^2 - 4x + 45)}{1536} + \frac{5x(2x^2 - x + 3)^{3/2}}{8} + \frac{1679\sqrt{2} \ln\left(2\sqrt{2x^2 - x + 3} + \frac{\sqrt{2}(4x-1)}{2}\right)}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(1/2)\*(3\*x + 5\*x^2 + 2),x)

[Out] (23\*2^(1/2)\*log((2\*x^2 - x + 3)^(1/2) + (2^(1/2)\*(2\*x - 1/2))/2))/256 + ((x/2 - 1/8)\*(2\*x^2 - x + 3)^(1/2))/8 + (73\*(2\*x^2 - x + 3)^(1/2)\*(32\*x^2 - 4\*x + 45))/1536 + (5\*x\*(2\*x^2 - x + 3)^(3/2))/8 + (1679\*2^(1/2)\*log(2\*(2\*x^2 - x + 3)^(1/2) + (2^(1/2)\*(4\*x - 1))/2))/2048

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2 - x + 3} (5x^2 + 3x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*(2\*x\*\*2-x+3)\*\*(1/2),x)

[Out] Integral(sqrt(2\*x\*\*2 - x + 3)\*(5\*x\*\*2 + 3\*x + 2), x)

$$3.59 \quad \int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2} dx$$

**Optimal.** Leaf size=174

$$\frac{1}{5} \sqrt{\frac{11}{31} (13 + 10\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{62(13+10\sqrt{2})}} ((20 + 13\sqrt{2})x + 7\sqrt{2} + 6)}{\sqrt{2x^2 - x + 3}} \right) - \frac{1}{5} \sqrt{\frac{11}{31} (10\sqrt{2} - 13)} \tanh^{-1} \left( \frac{\sqrt{\frac{11}{62(10\sqrt{2}-13)}} ((20 - 13\sqrt{2})x - 7\sqrt{2} + 6)}{\sqrt{2x^2 - x + 3}} \right)$$

**Rubi [A]** time = 0.44, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$ , Rules used = {989, 619, 215, 1035, 1029, 206, 204}

$$\frac{1}{5} \sqrt{\frac{11}{31} (13 + 10\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{62(13+10\sqrt{2})}} ((20 + 13\sqrt{2})x + 7\sqrt{2} + 6)}{\sqrt{2x^2 - x + 3}} \right) - \frac{1}{5} \sqrt{\frac{11}{31} (10\sqrt{2} - 13)} \tanh^{-1} \left( \frac{\sqrt{\frac{11}{62(10\sqrt{2}-13)}} ((20 - 13\sqrt{2})x - 7\sqrt{2} + 6)}{\sqrt{2x^2 - x + 3}} \right) - \frac{1}{5} \sqrt{2} \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2\*x^2]/(2 + 3\*x + 5\*x^2), x]

[Out] -(Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/5 + (Sqrt[(11\*(13 + 10\*Sqrt[2]))/31]\*ArcTan[(Sqrt[11/(62\*(13 + 10\*Sqrt[2]))])\*(6 + 7\*Sqrt[2] + (20 + 13\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/5 - (Sqrt[(11\*(-13 + 10\*Sqrt[2]))/31]\*ArcTanh[(Sqrt[11/(62\*(-13 + 10\*Sqrt[2]))])\*(6 - 7\*Sqrt[2] + (20 - 13\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/5

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 215**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

**Rule 619**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

**Rule 989**

Int[Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]/((d\_) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2), x\_Symbol] := Dist[c/f, Int[1/Sqrt[a + b\*x + c\*x^2], x], x] - Dist[1/f, Int[(c\*d - a\*f + (c\*e - b\*f)\*x)/(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0]

**Rule 1029**

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int
[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2} dx = -\left(\frac{1}{5} \int \frac{-11+11x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx\right) + \frac{2}{5} \int \frac{1}{\sqrt{3-x+2x^2}} dx$$

$$= \frac{1}{5} \sqrt{\frac{2}{23}} \text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}} dx, x, -1+4x\right) + \frac{\int \frac{121(2+\sqrt{2})-121\sqrt{2}x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{110\sqrt{2}} - \frac{\int \frac{121(2-\sqrt{2})+121\sqrt{2}x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{110\sqrt{2}}$$

$$= -\frac{1}{5} \sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) - \frac{1}{5} (1331(20-13\sqrt{2})) \text{Subst}\left(\int \frac{1}{-907742(13-10\sqrt{2})-11x^2} dx, x, -1+4x\right)$$

$$= -\frac{1}{5} \sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) + \frac{1}{5} \sqrt{\frac{11}{31}(13+10\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{62(13+10\sqrt{2})}}(6+7\sqrt{2}+(20+11\sqrt{2})x)}{\sqrt{3-x+2x^2}}\right)$$

Mathematica [C] time = 0.46, size = 185, normalized size = 1.06

$$-\frac{1}{5} \sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) - \frac{1}{5} i \sqrt{\frac{11}{62}} \left( \sqrt{13+i\sqrt{31}} \tanh^{-1}\left(\frac{-4i\sqrt{31}x-22x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) - \sqrt{13-i\sqrt{31}} \tanh^{-1}\left(\frac{4i\sqrt{31}x-22x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2), x]
[Out] -1/5*(Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]]) - (I/5)*Sqrt[11/62]*(Sqrt[13 + I
*Sqrt[31]]*ArcTanh[(63 + I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286
+ (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - Sqrt[13 - I*Sqrt[31]]*ArcTanh[(6
3 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqr
t[3 - x + 2*x^2])])
```

IntegrateAlgebraic [C] time = 0.35, size = 211, normalized size = 1.21

$$\frac{11}{5} \text{RootSum}\left[-5#1^4 + 6\sqrt{2}\#1^3 + 17\#1^2 - 26\sqrt{2}\#1 - 56\&, \frac{\#1^2 \log(-\#1 + \sqrt{2x^2-x+3} - \sqrt{2x}) + 2\sqrt{2}\#1 \log(-\#1 + \sqrt{2x^2-x+3} - \sqrt{2x}) - 2\log(-\#1 + \sqrt{2x^2-x+3} - \sqrt{2x})}{-10\#1^3 + 9\sqrt{2}\#1^2 + 17\#1 - 13\sqrt{2}}\right] - \frac{1}{5} \sqrt{2} \log(2\sqrt{2}\sqrt{2x^2-x+3} - 4x+1)$$

Antiderivative was successfully verified.



[In] IntegrateAlgebraic[Sqrt[3 - x + 2\*x^2]/(2 + 3\*x + 5\*x^2),x]

[Out]  $-1/5*(\text{Sqrt}[2]*\text{Log}[1 - 4*x + 2*\text{Sqrt}[2]*\text{Sqrt}[3 - x + 2*x^2]]) + (11*\text{RootSum}[-56 - 26*\text{Sqrt}[2]*\#1 + 17*\#1^2 + 6*\text{Sqrt}[2]*\#1^3 - 5*\#1^4 \& , (-2*\text{Log}[-(\text{Sqrt}[2]*x) + \text{Sqrt}[3 - x + 2*x^2] - \#1] + 2*\text{Sqrt}[2]*\text{Log}[-(\text{Sqrt}[2]*x) + \text{Sqrt}[3 - x + 2*x^2] - \#1]*\#1 + \text{Log}[-(\text{Sqrt}[2]*x) + \text{Sqrt}[3 - x + 2*x^2] - \#1]*\#1^2)/(-13*\text{Sqrt}[2] + 17*\#1 + 9*\text{Sqrt}[2]*\#1^2 - 10*\#1^3) \& )/5$

**fricas** [B] time = 0.99, size = 2016, normalized size = 11.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out]  $1/1550*6050^{(1/4)}*\text{sqrt}(31)*\text{sqrt}(5)*\text{sqrt}(2)*\text{sqrt}(13*\text{sqrt}(2) + 20)*\text{arctan}(1/89125*(460*\text{sqrt}(5)*(4*6050^{(3/4)}*\text{sqrt}(31)*(4702*x^7 - 19541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - \text{sqrt}(2)*(4028*x^7 - 14488*x^6 + 30919*x^5 - 46671*x^4 + 22688*x^3 - 9144*x^2 - 27648*x + 17280) - 34560*x + 27648) + 5*6050^{(1/4)}*\text{sqrt}(31)*(22836*x^7 - 355266*x^6 + 1914360*x^5 - 4475096*x^4 + 5840640*x^3 - 4011840*x^2 - \text{sqrt}(2)*(18463*x^7 - 280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - 3068928*x + 1990656) - 3981312*x + 3068928))*\text{sqrt}(2*x^2 - x + 3)*\text{sqrt}(13*\text{sqrt}(2) + 20) + 253000*\text{sqrt}(31)*\text{sqrt}(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \text{sqrt}(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*\text{sqrt}(10)*(\text{sqrt}(5)*(4*6050^{(3/4)}*\text{sqrt}(31)*(15454*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - \text{sqrt}(2)*(15438*x^7 - 22007*x^6 + 69837*x^5 - 21232*x^4 + 19368*x^3 + 44928*x^2 - 44928*x) - 13824*x) + 5*6050^{(1/4)}*\text{sqrt}(31)*(77254*x^7 - 1000024*x^6 + 3868360*x^5 - 5120640*x^4 + 7012800*x^3 + 2405376*x^2 - \text{sqrt}(2)*(69479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x^2 - 4478976*x) - 2405376*x))*\text{sqrt}(2*x^2 - x + 3)*\text{sqrt}(13*\text{sqrt}(2) + 20) + 550*\text{sqrt}(31)*\text{sqrt}(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - \text{sqrt}(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 25*\text{sqrt}(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*\text{sqrt}(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\text{sqrt}(-(6050^{(1/4)}*\text{sqrt}(5)*\text{sqrt}(2*x^2 - x + 3)*(\text{sqrt}(2)*(3*x + 5) - 8*x + 2)*\text{sqrt}(13*\text{sqrt}(2) + 20) - 245*x^2 - 220*\text{sqrt}(2)*(2*x^2 - x + 3) + 755*x - 1000)/x^2) + 2875*\text{sqrt}(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*\text{sqrt}(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 1/1550*6050^{(1/4)}*\text{sqrt}(31)*\text{sqrt}(5)*\text{sqrt}(2)*\text{sqrt}(13*\text{sqrt}(2) + 20)*\text{arctan}(1/89125*(460*\text{sqrt}(5)*(4*6050^{(3/4)}*\text{sqrt}(31)*(4702*x^7 - 19541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - \text{sqrt}(2)*(4028*x^7 - 14488*x^6 + 30919*x^5 - 46671*x^4 + 22688*x^3 - 9144*x^2 - 27648*x + 17280) - 34560*x + 27648) + 5*6050^{(1/4)}*\text{sqrt}(31)*(22836*x^7 - 355266*x^6 + 1914360*x^5 - 4475096*x^4 + 5840640*x^3 - 4011840*x^2 - \text{sqrt}(2)*(18463*x^7 - 280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - 3068928*x + 1990656) - 3981312*x + 3068928))*\text{sqrt}(2*x^2 - x + 3)*\text{sqrt}(13*\text{sqrt}(2) + 20) - 253000*\text{sqrt}(31)*\text{sqrt}(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \text{sqrt}(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*\text{sqrt}(10)*(\text{sqrt}(5)*(4*6050^{(3/4)}*\text{sqrt}(31)*(15454*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - \text{sqrt}(2)*(15438*x^7 - 22007*x^6 + 69837*x^5 - 21232*x^4 + 19368*x^3 + 44928*x^2 - 44928*x) - 13824*x) + 5*6050^{(1/4)}*\text{sqrt}(31)*(77254*x^7 - 10000$

```

24*x^6 + 3868360*x^5 - 5120640*x^4 + 7012800*x^3 + 2405376*x^2 - sqrt(2)*(6
9479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x
^2 - 4478976*x) - 2405376*x))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) - 5
50*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 +
396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2
44047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*
x) + 3276288*x) - 25*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90
866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(
4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944
*x) + 144820224*x))*sqrt((6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(
3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) + 245*x^2 + 220*sqrt(2)*(2*x^2 -
x + 3) - 755*x + 1000)/x^2) - 2875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53
385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 -
7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 556
8*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 -
4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34
615296*x^2 - 24772608*x + 18579456) - 1/6200*6050^(1/4)*sqrt(5)*sqrt(13*sq
rt(2) + 20)*(13*sqrt(2) - 20)*log(40*(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3
)*(sqrt(2)*(3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) + 245*x^2 + 220*sqrt(
2)*(2*x^2 - x + 3) - 755*x + 1000)/x^2) + 1/6200*6050^(1/4)*sqrt(5)*sqrt(13
*sqrt(2) + 20)*(13*sqrt(2) - 20)*log(-40*(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x
+ 3)*(sqrt(2)*(3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) - 245*x^2 - 220*s
qrt(2)*(2*x^2 - x + 3) + 755*x - 1000)/x^2) + 1/10*sqrt(2)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infini
ty]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity]
]proot error [1.0,infinity,infinity,infinity,infinity]Evaluation time: 5.57
Done
```

**maple** [B] time = 0.15, size = 2065, normalized size = 11.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/5*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-1/52855*(8*(2^(1/2)-1+x)^2/(2^(1
/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)*2^(
1/2)*(285*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549
362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2)))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+
24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(
2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^
(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+
1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+386*(-8866+6820*2^(1/2))
^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2)))*
(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2
^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/
2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^
(1/2))^(1/2)-274846*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/

```

$$2) * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 8 - 3 * 2^{(1/2)} \wedge (1/2) / (-8866 + 6820 * 2^{(1/2)}) \wedge (1/2) * 2^{(1/2)} - 1543366 * \operatorname{arctanh}(31/2 * (8 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 3 * 2^{(1/2)} * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 8 - 3 * 2^{(1/2)}) \wedge (1/2) / (-8866 + 6820 * 2^{(1/2)}) \wedge (1/2)) \wedge (1/2) / ((8 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 3 * 2^{(1/2)} * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 8 - 3 * 2^{(1/2)}) / (1 + (2^{(1/2)-1+x}) / (2^{(1/2)+1-x}))^2 \wedge (1/2) / (1 + (2^{(1/2)-1+x}) / (2^{(1/2)+1-x})) / (8 + 3 * 2^{(1/2)}) / (-8866 + 6820 * 2^{(1/2)}) \wedge (1/2) + 1/21142 * (8 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 3 * 2^{(1/2)} * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 8 - 3 * 2^{(1/2)}) \wedge (1/2) * 2^{(1/2)} * (151 * 2^{(1/2)} * (-8866 + 6820 * 2^{(1/2)}) \wedge (1/2) * \operatorname{arctan}(1/11692487 * (-775687 + 549362 * 2^{(1/2)}) \wedge (1/2) * (-23 * (8 + 3 * 2^{(1/2)}) * (-23 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 24 * 2^{(1/2)-41)) \wedge (1/2) * (6485 * 2^{(1/2)} * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 10368 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 22379 * 2^{(1/2)} + 32016) / (23 * (2^{(1/2)-1+x})^4 / (2^{(1/2)+1-x})^4 + 82 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 23) * (2^{(1/2)-1+x}) / (2^{(1/2)+1-x}) * (8 + 3 * 2^{(1/2)})) * (-775687 + 549362 * 2^{(1/2)}) \wedge (1/2) + 218 * (-8866 + 6820 * 2^{(1/2)}) \wedge (1/2) * \operatorname{arctan}(1/11692487 * (-775687 + 549362 * 2^{(1/2)}) \wedge (1/2) * (-23 * (8 + 3 * 2^{(1/2)}) * (-23 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 24 * 2^{(1/2)-41)) \wedge (1/2) * (6485 * 2^{(1/2)} * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 10368 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 22379 * 2^{(1/2)} + 32016) / (23 * (2^{(1/2)-1+x})^4 / (2^{(1/2)+1-x})^4 + 82 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 23) * (2^{(1/2)-1+x}) / (2^{(1/2)+1-x}) * (8 + 3 * 2^{(1/2)})) * (-775687 + 549362 * 2^{(1/2)}) \wedge (1/2) + 401698 * \operatorname{arctanh}(31/2 * (8 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 3 * 2^{(1/2)} * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 8 - 3 * 2^{(1/2)}) \wedge (1/2) / (-8866 + 6820 * 2^{(1/2)}) \wedge (1/2)) * 2^{(1/2)} - 63426 * \operatorname{arctanh}(31/2 * (8 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 3 * 2^{(1/2)} * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 8 - 3 * 2^{(1/2)}) \wedge (1/2) / (-8866 + 6820 * 2^{(1/2)}) \wedge (1/2)) / ((8 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 3 * 2^{(1/2)} * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 8 - 3 * 2^{(1/2)}) / (1 + (2^{(1/2)-1+x}) / (2^{(1/2)+1-x}))^2 \wedge (1/2) / (1 + (2^{(1/2)-1+x}) / (2^{(1/2)+1-x})) / (8 + 3 * 2^{(1/2)}) / (-8866 + 6820 * 2^{(1/2)}) \wedge (1/2) + 3/21142 * (8 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 3 * 2^{(1/2)} * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 8 - 3 * 2^{(1/2)}) \wedge (1/2) * 2^{(1/2)} * (369 * 2^{(1/2)} * (-8866 + 6820 * 2^{(1/2)}) \wedge (1/2)) \wedge (1/2) * \operatorname{arctan}(1/11692487 * (-775687 + 549362 * 2^{(1/2)}) \wedge (1/2) * (-23 * (8 + 3 * 2^{(1/2)}) * (-23 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 24 * 2^{(1/2)-41)) \wedge (1/2) * (6485 * 2^{(1/2)} * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 10368 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 22379 * 2^{(1/2)} + 32016) / (23 * (2^{(1/2)-1+x})^4 / (2^{(1/2)+1-x})^4 + 82 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 23) * (2^{(1/2)-1+x}) / (2^{(1/2)+1-x}) * (8 + 3 * 2^{(1/2)})) * (-775687 + 549362 * 2^{(1/2)}) \wedge (1/2) + 520 * (-8866 + 6820 * 2^{(1/2)}) \wedge (1/2) * \operatorname{arctan}(1/11692487 * (-775687 + 549362 * 2^{(1/2)}) \wedge (1/2) * (-23 * (8 + 3 * 2^{(1/2)}) * (-23 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 24 * 2^{(1/2)-41)) \wedge (1/2) * (6485 * 2^{(1/2)} * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 10368 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 22379 * 2^{(1/2)} + 32016) / (23 * (2^{(1/2)-1+x})^4 / (2^{(1/2)+1-x})^4 + 82 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 23) * (2^{(1/2)-1+x}) / (2^{(1/2)+1-x}) * (8 + 3 * 2^{(1/2)})) * (-775687 + 549362 * 2^{(1/2)}) \wedge (1/2) + 465124 * \operatorname{arctanh}(31/2 * (8 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 3 * 2^{(1/2)} * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 8 - 3 * 2^{(1/2)}) \wedge (1/2) / (-8866 + 6820 * 2^{(1/2)}) \wedge (1/2)) * 2^{(1/2)} - 866822 * \operatorname{arctanh}(31/2 * (8 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 3 * 2^{(1/2)} * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 8 - 3 * 2^{(1/2)}) \wedge (1/2) / (-8866 + 6820 * 2^{(1/2)}) \wedge (1/2)) / ((8 * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 3 * 2^{(1/2)} * (2^{(1/2)-1+x})^2 / (2^{(1/2)+1-x})^2 + 8 - 3 * 2^{(1/2)}) / (1 + (2^{(1/2)-1+x}) / (2^{(1/2)+1-x}))^2 \wedge (1/2) / (1 + (2^{(1/2)-1+x}) / (2^{(1/2)+1-x})) / (8 + 3 * 2^{(1/2)}) / (-8866 + 6820 * 2^{(1/2)}) \wedge (1/2)$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] integrate(sqrt(2\*x^2 - x + 3)/(5\*x^2 + 3\*x + 2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^2 - x + 3}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2), x)
```

```
[Out] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2), x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2), x)
```

$$3.60 \quad \int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=188

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{31(5x^2+3x+2)} + \frac{1}{62} \sqrt{\frac{1}{682}(70517+49942\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(70517+49942\sqrt{2})}}((973+696\sqrt{2})x+277\sqrt{2})}{\sqrt{2x^2-x+3}} \right)$$

Rubi [A] time = 0.39, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {971, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{31(5x^2+3x+2)} + \frac{1}{62} \sqrt{\frac{1}{682}(70517+49942\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(70517+49942\sqrt{2})}}((973+696\sqrt{2})x+277\sqrt{2}+419)}{\sqrt{2x^2-x+3}} \right) - \frac{1}{62} \sqrt{\frac{1}{682}(49942\sqrt{2}-70517)} \tanh^{-1} \left( \frac{\sqrt{\frac{11}{31(49942\sqrt{2}-70517)}}((973-696\sqrt{2})x-277\sqrt{2}+419)}{\sqrt{2x^2-x+3}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2\*x^2]/(2 + 3\*x + 5\*x^2)^2,x]

[Out] ((3 + 10\*x)\*Sqrt[3 - x + 2\*x^2])/(31\*(2 + 3\*x + 5\*x^2)) + (Sqrt[(70517 + 49942\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(70517 + 49942\*Sqrt[2]))])\*(419 + 277\*Sqrt[2] + (973 + 696\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/62 - (Sqrt[(-70517 + 49942\*Sqrt[2])/682]\*ArcTanh[(Sqrt[11/(31\*(-70517 + 49942\*Sqrt[2]))])\*(419 - 277\*Sqrt[2] + (973 - 696\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/62

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 971

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/((b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[2\*c\*d\*(2\*p + 3) + b\*e\*q + (2\*b\*f\*q + 2\*c\*e\*(2\*p + q + 3))\*x + 2\*c\*f\*(2\*p + 2\*q + 3)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

Rule 1029

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] :> Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2} dx = \frac{(3+10x)\sqrt{3-x+2x^2}}{31(2+3x+5x^2)} - \frac{1}{31} \int \frac{-\frac{63}{2} + 11x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31(2+3x+5x^2)} - \frac{\int \frac{\frac{11}{2}(85-63\sqrt{2}) - \frac{11}{2}(41-22\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{682\sqrt{2}} + \frac{\int \frac{\frac{11}{2}(85+63\sqrt{2}) - \frac{11}{2}(41+22\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{682\sqrt{2}}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31(2+3x+5x^2)} - \frac{1}{248} \left( 11(99884 - 70517\sqrt{2}) \right) \text{Subst} \left( \int \frac{1}{-\frac{3751}{4}(70517 - 4994\sqrt{2}) + x} dx \right)$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31(2+3x+5x^2)} + \frac{1}{62} \sqrt{\frac{1}{682}(70517 + 49942\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(70517+49942\sqrt{2})}}(4x+3)}{\sqrt{3-x+2x^2}} \right)$$

Mathematica [C] time = 1.04, size = 214, normalized size = 1.14

$$\frac{27280\sqrt{2x^2-x+3}(10x+3)}{5x^2+3x+2} + i\sqrt{286-22i\sqrt{31}}(973\sqrt{31}+1271i)\tanh^{-1}\left(\frac{4i\sqrt{31}x-22x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) - i\sqrt{286+22i\sqrt{31}}(973\sqrt{31}-1271i)\tanh^{-1}\left(\frac{-22-4i\sqrt{31}x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)$$

845680

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2, x]
```

```
[Out] ((27280*(3 + 10*x)*Sqrt[3 - x + 2*x^2])/(2 + 3*x + 5*x^2) + I*Sqrt[286 - (2
2*I)*Sqrt[31]]*(1271*I + 973*Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] - 22*x + (4
*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - I*Sq
rt[286 + (22*I)*Sqrt[31]]*(-1271*I + 973*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31]
+ (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*
x^2])])/845680
```

IntegrateAlgebraic [C] time = 0.70, size = 423, normalized size = 2.25

$$\frac{651\text{RootSum}\left[56t^4 + 6\sqrt{2}t^3 + 17t^2 - 26\sqrt{2}t - 56\right]}{1550} + \frac{\log\left(\frac{41 + \sqrt{27-13}\sqrt{2}t}{100\sqrt{2}t^2 - 10t - 13\sqrt{2}}\right)}{25\text{RootSum}\left[56t^4 + 6\sqrt{2}t^3 + 17t^2 - 26\sqrt{2}t - 56\right]} - \frac{10\sqrt{2}\log\left(\frac{-41 + \sqrt{27-13}\sqrt{2}t}{-108t^2 + 9\sqrt{2}t^2 + 1791 - 13\sqrt{2}}\right) + 41\log\left(\frac{41 + \sqrt{27-13}\sqrt{2}t}{-108t^2 + 9\sqrt{2}t^2 + 1791 - 13\sqrt{2}}\right)}{1550\text{RootSum}\left[56t^4 + 6\sqrt{2}t^3 + 17t^2 - 26\sqrt{2}t - 56\right]} - \frac{558t^2\log\left(\frac{-41 + \sqrt{27-13}\sqrt{2}t}{-108t^2 + 9\sqrt{2}t^2 + 1791 - 13\sqrt{2}}\right) - 191\sqrt{2}\log\left(\frac{-41 + \sqrt{27-13}\sqrt{2}t}{-108t^2 + 9\sqrt{2}t^2 + 1791 - 13\sqrt{2}}\right)}{31(5t^2 + 3t + 2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2, x]
```

```
[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(31*(2 + 3*x + 5*x^2)) - (6151*RootSum[-56
- 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , Log[-(Sqrt[2]*x) +
Sqrt[3 - x + 2*x^2] - #1]/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3)
& ])/1550 + (2*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*
#1^4 & , (49*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 10*Sqrt[2]*Log[
-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[
```

2]\*#1^2 - 10\*#1^3) & ])/25 - RootSum[-56 - 26\*Sqrt[2]\*#1 + 17\*#1^2 + 6\*Sqrt[2]\*#1^3 - 5\*#1^4 & , (-191\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1 - 55\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1^2)/(-13\*Sqrt[2] + 17\*#1 + 9\*Sqrt[2]\*#1^2 - 10\*#1^3) & ]/155

**fricas** [B] time = 1.19, size = 2102, normalized size = 11.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] -1/186703822445536\*(88412\*4988406728^(1/4)\*sqrt(24971)\*sqrt(341)\*sqrt(2)\*(5\*x^2 + 3\*x + 2)\*sqrt(70517\*sqrt(2) + 99884)\*arctan(1/10668926457462302923\*(3096404\*sqrt(24971)\*(11\*4988406728^(3/4)\*sqrt(341)\*(537184\*x^7 - 2047820\*x^6 + 4310846\*x^5 - 6853210\*x^4 + 3421536\*x^3 - 1589328\*x^2 - sqrt(2)\*(370014\*x^7 - 1438653\*x^6 + 3014868\*x^5 - 4873381\*x^4 + 2452952\*x^3 - 1184616\*x^2 - 2633472\*x + 1893888) - 3787776\*x + 2633472) + 774101\*4988406728^(1/4)\*sqrt(341)\*(40625\*x^7 - 622509\*x^6 + 3280912\*x^5 - 7459052\*x^4 + 9621216\*x^3 - 5992992\*x^2 - sqrt(2)\*(28204\*x^7 - 433677\*x^6 + 2297444\*x^5 - 5257628\*x^4 + 6800832\*x^3 - 4341024\*x^2 - 4810752\*x + 3442176) - 6884352\*x + 4810752))\*sqrt(2\*x^2 - x + 3)\*sqrt(70517\*sqrt(2) + 99884) + 30285984782473634104\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x^8 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752088\*x^3 + 396144\*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) - 2\*sqrt(49942)\*(sqrt(24971)\*(11\*4988406728^(3/4)\*sqrt(341)\*(84604\*x^7 - 121310\*x^6 + 389610\*x^5 - 147168\*x^4 + 168912\*x^3 + 186624\*x^2 - sqrt(2)\*(57082\*x^7 - 82029\*x^6 + 264639\*x^5 - 107216\*x^4 + 130104\*x^3 + 110592\*x^2 - 110592\*x) - 186624\*x) + 774101\*4988406728^(1/4)\*sqrt(341)\*(6379\*x^7 - 82508\*x^6 + 318020\*x^5 - 410688\*x^4 + 523872\*x^3 + 331776\*x^2 - sqrt(2)\*(4365\*x^7 - 56468\*x^6 + 217820\*x^5 - 282816\*x^4 + 366624\*x^3 + 207360\*x^2 - 207360\*x) - 331776\*x))\*sqrt(2\*x^2 - x + 3)\*sqrt(70517\*sqrt(2) + 99884) + 425261673562\*sqrt(31)\*sqrt(2)\*(123408\*x^8 - 914152\*x^7 + 1578888\*x^6 - 3293072\*x^5 + 396480\*x^4 + 798336\*x^3 - 3822336\*x^2 - sqrt(2)\*(15550\*x^8 - 118051\*x^7 + 244047\*x^6 - 707374\*x^5 + 1053960\*x^4 - 1667952\*x^3 + 1209600\*x^2 - 1036800\*x) + 3276288\*x) + 19330076071\*sqrt(31)\*(254591\*x^8 - 4815126\*x^7 + 32303580\*x^6 - 90866808\*x^5 + 108781920\*x^4 - 74219328\*x^3 - 168956928\*x^2 - 15488\*sqrt(2)\*(4\*x^8 - 76\*x^7 + 517\*x^6 - 1536\*x^5 + 2385\*x^4 - 3618\*x^3 + 2268\*x^2 - 1944\*x) + 144820224\*x))\*sqrt(-(4988406728^(1/4)\*sqrt(24971)\*sqrt(341)\*sqrt(31)\*sqrt(2\*x^2 - x + 3)\*(sqrt(2)\*(10\*x + 3) - 13\*x - 7)\*sqrt(70517\*sqrt(2) + 99884) - 1175859419\*x^2 - 1055873764\*sqrt(2)\*(2\*x^2 - x + 3) + 3623566781\*x - 4799426200)/x^2) + 344158917982654933\*sqrt(31)\*(2828123\*x^8 - 9696916\*x^7 + 53385560\*x^6 - 142835344\*x^5 + 254146592\*x^4 - 249300096\*x^3 + 37981440\*x^2 - 7744\*sqrt(2)\*(1348\*x^8 - 2692\*x^7 + 9789\*x^6 - 10070\*x^5 + 15569\*x^4 - 5568\*x^3 + 1080\*x^2 + 4320\*x - 5184) + 223064064\*x - 94887936))/(2585191\*x^8 - 4661200\*x^7 + 14191920\*x^6 + 490880\*x^5 - 13562944\*x^4 + 44249088\*x^3 - 34615296\*x^2 - 24772608\*x + 18579456)) + 88412\*4988406728^(1/4)\*sqrt(24971)\*sqrt(341)\*sqrt(2)\*(5\*x^2 + 3\*x + 2)\*sqrt(70517\*sqrt(2) + 99884)\*arctan(1/10668926457462302923\*(3096404\*sqrt(24971)\*(11\*4988406728^(3/4)\*sqrt(341)\*(537184\*x^7 - 2047820\*x^6 + 4310846\*x^5 - 6853210\*x^4 + 3421536\*x^3 - 1589328\*x^2 - sqrt(2)\*(370014\*x^7 - 1438653\*x^6 + 3014868\*x^5 - 4873381\*x^4 + 2452952\*x^3 - 1184616\*x^2 - 2633472\*x + 1893888) - 3787776\*x + 2633472) + 774101\*4988406728^(1/4)\*sqrt(341)\*(40625\*x^7 - 622509\*x^6 + 3280912\*x^5 - 7459052\*x^4 + 9621216\*x^3 - 5992992\*x^2 - sqrt(2)\*(28204\*x^7 - 433677\*x^6 + 2297444\*x^5 - 5257628\*x^4 + 6800832\*x^3 - 4341024\*x^2 - 4810752\*x + 3442176) - 6884352\*x + 4810752))\*sqrt(2\*x^2 - x + 3)\*sqrt(70517\*sqrt(2) + 99884) - 30285984782473634104\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x^8 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752088\*x^3 + 396144\*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) - 2\*sq

```

rt(49942)*(sqrt(24971))*(11*4988406728^(3/4)*sqrt(341)*(84604*x^7 - 121310*x
^6 + 389610*x^5 - 147168*x^4 + 168912*x^3 + 186624*x^2 - sqrt(2)*(57082*x^7
- 82029*x^6 + 264639*x^5 - 107216*x^4 + 130104*x^3 + 110592*x^2 - 110592*x
) - 186624*x) + 774101*4988406728^(1/4)*sqrt(341)*(6379*x^7 - 82508*x^6 + 3
18020*x^5 - 410688*x^4 + 523872*x^3 + 331776*x^2 - sqrt(2)*(4365*x^7 - 5646
8*x^6 + 217820*x^5 - 282816*x^4 + 366624*x^3 + 207360*x^2 - 207360*x) - 331
776*x))*sqrt(2*x^2 - x + 3)*sqrt(70517*sqrt(2) + 99884) - 425261673562*sqrt
(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*
x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x
^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32
76288*x) - 19330076071*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 -
90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)
*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 19
44*x) + 144820224*x))*sqrt((4988406728^(1/4)*sqrt(24971)*sqrt(341)*sqrt(31)
*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*sqrt(70517*sqrt(2) + 9
9884) + 1175859419*x^2 + 1055873764*sqrt(2)*(2*x^2 - x + 3) - 3623566781*x
+ 4799426200)/x^2) - 344158917982654933*sqrt(31)*(2828123*x^8 - 9696916*x^7
+ 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*
x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4
- 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*
x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3
- 34615296*x^2 - 24772608*x + 18579456)) - 4988406728^(1/4)*sqrt(24971)*sq
rt(341)*sqrt(31)*(499420*x^2 - 70517*sqrt(2)*(5*x^2 + 3*x + 2) + 299652*x +
199768)*sqrt(70517*sqrt(2) + 99884)*log(199768*(4988406728^(1/4)*sqrt(2497
1)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*s
qrt(70517*sqrt(2) + 99884) + 1175859419*x^2 + 1055873764*sqrt(2)*(2*x^2 - x
+ 3) - 3623566781*x + 4799426200)/x^2) + 4988406728^(1/4)*sqrt(24971)*sqrt
(341)*sqrt(31)*(499420*x^2 - 70517*sqrt(2)*(5*x^2 + 3*x + 2) + 299652*x + 1
99768)*sqrt(70517*sqrt(2) + 99884)*log(-199768*(4988406728^(1/4)*sqrt(24971
)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*sq
rt(70517*sqrt(2) + 99884) - 1175859419*x^2 - 1055873764*sqrt(2)*(2*x^2 - x
+ 3) + 3623566781*x - 4799426200)/x^2) - 6022703949856*sqrt(2*x^2 - x + 3)*
(10*x + 3))/(5*x^2 + 3*x + 2)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infini
ty]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 17.67Done
```

**maple** [B] time = 0.23, size = 16357, normalized size = 87.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```



**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] integrate(sqrt(2\*x^2 - x + 3)/(5\*x^2 + 3\*x + 2)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(1/2)/(3\*x + 5\*x^2 + 2)^2,x)

[Out] int((2\*x^2 - x + 3)^(1/2)/(3\*x + 5\*x^2 + 2)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(1/2)/(5\*x\*\*2+3\*x+2)\*\*2,x)

[Out] Integral(sqrt(2\*x\*\*2 - x + 3)/(5\*x\*\*2 + 3\*x + 2)\*\*2, x)

**3.61**  $\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3} dx$

**Optimal.** Leaf size=223

$$\frac{\sqrt{2x^2 - x + 3} (10x + 3)}{62 (5x^2 + 3x + 2)^2} + \frac{(13665x + 3464)\sqrt{2x^2 - x + 3}}{84568 (5x^2 + 3x + 2)} + \frac{\sqrt{\frac{1}{682} (112285869463 + 79399380740\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{1}{31} (112285869463 + 79399380740\sqrt{2})}}{\sqrt{2x^2 - x + 3}} \right)}{169136}$$

**Rubi [A]** time = 0.46, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {971, 1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2 - x + 3} (10x + 3)}{62 (5x^2 + 3x + 2)^2} + \frac{(13665x + 3464)\sqrt{2x^2 - x + 3}}{84568 (5x^2 + 3x + 2)} + \frac{\sqrt{\frac{1}{682} (112285869463 + 79399380740\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31} (112285869463 + 79399380740\sqrt{2})}}{\sqrt{2x^2 - x + 3}} \right)}{169136} - \frac{\sqrt{\frac{1}{682} (79399380740\sqrt{2} - 112285869463)} \tanh^{-1} \left( \frac{\sqrt{\frac{11}{31} (112285869463 + 79399380740\sqrt{2})}}{\sqrt{2x^2 - x + 3}} \right)}{169136}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(62*(2 + 3*x + 5*x^2)^2) + ((3464 + 13665*x)*Sqrt[3 - x + 2*x^2])/(84568*(2 + 3*x + 5*x^2)) + (Sqrt[(112285869463 + 79399380740*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(112285869463 + 79399380740*Sqrt[2]))])*(509587 + 362788*Sqrt[2] + (1235163 + 872375*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2]])/169136 - (Sqrt[(-112285869463 + 79399380740*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-112285869463 + 79399380740*Sqrt[2]))])*(509587 - 362788*Sqrt[2] + (1235163 - 872375*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2]])/169136
```

**Rule 204**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

**Rule 206**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

**Rule 971**

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p + 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]
```

**Rule 1029**

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]
```

, 0]

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)
^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3} dx = \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} - \frac{1}{62} \int \frac{-\frac{183}{2} + 31x - 40x^2}{\sqrt{3-x+2x^2}(2+3x+5x^2)^2} dx$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568(2+3x+5x^2)} - \frac{\int \frac{-213004 + \frac{358655x}{4}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{465124}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568(2+3x+5x^2)} - \frac{\int \frac{\frac{121}{4}(110061-77456\sqrt{2}) - \frac{121}{4}(448)}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{10232728\sqrt{2}}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568(2+3x+5x^2)} - \frac{(11(158798761480 - 11228}}{10232728\sqrt{2}}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568(2+3x+5x^2)} + \frac{\sqrt{\frac{1}{682}}(112285869463 + 793}}{10232728\sqrt{2}}$$

**Mathematica [C]** time = 2.08, size = 299, normalized size = 1.34

$$\frac{5 \left( \frac{i \sqrt{286+22i\sqrt{31}} (258253\sqrt{31}+1004586i) \tanh^{-1}\left(\frac{-22-4i\sqrt{31}+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{(\sqrt{31}-13i)^2} + \frac{2000(1364(\sqrt{31}+13i)\sqrt{2x^2-x+3}(68325x^3+58315x^2+51362x+11020)-5\sqrt{286-22i\sqrt{31}}(174475\sqrt{31}-202151i)(5x^2+3x+2)^2 \tanh^{-1}\left(\frac{-22+4i\sqrt{31}-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right))}{(\sqrt{31}-13i)(\sqrt{31}+13i)^2(-10ix+\sqrt{31}-3i)^2(10ix+\sqrt{31}+3i)^2} \right)}{14418844}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
[Out] (5*((I*Sqrt[286 + (22*I)*Sqrt[31]]*(1004586*I + 258253*Sqrt[31]))*ArcTanh[(6
3 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x]/(2*Sqrt[286 + (22*I)*Sqrt[31]]*S
qrt[3 - x + 2*x^2])))/(-13*I + Sqrt[31])^2 + (2000*(1364*(13*I + Sqrt[31])*
Sqrt[3 - x + 2*x^2]*(11020 + 51362*x + 58315*x^2 + 68325*x^3) - 5*Sqrt[286
- (22*I)*Sqrt[31]]*(-202151*I + 174475*Sqrt[31]))*(2 + 3*x + 5*x^2)^2*ArcTan
h[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x]/(2*Sqrt[286 - (22*I)*Sqrt[31
]]*Sqrt[3 - x + 2*x^2])))/((-13*I + Sqrt[31])*(13*I + Sqrt[31])^2*(-3*I +
Sqrt[31] - (10*I)*x)^2*(3*I + Sqrt[31] + (10*I)*x)^2))/14418844
```

**IntegrateAlgebraic [C]** time = 0.82, size = 396, normalized size = 1.78

$$\frac{\text{RootSum}\left[-561^2 + 6\sqrt{2}81^2 + 1781^2 - 26\sqrt{2}81 - 566, \frac{-492142797\sqrt{2}\sqrt{31}\sqrt{2x^2-x+3} + \sqrt{286-22i\sqrt{31}}(174475\sqrt{31}-202151i)(5x^2+3x+2)^2 \tanh^{-1}\left(\frac{-22+4i\sqrt{31}-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{-19i^2+5\sqrt{2}81^2-1781-566}\right] + \text{RootSum}\left[-561^2 + 6\sqrt{2}81^2 + 1781^2 - 26\sqrt{2}81 - 566, \frac{-22843297\sqrt{2}\sqrt{31}\sqrt{2x^2-x+3} + \sqrt{286+22i\sqrt{31}}(174475\sqrt{31}+202151i)(5x^2+3x+2)^2 \tanh^{-1}\left(\frac{-22-4i\sqrt{31}+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{-19i^2+5\sqrt{2}81^2-1781-566}\right]}{84568(3x^2+3x+2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
[Out] (Sqrt[3 - x + 2*x^2]*(11020 + 51362*x + 58315*x^2 + 68325*x^3))/(84568*(2 +
3*x + 5*x^2)^2) + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 -
5*#1^4 & , (-537295920831*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 1
20146195680*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 45923
442075*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17
*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/55920590000 - RootSum[-56 - 26*Sqrt[2]*
#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-2139373897*Log[-(Sqrt[2]*x) +
Sqrt[3 - x + 2*x^2] - #1] + 277937160*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x
+ 2*x^2] - #1]*#1 - 228643025*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]
*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/225486250
```

**fricas** [B] time = 1.44, size = 2182, normalized size = 9.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out]  $\frac{1}{65052151896952926425996714240} \cdot (14205421276 \cdot 788032707736935368450^{1/4} \cdot \sqrt{39699690370} \cdot \sqrt{341} \cdot \sqrt{2} \cdot (25x^4 + 30x^3 + 29x^2 + 12x + 4) \cdot \sqrt{112285869463 \sqrt{2} + 158798761480}) \cdot \arctan\left(\frac{1}{861047662213971287591057659551879544939625} \cdot (2461380802940 \cdot \sqrt{39699690370}) \cdot (22 \cdot 788032707736935368450^{3/4} \cdot \sqrt{341} \cdot (667937076x^7 - 2573871186x^6 + 5404850058x^5 - 8671430212x^4 + 4348809776x^3 - 2064441888x^2 - \sqrt{2} \cdot (473555282x^7 - 1821195871x^6 + 3826055542x^5 - 6128133137x^4 + 3070797960x^3 - 1452037320x^2 - 3352976640x + 2366869248) - 4733738496x + 3352976640) + 615345200735 \cdot 788032707736935368450^{1/4} \cdot \sqrt{341} \cdot (50730703x^7 - 778833417x^6 + 4116367112x^5 - 9392273180x^4 + 12133646496x^3 - 7660912032x^2 - \sqrt{2} \cdot (35938543x^7 - 551546778x^6 + 2913578540x^5 - 6643469608x^4 + 8580088800x^3 - 5403919680x^2 - 6107913216x + 4313793024) - 8627586048x + 6107913216)\right) \cdot \sqrt{2x^2 - x + 3} \cdot \sqrt{112285869463 \sqrt{2} + 158798761480} + 2444264331446112042193970130340819353377000 \cdot \sqrt{31} \cdot \sqrt{2} \cdot (28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2} \cdot (8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - 2 \cdot \sqrt{39699690370/160673} \cdot (\sqrt{39699690370}) \cdot (22 \cdot 788032707736935368450^{3/4} \cdot \sqrt{341} \cdot (104024992x^7 - 149335248x^6 + 480784368x^5 - 188730368x^4 + 223535232x^3 + 214417152x^2 - \sqrt{2} \cdot (73906058x^7 - 106073653x^6 + 341348823x^5 - 133050960x^4 + 156704760x^3 + 154338048x^2 - 154338048x) - 214417152x) + 615345200735 \cdot 788032707736935368450^{1/4} \cdot \sqrt{341} \cdot (7903323x^7 - 102233612x^6 + 394216580x^5 - 510585408x^4 + 657060192x^3 + 391744512x^2 - 4 \cdot \sqrt{2} \cdot (1401761x^7 - 18132196x^6 + 69912940x^5 - 90501120x^4 + 116274240x^3 + 70118784x^2 - 70118784x) - 391744512x) \cdot \sqrt{2x^2 - x + 3} \cdot \sqrt{112285869463 \sqrt{2} + 158798761480} + 43175912524323866211143695850 \cdot \sqrt{31} \cdot \sqrt{2} \cdot (123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2} \cdot (15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) + 1962541478378357555051986175 \cdot \sqrt{31} \cdot (254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488 \cdot \sqrt{2} \cdot (4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x) \cdot \sqrt{-(788032707736935368450^{1/4} \cdot \sqrt{39699690370} \cdot \sqrt{341} \cdot \sqrt{31} \cdot \sqrt{2x^2 - x + 3} \cdot (\sqrt{2} \cdot (12053x + 5138) - 17191x - 6915) \cdot \sqrt{112285869463 \sqrt{2} + 158798761480}) - 150182556985858180945x^2 - 134857806273015509420 \cdot \sqrt{2} \cdot (2x^2 - x + 3) + 462807471527848680055x - 612990028513706861000)/x^2) + 27775731039160364115840569662963856288375 \cdot \sqrt{31} \cdot (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744 \cdot \sqrt{2} \cdot (1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456) + 14205421276 \cdot 788032707736935368450^{1/4} \cdot \sqrt{39699690370} \cdot \sqrt{341} \cdot \sqrt{2} \cdot (25x^4 + 30x^3 + 29x^2 + 12x + 4) \cdot \sqrt{112285869463 \sqrt{2} + 158798761480}) \cdot \arctan\left(\frac{1}{861047662213971287591057659551879544939625} \cdot (2461380802940 \cdot \sqrt{39699690370}) \cdot (22 \cdot 788032707736935368450^{3/4} \cdot \sqrt{341} \cdot (667937076x^7 - 2573871186x^6 + 5404850058x^5 - 8671430212x^4 + 4348809776x^3 - 2064441888x^2 - \sqrt{2} \cdot (473555282x^7 - 1821195871x^6 + 3826055542x^5 - 6128133137x^4 + 3070797960x^3 - 1452037320x^2 - 3352976640x + 2366869248) - 4733738496x + 3352976640) + 615345200735 \cdot 788032707736935368450^{1/4} \cdot \sqrt{341} \cdot (50730703x^7 - 778833417x^6 + 4116367112x^5 - 9392273180x^4 + 12133646496x^3 - 7660912032x^2 - \sqrt{2} \cdot (35938543x^7 - 551546778x^6 + 2913578540x^5 - 6643469608x^4 + 8580088800x^3 - 540$

```

3919680*x^2 - 6107913216*x + 4313793024) - 8627586048*x + 6107913216))*sqrt
(2*x^2 - x + 3)*sqrt(112285869463*sqrt(2) + 158798761480) - 244426433144611
2042193970130340819353377000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704
270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(874
6*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3
96144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(39699690370/1
60673)*(sqrt(39699690370)*(22*788032707736935368450^(3/4)*sqrt(341)*(104024
992*x^7 - 149335248*x^6 + 480784368*x^5 - 188730368*x^4 + 223535232*x^3 + 2
14417152*x^2 - sqrt(2)*(73906058*x^7 - 106073653*x^6 + 341348823*x^5 - 1330
50960*x^4 + 156704760*x^3 + 154338048*x^2 - 154338048*x) - 214417152*x) + 6
15345200735*788032707736935368450^(1/4)*sqrt(341)*(7903323*x^7 - 102233612*
x^6 + 394216580*x^5 - 510585408*x^4 + 657060192*x^3 + 391744512*x^2 - 4*sqrt
(2)*(1401761*x^7 - 18132196*x^6 + 69912940*x^5 - 90501120*x^4 + 116274240*
x^3 + 70118784*x^2 - 70118784*x) - 391744512*x))*sqrt(2*x^2 - x + 3)*sqrt(1
12285869463*sqrt(2) + 158798761480) - 43175912524323866211143695850*sqrt(31
)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4
+ 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
- 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32762
88*x) - 1962541478378357555051986175*sqrt(31)*(254591*x^8 - 4815126*x^7 + 3
2303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 -
15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 +
2268*x^2 - 1944*x) + 144820224*x))*sqrt((788032707736935368450^(1/4)*sqrt(
39699690370)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(12053*x + 513
8) - 17191*x - 6915)*sqrt(112285869463*sqrt(2) + 158798761480) + 1501825569
85858180945*x^2 + 134857806273015509420*sqrt(2)*(2*x^2 - x + 3) - 462807471
527848680055*x + 612990028513706861000)/x^2) - 2777573103916036411584056966
2963856288375*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14283534
4*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x
^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 43
20*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 141919
20*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608
*x + 18579456)) + 788032707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*s
qrt(31)*(3969969037000*x^4 + 4763962844400*x^3 + 4605164082920*x^2 - 112285
869463*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1905585137760*x + 63
5195045920)*sqrt(112285869463*sqrt(2) + 158798761480)*log(635195045920/1606
73*(788032707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*sqrt(2
*x^2 - x + 3)*(sqrt(2)*(12053*x + 5138) - 17191*x - 6915)*sqrt(112285869463
*sqrt(2) + 158798761480) + 150182556985858180945*x^2 + 13485780627301550942
0*sqrt(2)*(2*x^2 - x + 3) - 462807471527848680055*x + 612990028513706861000
)/x^2) - 788032707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*(
3969969037000*x^4 + 4763962844400*x^3 + 4605164082920*x^2 - 112285869463*sq
rt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1905585137760*x + 63519504592
0)*sqrt(112285869463*sqrt(2) + 158798761480)*log(-635195045920/160673*(7880
32707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x
+ 3)*(sqrt(2)*(12053*x + 5138) - 17191*x - 6915)*sqrt(112285869463*sqrt(2)
+ 158798761480) - 150182556985858180945*x^2 - 134857806273015509420*sqrt(2
)*(2*x^2 - x + 3) + 462807471527848680055*x - 612990028513706861000)/x^2) +
769228926981280465731680*(68325*x^3 + 58315*x^2 + 51362*x + 11020)*sqrt(2*
x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)

```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,inf
```

nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity  
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai  
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity  
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf  
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit  
y]Evaluation time: 48.06Done

**maple [B]** time = 0.37, size = 44343, normalized size = 198.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)^3,x)

[Out] result too large to display

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(1/2)/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] integrate(sqrt(2\*x^2 - x + 3)/(5\*x^2 + 3\*x + 2)^3, x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(1/2)/(3\*x + 5\*x^2 + 2)^3,x)

[Out] int((2\*x^2 - x + 3)^(1/2)/(3\*x + 5\*x^2 + 2)^3, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(1/2)/(5\*x\*\*2+3\*x+2)\*\*3,x)

[Out] Integral(sqrt(2\*x\*\*2 - x + 3)/(5\*x\*\*2 + 3\*x + 2)\*\*3, x)

$$3.62 \quad \int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2)^4 dx$$

**Optimal.** Leaf size=231

$$\frac{56422489(2x^2 - x + 3)^{5/2} x^2}{8257536} + \frac{48669967(2x^2 - x + 3)^{5/2} x}{22020096} + \frac{2124689283(2x^2 - x + 3)^{5/2}}{146800640} - \frac{382121949(1 - 4x)}{134217728}$$

**Rubi [A]** time = 0.34, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 27, number of rules / integrand size = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{625}{24} (2x^2 - x + 3)^{5/2} x^2 + \frac{7625}{96} (2x^2 - x + 3)^{5/2} x + \frac{95165}{768} (2x^2 - x + 3)^{5/2} + \frac{941905}{9216} (2x^2 - x + 3)^{5/2} x^4 + \frac{10444117}{294912} (2x^2 - x + 3)^{5/2} x^3 + \frac{56422489}{8257536} (2x^2 - x + 3)^{5/2} x^2 + \frac{48669967}{22020096} (2x^2 - x + 3)^{5/2} x + \frac{2124689283}{146800640} (2x^2 - x + 3)^{5/2} - \frac{382121949(1 - 4x)}{134217728} - \frac{26366414481(1 - 4x)\sqrt{2x^2 - x + 3}}{2147483648} - \frac{606427533063 \operatorname{arsinh}\left(\frac{1 - 4x}{\sqrt{23}}\right)}{4294967296\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] (-26366414481\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/2147483648 - (382121949\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(3/2))/134217728 + (2124689283\*(3 - x + 2\*x^2)^(5/2))/146800640 + (48669967\*x\*(3 - x + 2\*x^2)^(5/2))/22020096 - (56422489\*x^2\*(3 - x + 2\*x^2)^(5/2))/8257536 + (10444117\*x^3\*(3 - x + 2\*x^2)^(5/2))/294912 + (941905\*x^4\*(3 - x + 2\*x^2)^(5/2))/9216 + (95165\*x^5\*(3 - x + 2\*x^2)^(5/2))/768 + (7625\*x^6\*(3 - x + 2\*x^2)^(5/2))/96 + (625\*x^7\*(3 - x + 2\*x^2)^(5/2))/24 - (606427533063\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(4294967296\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[Rt[b, 2]\*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 612

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]



Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^4 dx &= \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{24} \int (3-x+2x^2)^{3/2} (384+2304x+9024x^2) dx \\
&= \frac{7625}{96} x^6 (3-x+2x^2)^{5/2} + \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (384+2304x+9024x^2) dx \\
&= \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{7625}{96} x^6 (3-x+2x^2)^{5/2} + \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (384+2304x+9024x^2) dx \\
&= \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{7625}{96} x^6 (3-x+2x^2)^{5/2} + \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (384+2304x+9024x^2) dx \\
&= \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{7625}{96} x^6 (3-x+2x^2)^{5/2} + \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (384+2304x+9024x^2) dx \\
&= -\frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{7625}{96} x^6 (3-x+2x^2)^{5/2} + \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (384+2304x+9024x^2) dx \\
&= \frac{48669967x (3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{7625}{96} x^6 (3-x+2x^2)^{5/2} + \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (384+2304x+9024x^2) dx \\
&= \frac{2124689283 (3-x+2x^2)^{5/2}}{146800640} + \frac{48669967x (3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{7625}{96} x^6 (3-x+2x^2)^{5/2} + \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (384+2304x+9024x^2) dx \\
&= -\frac{382121949(1-4x) (3-x+2x^2)^{3/2}}{134217728} + \frac{2124689283 (3-x+2x^2)^{5/2}}{146800640} + \frac{48669967x (3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{7625}{96} x^6 (3-x+2x^2)^{5/2} + \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (384+2304x+9024x^2) dx \\
&= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648} - \frac{382121949(1-4x) (3-x+2x^2)^{3/2}}{134217728} + \frac{2124689283 (3-x+2x^2)^{5/2}}{146800640} + \frac{48669967x (3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{7625}{96} x^6 (3-x+2x^2)^{5/2} + \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (384+2304x+9024x^2) dx \\
&= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648} - \frac{382121949(1-4x) (3-x+2x^2)^{3/2}}{134217728} + \frac{2124689283 (3-x+2x^2)^{5/2}}{146800640} + \frac{48669967x (3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{7625}{96} x^6 (3-x+2x^2)^{5/2} + \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (384+2304x+9024x^2) dx \\
&= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648} - \frac{382121949(1-4x) (3-x+2x^2)^{3/2}}{134217728} + \frac{2124689283 (3-x+2x^2)^{5/2}}{146800640} + \frac{48669967x (3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{7625}{96} x^6 (3-x+2x^2)^{5/2} + \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (384+2304x+9024x^2) dx
\end{aligned}$$

**Mathematica [A]** time = 0.37, size = 95, normalized size = 0.41

$$\frac{4\sqrt{2^2-x+3} (70464307200000x^{11} + 144451829760000x^{10} + 349379651174400x^9 + 534038708224000x^8 + 745133229998080x^7 + 765087080448000x^6 + 675479464714240x^5 + 451581382260736x^4 + 23902184223104x^3 + 65151998063712x^2 + 12971175524316x + 7403200951418) - 191024672914845\sqrt{2} \operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2705829396480}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] (4\*sqrt[3 - x + 2\*x^2]\*(74032009514181 + 12971175524316\*x + 65151998063712\*x^2 + 239021184223104\*x^3 + 451581382260736\*x^4 + 675479464714240\*x^5 + 765087080448000\*x^6 + 745133229998080\*x^7 + 534038708224000\*x^8 + 349379651174400\*x^9 + 144451829760000\*x^10 + 70464307200000\*x^11) - 191024672914845\*sqrt[2]\*ArcSinh[(1 - 4\*x)/sqrt[23]])/2705829396480

**IntegrateAlgebraic [A]** time = 1.37, size = 110, normalized size = 0.48

$$\frac{4\sqrt{2^2-x+3} (70464307200000x^{11} + 144451829760000x^{10} + 349379651174400x^9 + 534038708224000x^8 + 745133229998080x^7 + 765087080448000x^6 + 675479464714240x^5 + 451581382260736x^4 + 23902184223104x^3 + 65151998063712x^2 + 12971175524316x + 7403200951418) - 606427533063 \log\left(\frac{2\sqrt{2} \sqrt{2^2-x+3} - 4x + 1}{\sqrt{23}}\right)}{676457349120}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(74032009514181 + 12971175524316\*x + 65151998063712\*x^2 + 239021184223104\*x^3 + 451581382260736\*x^4 + 675479464714240\*x^5 + 765087080448000\*x^6 + 745133229998080\*x^7 + 534038708224000\*x^8 + 349379651174400\*x^9 + 144451829760000\*x^10 + 70464307200000\*x^11))/676457349120 - (606427533063\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(4294967296\*Sqrt[2])

**fricas** [A] time = 0.48, size = 108, normalized size = 0.47

$$\frac{1}{676457349120} (70464307200000x^{11} + 144451829760000x^{10} + 349379651174400x^9 + 534038708224000x^8 + 745133229998080x^7 + 765087080448000x^6 + 675479464714240x^5 + 451581382260736x^4 + 239021184223104x^3 + 65151998063712x^2 + 12971175524316x + 74032009514181) \sqrt{2x^2 - x + 3} - \frac{606427533063}{1719869184} \sqrt{2} \log\left(\frac{-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25}{1719869184}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^4,x, algorithm="fricas")

[Out] 1/676457349120\*(70464307200000\*x^11 + 144451829760000\*x^10 + 349379651174400\*x^9 + 534038708224000\*x^8 + 745133229998080\*x^7 + 765087080448000\*x^6 + 675479464714240\*x^5 + 451581382260736\*x^4 + 239021184223104\*x^3 + 65151998063712\*x^2 + 12971175524316\*x + 74032009514181)\*sqrt(2\*x^2 - x + 3) + 606427533063/1719869184\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.25, size = 103, normalized size = 0.45

$$\frac{1}{676457349120} (4(8(4(16(20(8(28(160(12(200(20x + 41)x + 19833)x + 363785)x + 81213077)x + 2334860475)x + 16491197869)x + 220498721807)x + 1867353001743)x + 2035999939491)x + 3242793881079)x + 74032009514181) \sqrt{2x^2 - x + 3} - \frac{606427533063}{8589934592} \sqrt{2} \log\left(\frac{-2\sqrt{2}(\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25)}{8589934592}\right) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^4,x, algorithm="giac")

[Out] 1/676457349120\*(4\*(8\*(4\*(16\*(20\*(8\*(28\*(160\*(12\*(200\*(20\*x + 41)\*x + 19833)\*x + 363785)\*x + 81213077)\*x + 2334860475)\*x + 16491197869)\*x + 220498721807)\*x + 1867353001743)\*x + 2035999939491)\*x + 3242793881079)\*x + 74032009514181)\*sqrt(2\*x^2 - x + 3) - 606427533063/8589934592\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.04, size = 185, normalized size = 0.80

$$\frac{625(2x^2 - x + 3)^{5/2}}{24} + \frac{7625(2x^2 - x + 3)^{3/2}}{96} + \frac{95165(2x^2 - x + 3)^{1/2}}{768} + \frac{941905(2x^2 - x + 3)^{1/2}}{9216} + \frac{10444117(2x^2 - x + 3)^{1/2}}{294912} - \frac{56422489(2x^2 - x + 3)^{1/2}}{8257536} + \frac{48669967(2x^2 - x + 3)^{1/2}}{22020096} + \frac{606427533063\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{2}(x-1)}{21}\right)}{8589934592} + \frac{2124689283(2x^2 - x + 3)^{1/2}}{146800640} + \frac{26366414481(4x - 1)\sqrt{2x^2 - x + 3}}{2147483648} + \frac{382121949(4x - 1)(2x^2 - x + 3)^{1/2}}{134217728}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^4,x)

[Out] 625/24\*x^7\*(2\*x^2-x+3)^(5/2)+7625/96\*x^6\*(2\*x^2-x+3)^(5/2)+2124689283/146800640\*(2\*x^2-x+3)^(5/2)+95165/768\*x^5\*(2\*x^2-x+3)^(5/2)+941905/9216\*x^4\*(2\*x^2-x+3)^(5/2)+10444117/294912\*x^3\*(2\*x^2-x+3)^(5/2)-56422489/8257536\*x^2\*(2\*x^2-x+3)^(5/2)+48669967/22020096\*x\*(2\*x^2-x+3)^(5/2)+606427533063/8589934592\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+26366414481/2147483648\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)+382121949/134217728\*(4\*x-1)\*(2\*x^2-x+3)^(3/2)

**maxima** [A] time = 1.02, size = 206, normalized size = 0.89

$$\frac{625}{24} (2x^2 - x + 3)^{5/2} + \frac{7625}{96} (2x^2 - x + 3)^{3/2} + \frac{95165}{768} (2x^2 - x + 3)^{1/2} + \frac{941905}{9216} (2x^2 - x + 3)^{1/2} + \frac{10444117}{294912} (2x^2 - x + 3)^{1/2} - \frac{56422489}{8257536} (2x^2 - x + 3)^{1/2} + \frac{48669967}{22020096} (2x^2 - x + 3)^{1/2} + \frac{2124689283}{146800640} (2x^2 - x + 3)^{1/2} + \frac{382121949}{134217728} (2x^2 - x + 3)^{1/2} + \frac{26366414481}{3589934592} \sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{2}(x-1)}{21}\right) + \frac{2124689283}{146800640} \sqrt{2x^2 - x + 3} + \frac{606427533063}{8589934592} \sqrt{2} \operatorname{arcsinh}\left(\frac{1}{25} \sqrt{25}(4x - 1)\right) + \frac{26366414481}{2147483648} \sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^4,x, algorithm="maxima")

[Out] 625/24\*(2\*x^2 - x + 3)^(5/2)\*x^7 + 7625/96\*(2\*x^2 - x + 3)^(5/2)\*x^6 + 95165/768\*(2\*x^2 - x + 3)^(5/2)\*x^5 + 941905/9216\*(2\*x^2 - x + 3)^(5/2)\*x^4 + 10444117/294912\*(2\*x^2 - x + 3)^(5/2)\*x^3 - 56422489/8257536\*(2\*x^2 - x + 3)^(5/2)

$$\begin{aligned} & ^{(5/2)}x^2 + 48669967/22020096*(2*x^2 - x + 3)^{(5/2)}*x + 2124689283/1468006 \\ & 40*(2*x^2 - x + 3)^{(5/2)} + 382121949/33554432*(2*x^2 - x + 3)^{(3/2)}*x - 382 \\ & 121949/134217728*(2*x^2 - x + 3)^{(3/2)} + 26366414481/536870912*\text{sqrt}(2*x^2 - \\ & x + 3)*x + 606427533063/8589934592*\text{sqrt}(2)*\text{arcsinh}(1/23*\text{sqrt}(23))*(4*x - 1) \\ & ) - 26366414481/2147483648*\text{sqrt}(2*x^2 - x + 3) \end{aligned}$$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (2x^2 - x + 3)^{3/2} (5x^2 + 3x + 2)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)^4,x)

[Out] int((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)^4, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(3/2)\*(5\*x\*\*2+3\*x+2)\*\*4,x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(3/2)\*(5\*x\*\*2 + 3\*x + 2)\*\*4, x)

$$3.63 \quad \int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2)^3 dx$$

**Optimal.** Leaf size=189

$$\frac{384739(2x^2 - x + 3)^{5/2} x^2}{43008} - \frac{81685(2x^2 - x + 3)^{5/2} x}{114688} - \frac{4625907(2x^2 - x + 3)^{5/2}}{2293760} - \frac{667795(1 - 4x)(2x^2 - x + 3)^{3/2}}{2097152}$$

**Rubi [A]** time = 0.19, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{4}(2x^2 - x + 3)^{5/2} x^3 + \frac{725}{48}(2x^2 - x + 3)^{5/2} x^2 + \frac{27785(2x^2 - x + 3)^{5/2} x}{1536} + \frac{384739(2x^2 - x + 3)^{5/2} x^2}{43008} - \frac{81685(2x^2 - x + 3)^{5/2} x}{114688} - \frac{4625907(2x^2 - x + 3)^{5/2}}{2293760} - \frac{667795(1 - 4x)(2x^2 - x + 3)^{3/2}}{2097152} - \frac{46077855(1 - 4x)\sqrt{2x^2 - x + 3}}{33554432} - \frac{1059790665 \operatorname{sinh}^{-1}\left(\frac{1 - 4x}{\sqrt{23}}\right)}{67108864\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] (-46077855\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/33554432 - (667795\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(3/2))/2097152 - (4625907\*(3 - x + 2\*x^2)^(5/2))/2293760 - (81685\*x\*(3 - x + 2\*x^2)^(5/2))/114688 + (384739\*x^2\*(3 - x + 2\*x^2)^(5/2))/43008 + (27785\*x^3\*(3 - x + 2\*x^2)^(5/2))/1536 + (725\*x^4\*(3 - x + 2\*x^2)^(5/2))/48 + (25\*x^5\*(3 - x + 2\*x^2)^(5/2))/4 - (1059790665\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(67108864\*Sqrt[2])

Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1661

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^3 dx &= \frac{25}{4}x^5(3-x+2x^2)^{5/2} + \frac{1}{20} \int (3-x+2x^2)^{3/2} (160+720x+2280x^2) dx \\
&= \frac{725}{48}x^4(3-x+2x^2)^{5/2} + \frac{25}{4}x^5(3-x+2x^2)^{5/2} + \frac{1}{360} \int (3-x+2x^2)^{3/2} (160+720x+2280x^2) dx \\
&= \frac{27785x^3(3-x+2x^2)^{5/2}}{1536} + \frac{725}{48}x^4(3-x+2x^2)^{5/2} + \frac{25}{4}x^5(3-x+2x^2)^{5/2} \\
&= \frac{384739x^2(3-x+2x^2)^{5/2}}{43008} + \frac{27785x^3(3-x+2x^2)^{5/2}}{1536} + \frac{725}{48}x^4(3-x+2x^2)^{5/2} \\
&= -\frac{81685x(3-x+2x^2)^{5/2}}{114688} + \frac{384739x^2(3-x+2x^2)^{5/2}}{43008} + \frac{27785x^3(3-x+2x^2)^{5/2}}{1536} \\
&= -\frac{4625907(3-x+2x^2)^{5/2}}{2293760} - \frac{81685x(3-x+2x^2)^{5/2}}{114688} + \frac{384739x^2(3-x+2x^2)^{5/2}}{43008} \\
&= -\frac{667795(1-4x)(3-x+2x^2)^{3/2}}{2097152} - \frac{4625907(3-x+2x^2)^{5/2}}{2293760} - \frac{81685x(3-x+2x^2)^{5/2}}{114688} \\
&= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432} - \frac{667795(1-4x)(3-x+2x^2)^{3/2}}{2097152} \\
&= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432} - \frac{667795(1-4x)(3-x+2x^2)^{3/2}}{2097152} \\
&= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432} - \frac{667795(1-4x)(3-x+2x^2)^{3/2}}{2097152}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 85, normalized size = 0.45

$$\frac{4\sqrt{2x^2-x+3} (88080384000x^9 + 124780544000x^8 + 328328806400x^7 + 430820229120x^6 + 571298324480x^5 + 487891884032x^4 + 389257196928x^3 + 199615064544x^2 + 53985432012x - 72152399943) - 111278019825\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{3}}\right)}{14092861440}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] (4\*sqrt[3 - x + 2\*x^2]\*(-72152399943 + 53985432012\*x + 199615064544\*x^2 + 389257196928\*x^3 + 487891884032\*x^4 + 571298324480\*x^5 + 430820229120\*x^6 + 328328806400\*x^7 + 124780544000\*x^8 + 88080384000\*x^9) - 111278019825\*sqrt[2]\*ArcSinh[(1 - 4\*x)/sqrt[23]])/14092861440

**IntegrateAlgebraic [A]** time = 1.06, size = 100, normalized size = 0.53

$$\frac{\sqrt{2x^2-x+3} (88080384000x^9 + 124780544000x^8 + 328328806400x^7 + 430820229120x^6 + 571298324480x^5 + 487891884032x^4 + 389257196928x^3 + 199615064544x^2 + 53985432012x - 72152399943) - 1059790665 \log(2\sqrt{2x^2-x+3} - 4x + 1)}{3523215360} - \frac{67108864\sqrt{2}}{67108864\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] (sqrt[3 - x + 2\*x^2]\*(-72152399943 + 53985432012\*x + 199615064544\*x^2 + 389257196928\*x^3 + 487891884032\*x^4 + 571298324480\*x^5 + 430820229120\*x^6 + 328328806400\*x^7 + 124780544000\*x^8 + 88080384000\*x^9))/3523215360 - (1059790665\*log[1 - 4\*x + 2\*sqrt[2]\*sqrt[3 - x + 2\*x^2]])/(67108864\*sqrt[2])

**fricas [A]** time = 0.46, size = 98, normalized size = 0.52

$$\frac{1}{3523215360} (88080384000x^9 + 124780544000x^8 + 328328806400x^7 + 430820229120x^6 + 571298324480x^5 + 487891884032x^4 + 389257196928x^3 + 199615064544x^2 + 53985432012x - 72152399943)\sqrt{2x^2-x+3} + \frac{1059790665}{268435456} \sqrt{2} \log(-4\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/3523215360\*(88080384000\*x^9 + 124780544000\*x^8 + 328328806400\*x^7 + 430820229120\*x^6 + 571298324480\*x^5 + 487891884032\*x^4 + 389257196928\*x^3 + 199615064544\*x^2 + 53985432012\*x - 72152399943)\*sqrt(2\*x^2 - x + 3) + 1059790665/5/268435456\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.26, size = 93, normalized size = 0.49

$$\frac{1}{3523215360} \left( (8(4(16(20(8(140(160(12x+17)x+7157)x+1314759)x+13947713)x+238228459)x+3041071851)x+6237970767)x+13496358003)x-72152399943) \sqrt{2x^2-x+3} - \frac{1059790665}{134217728} \sqrt{2} \log(-2\sqrt{2}(\sqrt{2x-x+3}+1)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 1/3523215360\*(4\*(8\*(4\*(16\*(20\*(8\*(140\*(160\*(12\*x + 17)\*x + 7157)\*x + 1314759)\*x + 13947713)\*x + 238228459)\*x + 3041071851)\*x + 6237970767)\*x + 13496358003)\*x - 72152399943)\*sqrt(2\*x^2 - x + 3) - 1059790665/134217728\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 151, normalized size = 0.80

$$\frac{25(2x^2-x+3)^{5/2}}{4} + \frac{725(2x^2-x+3)^{5/2}x^4}{48} + \frac{27785(2x^2-x+3)^{5/2}x^3}{1536} + \frac{384739(2x^2-x+3)^{5/2}x^2}{43008} - \frac{81685(2x^2-x+3)^{5/2}x}{114688} + \frac{1059790665\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}(x-1)}{23}\right)}{134217728} - \frac{4625907(2x^2-x+3)^{5/2}}{2293760} + \frac{46077855(4x-1)\sqrt{2x^2-x+3}}{33554432} + \frac{667795(4x-1)(2x^2-x+3)^{3/2}}{2097152}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^3,x)

[Out] -4625907/2293760\*(2\*x^2-x+3)^(5/2)+25/4\*(2\*x^2-x+3)^(5/2)\*x^5+725/48\*(2\*x^2-x+3)^(5/2)\*x^4+27785/1536\*(2\*x^2-x+3)^(5/2)\*x^3+384739/43008\*(2\*x^2-x+3)^(5/2)\*x^2-81685/114688\*(2\*x^2-x+3)^(5/2)\*x+1059790665/134217728\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+46077855/33554432\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)+667795/2097152\*(4\*x-1)\*(2\*x^2-x+3)^(3/2)

**maxima** [A] time = 0.99, size = 172, normalized size = 0.91

$$\frac{25}{4}(2x^2-x+3)^{5/2} + \frac{725}{48}(2x^2-x+3)^{5/2}x^4 + \frac{27785}{1536}(2x^2-x+3)^{5/2}x^3 + \frac{384739}{43008}(2x^2-x+3)^{5/2}x^2 - \frac{81685}{114688}(2x^2-x+3)^{5/2}x - \frac{4625907}{2293760}(2x^2-x+3)^{5/2} - \frac{667795}{524288}(2x^2-x+3)^{3/2} - \frac{667795}{2097152}(2x^2-x+3)^{3/2} + \frac{46077855}{8388608}\sqrt{2x^2-x+3} + \frac{1059790665}{134217728}\sqrt{2} \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{46077855}{33554432}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 25/4\*(2\*x^2 - x + 3)^(5/2)\*x^5 + 725/48\*(2\*x^2 - x + 3)^(5/2)\*x^4 + 27785/1536\*(2\*x^2 - x + 3)^(5/2)\*x^3 + 384739/43008\*(2\*x^2 - x + 3)^(5/2)\*x^2 - 81685/114688\*(2\*x^2 - x + 3)^(5/2)\*x - 4625907/2293760\*(2\*x^2 - x + 3)^(5/2) + 667795/524288\*(2\*x^2 - x + 3)^(3/2)\*x - 667795/2097152\*(2\*x^2 - x + 3)^(3/2) + 46077855/8388608\*sqrt(2\*x^2 - x + 3)\*x + 1059790665/134217728\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 46077855/33554432\*sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x^2 - x + 3)^{3/2} (5x^2 + 3x + 2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)^3,x)

[Out] int((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**3, x)
```

$$3.64 \quad \int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2)^2 dx$$

Optimal. Leaf size=147

$$\frac{1235}{448} (2x^2 - x + 3)^{5/2} x^2 + \frac{24499 (2x^2 - x + 3)^{5/2} x}{10752} + \frac{73861 (2x^2 - x + 3)^{5/2}}{215040} + \frac{24293(1 - 4x)(2x^2 - x + 3)^{3/2}}{196608} + \frac{558739(1 - 4x)\sqrt{2x^2 - x + 3}}{1048576} + \frac{12850997 \operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2097152\sqrt{2}}$$

**Rubi [A]** time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{16} (2x^2 - x + 3)^{5/2} x^3 + \frac{1235}{448} (2x^2 - x + 3)^{5/2} x^2 + \frac{24499 (2x^2 - x + 3)^{5/2} x}{10752} + \frac{73861 (2x^2 - x + 3)^{5/2}}{215040} + \frac{24293(1 - 4x)(2x^2 - x + 3)^{3/2}}{196608} + \frac{558739(1 - 4x)\sqrt{2x^2 - x + 3}}{1048576} + \frac{12850997 \operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2097152\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] (558739\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/1048576 + (24293\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(3/2))/196608 + (73861\*(3 - x + 2\*x^2)^(5/2))/215040 + (24499\*x\*(3 - x + 2\*x^2)^(5/2))/10752 + (1235\*x^2\*(3 - x + 2\*x^2)^(5/2))/448 + (25\*x^3\*(3 - x + 2\*x^2)^(5/2))/16 + (12850997\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(2097152\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

#### Rubi steps



$$\begin{aligned}
\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^2 dx &= \frac{25}{16}x^3(3-x+2x^2)^{5/2} + \frac{1}{16} \int (3-x+2x^2)^{3/2} (64+192x+239x^2+ \\
&= \frac{1235}{448}x^2(3-x+2x^2)^{5/2} + \frac{25}{16}x^3(3-x+2x^2)^{5/2} + \frac{1}{224} \int (3-x+2 \\
&= \frac{24499x(3-x+2x^2)^{5/2}}{10752} + \frac{1235}{448}x^2(3-x+2x^2)^{5/2} + \frac{25}{16}x^3(3-x+2 \\
&= \frac{73861(3-x+2x^2)^{5/2}}{215040} + \frac{24499x(3-x+2x^2)^{5/2}}{10752} + \frac{1235}{448}x^2(3-x+2 \\
&= \frac{24293(1-4x)(3-x+2x^2)^{3/2}}{196608} + \frac{73861(3-x+2x^2)^{5/2}}{215040} + \frac{24499x(3-x+2 \\
&= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)(3-x+2x^2)^{3/2}}{196608} + \frac{73861(3-x+2x^2)^{5/2}}{215040} \\
&= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)(3-x+2x^2)^{3/2}}{196608} + \frac{73861(3-x+2x^2)^{5/2}}{215040} \\
&= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)(3-x+2x^2)^{3/2}}{196608} + \frac{73861(3-x+2x^2)^{5/2}}{215040}
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 75, normalized size = 0.51

$$\frac{4\sqrt{2x^2-x+3}(688128000x^7+525926400x^6+2025840640x^5+2061273088x^4+2728413312x^3+1799647136x^2+1619403428x+439831323)+1349354685\sqrt{2}\operatorname{sinh}^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{440401920}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] (4\*sqrt[3 - x + 2\*x^2]\*(439831323 + 1619403428\*x + 1799647136\*x^2 + 2728413312\*x^3 + 2061273088\*x^4 + 2025840640\*x^5 + 525926400\*x^6 + 688128000\*x^7) + 1349354685\*sqrt[2]\*ArcSinh[(1 - 4\*x)/sqrt[23]])/440401920

**IntegrateAlgebraic [A]** time = 0.82, size = 90, normalized size = 0.61

$$\frac{12850997 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{2097152\sqrt{2}} + \frac{\sqrt{2x^2-x+3}(688128000x^7+525926400x^6+2025840640x^5+2061273088x^4+2728413312x^3+1799647136x^2+1619403428x+439831323)}{110100480}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] (sqrt[3 - x + 2\*x^2]\*(439831323 + 1619403428\*x + 1799647136\*x^2 + 2728413312\*x^3 + 2061273088\*x^4 + 2025840640\*x^5 + 525926400\*x^6 + 688128000\*x^7))/110100480 + (12850997\*log[1 - 4\*x + 2\*sqrt[2]\*sqrt[3 - x + 2\*x^2]])/(2097152\*sqrt[2])

**fricas [A]** time = 0.45, size = 88, normalized size = 0.60

$$\frac{1}{110100480}(688128000x^7+525926400x^6+2025840640x^5+2061273088x^4+2728413312x^3+1799647136x^2+1619403428x+439831323)\sqrt{2x^2-x+3} + \frac{12850997}{8388608}\sqrt{2}\log(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

```
[Out] 1/110100480*(688128000*x^7 + 525926400*x^6 + 2025840640*x^5 + 2061273088*x^4 + 2728413312*x^3 + 1799647136*x^2 + 1619403428*x + 439831323)*sqrt(2*x^2 - x + 3) + 12850997/8388608*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

**giac** [A] time = 0.26, size = 83, normalized size = 0.56

$$\frac{1}{110100480} (4 (8 (4 (16 (20 (120 (140 x + 107) x + 49459) x + 1006481) x + 21315729) x + 56238973) x + 404850857) x + 439831323) \sqrt{2x^2 - x + 3} + \frac{12850997}{4194304} \sqrt{2} \log(-2\sqrt{2}(\sqrt{2x^2 - x + 3}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 1/110100480*(4*(8*(4*(16*(20*(120*(140*x + 107)*x + 49459)*x + 1006481)*x + 21315729)*x + 56238973)*x + 404850857)*x + 439831323)*sqrt(2*x^2 - x + 3) + 12850997/4194304*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

**maple** [A] time = 0.01, size = 117, normalized size = 0.80

$$\frac{25(2x^2-x+3)^{\frac{5}{2}}x^3}{16} + \frac{1235(2x^2-x+3)^{\frac{5}{2}}x^2}{448} + \frac{24499(2x^2-x+3)^{\frac{5}{2}}x}{10752} - \frac{12850997\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304} + \frac{73861(2x^2-x+3)^{\frac{5}{2}}}{215040} - \frac{558739(4x-1)\sqrt{2x^2-x+3}}{1048576} - \frac{24293(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{196608}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x)
```

```
[Out] 73861/215040*(2*x^2-x+3)^(5/2)+25/16*(2*x^2-x+3)^(5/2)*x^3+1235/448*(2*x^2-x+3)^(5/2)*x^2+24499/10752*(2*x^2-x+3)^(5/2)*x-12850997/4194304*2^(1/2)*arc sinh(4/23*23^(1/2)*(x-1/4))-558739/1048576*(4*x-1)*(2*x^2-x+3)^(1/2)-24293/196608*(4*x-1)*(2*x^2-x+3)^(3/2)
```

**maxima** [A] time = 0.98, size = 138, normalized size = 0.94

$$\frac{25}{16}(2x^2-x+3)^{\frac{5}{2}}x^3 + \frac{1235}{448}(2x^2-x+3)^{\frac{5}{2}}x^2 + \frac{24499}{10752}(2x^2-x+3)^{\frac{5}{2}}x + \frac{73861}{215040}(2x^2-x+3)^{\frac{5}{2}} - \frac{24293}{49152}(2x^2-x+3)^{\frac{3}{2}}x + \frac{24293}{196608}(2x^2-x+3)^{\frac{3}{2}} - \frac{558739}{262144}\sqrt{2x^2-x+3} - \frac{12850997}{4194304}\sqrt{2}\operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{558739}{1048576}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 25/16*(2*x^2 - x + 3)^(5/2)*x^3 + 1235/448*(2*x^2 - x + 3)^(5/2)*x^2 + 24499/10752*(2*x^2 - x + 3)^(5/2)*x + 73861/215040*(2*x^2 - x + 3)^(5/2) - 24293/49152*(2*x^2 - x + 3)^(3/2)*x + 24293/196608*(2*x^2 - x + 3)^(3/2) - 558739/262144*sqrt(2*x^2 - x + 3)*x - 12850997/4194304*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 558739/1048576*sqrt(2*x^2 - x + 3)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x^2 - x + 3)^{3/2} (5x^2 + 3x + 2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**2, x)
```

$$3.65 \quad \int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2) dx$$

Optimal. Leaf size=105

$$\frac{5}{12}x(2x^2 - x + 3)^{5/2} + \frac{107}{240}(2x^2 - x + 3)^{5/2} - \frac{179(1 - 4x)(2x^2 - x + 3)^{3/2}}{1536} - \frac{4117(1 - 4x)\sqrt{2x^2 - x + 3}}{8192} - \frac{94691}{16}$$

**Rubi [A]** time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{12}x(2x^2 - x + 3)^{5/2} + \frac{107}{240}(2x^2 - x + 3)^{5/2} - \frac{179(1 - 4x)(2x^2 - x + 3)^{3/2}}{1536} - \frac{4117(1 - 4x)\sqrt{2x^2 - x + 3}}{8192} - \frac{94691 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2), x]

[Out] (-4117\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/8192 - (179\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(3/2))/1536 + (107\*(3 - x + 2\*x^2)^(5/2))/240 + (5\*x\*(3 - x + 2\*x^2)^(5/2))/12 - (94691\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(16384\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{3/2} (2+3x+5x^2) dx &= \frac{5}{12}x(3-x+2x^2)^{5/2} + \frac{1}{12} \int \left(9 + \frac{107x}{2}\right) (3-x+2x^2)^{3/2} dx \\
&= \frac{107}{240} (3-x+2x^2)^{5/2} + \frac{5}{12}x(3-x+2x^2)^{5/2} + \frac{179}{96} \int (3-x+2x^2)^{3/2} dx \\
&= -\frac{179(1-4x)(3-x+2x^2)^{3/2}}{1536} + \frac{107}{240} (3-x+2x^2)^{5/2} + \frac{5}{12}x(3-x+2x^2)^{5/2} \\
&= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192} - \frac{179(1-4x)(3-x+2x^2)^{3/2}}{1536} + \frac{107}{240} (3-x+2x^2)^{5/2} \\
&= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192} - \frac{179(1-4x)(3-x+2x^2)^{3/2}}{1536} + \frac{107}{240} (3-x+2x^2)^{5/2} \\
&= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192} - \frac{179(1-4x)(3-x+2x^2)^{3/2}}{1536} + \frac{107}{240} (3-x+2x^2)^{5/2}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 65, normalized size = 0.62

$$\frac{4\sqrt{2x^2-x+3} (204800x^5 + 14336x^4 + 561024x^3 + 319072x^2 + 565276x + 388341) - 1420365\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{491520}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2), x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(388341 + 565276\*x + 319072\*x^2 + 561024\*x^3 + 14336\*x^4 + 204800\*x^5) - 1420365\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/491520

**IntegrateAlgebraic [A]** time = 0.58, size = 80, normalized size = 0.76

$$\frac{\sqrt{2x^2-x+3} (204800x^5 + 14336x^4 + 561024x^3 + 319072x^2 + 565276x + 388341)}{122880} - \frac{94691 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{16384\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2), x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(388341 + 565276\*x + 319072\*x^2 + 561024\*x^3 + 14336\*x^4 + 204800\*x^5))/122880 - (94691\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(16384\*Sqrt[2])

**fricas [A]** time = 0.41, size = 78, normalized size = 0.74

$$\frac{1}{122880} (204800x^5 + 14336x^4 + 561024x^3 + 319072x^2 + 565276x + 388341)\sqrt{2x^2-x+3} + \frac{94691}{65536} \sqrt{2} \log(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2), x, algorithm="fricas")

[Out] 1/122880\*(204800\*x^5 + 14336\*x^4 + 561024\*x^3 + 319072\*x^2 + 565276\*x + 388341)\*sqrt(2\*x^2 - x + 3) + 94691/65536\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac [A]** time = 0.23, size = 73, normalized size = 0.70

$$\frac{1}{122880} (4(8(4(16(100x+7)x+4383)x+9971)x+141319)x+388341)\sqrt{2x^2-x+3} - \frac{94691}{32768} \sqrt{2} \log(-2\sqrt{2}(\sqrt{2}x - \sqrt{2x^2-x+3}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] 1/122880\*(4\*(8\*(4\*(16\*(100\*x + 7)\*x + 4383)\*x + 9971)\*x + 141319)\*x + 38834  
1)\*sqrt(2\*x^2 - x + 3) - 94691/32768\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sq  
rt(2\*x^2 - x + 3)) + 1)

**maple [A]** time = 0.01, size = 83, normalized size = 0.79

$$\frac{5(2x^2-x+3)^{\frac{5}{2}}x}{12} + \frac{94691\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}(x-\frac{1}{4})}{23}\right)}{32768} + \frac{107(2x^2-x+3)^{\frac{5}{2}}}{240} + \frac{179(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{1536} + \frac{4117(4x-1)\sqrt{2x^2-x+3}}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2),x)

[Out] 5/12\*(2\*x^2-x+3)^(5/2)\*x+107/240\*(2\*x^2-x+3)^(5/2)+179/1536\*(4\*x-1)\*(2\*x^2-  
x+3)^(3/2)+4117/8192\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)+94691/32768\*2^(1/2)\*arcsinh(  
4/23\*23^(1/2)\*(x-1/4))

**maxima [A]** time = 0.96, size = 104, normalized size = 0.99

$$\frac{5}{12}(2x^2-x+3)^{\frac{5}{2}}x + \frac{107}{240}(2x^2-x+3)^{\frac{5}{2}} + \frac{179}{384}(2x^2-x+3)^{\frac{3}{2}}x - \frac{179}{1536}(2x^2-x+3)^{\frac{3}{2}} + \frac{4117}{2048}\sqrt{2x^2-x+3}x + \frac{94691}{32768}\sqrt{2} \operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{4117}{8192}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] 5/12\*(2\*x^2 - x + 3)^(5/2)\*x + 107/240\*(2\*x^2 - x + 3)^(5/2) + 179/384\*(2\*x  
^2 - x + 3)^(3/2)\*x - 179/1536\*(2\*x^2 - x + 3)^(3/2) + 4117/2048\*sqrt(2\*x^2  
- x + 3)\*x + 94691/32768\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 4117/8  
192\*sqrt(2\*x^2 - x + 3)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (2x^2 - x + 3)^{3/2} (5x^2 + 3x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2),x)

[Out] int((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(3/2)\*(5\*x\*\*2+3\*x+2),x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(3/2)\*(5\*x\*\*2 + 3\*x + 2), x)

$$3.66 \quad \int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2} dx$$

**Optimal.** Leaf size=197

$$-\frac{1}{100}\sqrt{2x^2-x+3}(49-20x)+\frac{11}{125}\sqrt{\frac{11}{31}(247+500\sqrt{2})}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62(247+500\sqrt{2})}}((130+69\sqrt{2})x+61\sqrt{2}+8)}{\sqrt{2x^2-x+3}}\right)$$

**Rubi [A]** time = 0.49, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$ , Rules used = {977, 1076, 619, 215, 1035, 1029, 206, 204}

$$-\frac{1}{100}\sqrt{2x^2-x+3}(49-20x)+\frac{11}{125}\sqrt{\frac{11}{31}(247+500\sqrt{2})}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62(247+500\sqrt{2})}}((130+69\sqrt{2})x+61\sqrt{2}+8)}{\sqrt{2x^2-x+3}}\right)-\frac{11}{125}\sqrt{\frac{11}{31}(500\sqrt{2}-247)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62(500\sqrt{2}-247)}}((130-69\sqrt{2})x-61\sqrt{2}+8)}{\sqrt{2x^2-x+3}}\right)-\frac{2203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(3/2)/(2 + 3\*x + 5\*x^2), x]

[Out] -((49 - 20\*x)\*Sqrt[3 - x + 2\*x^2])/100 - (2203\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(1000\*Sqrt[2]) + (11\*Sqrt[(11\*(247 + 500\*Sqrt[2]))/31]\*ArcTan[(Sqrt[11/(62\*(247 + 500\*Sqrt[2]))])\*(8 + 61\*Sqrt[2] + (130 + 69\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/125 - (11\*Sqrt[(11\*(-247 + 500\*Sqrt[2]))/31]\*ArcTanh[(Sqrt[11/(62\*(-247 + 500\*Sqrt[2]))])\*(8 - 61\*Sqrt[2] + (130 - 69\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/125

#### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 977

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b\*f\*(3\*p + 2\*q) - c\*e\*(2\*p + q) + 2\*c\*f\*(p + q)\*x)\*(a + b\*x + c\*x^2)^(p - 1)\*(d + e\*x + f\*x^2)^(q + 1))/(2\*f^2\*(p + q)\*(2\*p + 2\*q + 1)), x] - Dist[1/(2\*f^2\*(p + q)\*(2\*p + 2\*q + 1)), Int[(a + b\*x + c\*x^2)^(p - 2)\*(d + e\*x + f\*x^2)^q\*Simp[(b\*d - a\*e)\*(c\*e - b\*f)\*(1 - p)\*(2\*p + q) - (p + q)\*(b^2\*d\*f\*(1 - p) - a\*(f\*(b\*e - 2\*a\*f)\*(2\*p + 2\*q + 1) + c\*(2\*d\*f - e^2\*(2\*p + q)))] + (2\*(c\*d - a\*f)\*(c\*e - b\*f)\*(1 - p)\*(2\*p + q

```
) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(
2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(p
+ q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
- 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*
q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

### Rule 1029

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

### Rule 1035

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

### Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2} dx &= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{1}{50} \int \frac{-\frac{731}{2} + \frac{1195x}{4} - \frac{2203x^2}{4}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx \\
&= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{1}{250} \int \frac{-726+3146x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx + \frac{2203}{1000} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} + \frac{\int \frac{2662(16+3\sqrt{2})+2662(10-13\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{5500\sqrt{2}} - \frac{\int \frac{2662(16-3\sqrt{2})+2662(10-13\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{5500\sqrt{2}} \\
&= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} - \frac{1}{125} \left(322102 \left(1000 - 247\sqrt{2}\right)\right) S \\
&= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} + \frac{11}{125} \sqrt{\frac{11}{31} \left(247 + 500\sqrt{2}\right)} \tan^{-1}
\end{aligned}$$

**Mathematica [C]** time = 0.69, size = 310, normalized size = 1.57

$$\frac{400\sqrt{31}\sqrt{2x^2-x+3} - 980\sqrt{31}\sqrt{2x^2-x+3} + 44\sqrt{286+22i\sqrt{31}}(\sqrt{31}-13i)\tanh^{-1}\left(\frac{-4\sqrt{31}+22i+\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) + 572i\sqrt{286-22i\sqrt{31}}\tanh^{-1}\left(\frac{4\sqrt{31}-22i-\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) + 44\sqrt{682(13-i\sqrt{31})}\tanh^{-1}\left(\frac{4\sqrt{31}-22i-\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) - 2203\sqrt{62}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2000\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(3/2)/(2 + 3\*x + 5\*x^2), x]

[Out] (-980\*Sqrt[31]\*Sqrt[3 - x + 2\*x^2] + 400\*Sqrt[31]\*x\*Sqrt[3 - x + 2\*x^2] - 2203\*Sqrt[62]\*ArcSinh[(1 - 4\*x)/Sqrt[23]] + 44\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*(-13\*I + Sqrt[31])\*ArcTanh[(63 + I\*Sqrt[31] - 22\*x - (4\*I)\*Sqrt[31]\*x)/(2\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])] + 44\*Sqrt[682\*(13 - I\*Sqrt[31])]\*ArcTanh[(63 - I\*Sqrt[31] - 22\*x + (4\*I)\*Sqrt[31]\*x)/(2\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])] + (572\*I)\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*ArcTanh[(63 - I\*Sqrt[31] - 22\*x + (4\*I)\*Sqrt[31]\*x)/(2\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])]/(2000\*Sqrt[31])

**IntegrateAlgebraic [C]** time = 0.53, size = 235, normalized size = 1.19

$$\frac{121}{125}\text{RootSum}\left[-5\#1^4 + 6\sqrt{2}\#1^3 + 17\#1^2 - 26\sqrt{2}\#1 - 56\&, \frac{13\#1^2 \log(-\#1 + \sqrt{2x^2-x+3} - \sqrt{2}x) + 6\sqrt{2}\#1 \log(-\#1 + \sqrt{2x^2-x+3} - \sqrt{2}x) - 36 \log(-\#1 + \sqrt{2x^2-x+3} - \sqrt{2}x)}{-10\#1^3 + 9\sqrt{2}\#1^2 + 17\#1 - 13\sqrt{2}}\&\right] + \frac{1}{100}\sqrt{2x^2-x+3}(20x-49) - \frac{2203 \log(2\sqrt{2}\sqrt{2x^2-x+3} - 4x+1)}{1000\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^(3/2)/(2 + 3\*x + 5\*x^2), x]

[Out] ((-49 + 20\*x)\*Sqrt[3 - x + 2\*x^2])/100 - (2203\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(1000\*Sqrt[2]) + (121\*RootSum[-56 - 26\*Sqrt[2]\*#1 + 17\*#1^2 + 6\*Sqrt[2]\*#1^3 - 5\*#1^4 &, (-36\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1] + 6\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1 + 13\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1^2)/(-13\*Sqrt[2] + 17\*#1 + 9\*Sqrt[2]\*#1^2 - 10\*#1^3) & ])/125

**fricas [B]** time = 0.99, size = 2027, normalized size = 10.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2),x, algorithm="fricas")



[Out]  $11/77500 \cdot 24200^{1/4} \cdot \sqrt{31} \cdot \sqrt{10} \cdot \sqrt{2} \cdot \sqrt{247 \sqrt{2} + 1000} \cdot \arctan\left(\frac{1}{10605875} \cdot (230 \sqrt{10}) \cdot (2 \cdot 24200^{3/4} \sqrt{31}) \cdot (20846x^7 - 109153x^6 + 215386x^5 - 427391x^4 + 234360x^3 - 156600x^2 - \sqrt{2} \cdot (28854x^7 - 90639x^6 + 200187x^5 - 262838x^4 + 117544x^3 - 23472x^2 - 186624x + 86400) - 172800x + 186624) + 5 \cdot 24200^{1/4} \sqrt{31} \cdot (112238x^7 - 1817988x^6 + 10351960x^5 - 25791248x^4 + 34522560x^3 - 28368000x^2 - \sqrt{2} \cdot (125839x^7 - 1864281x^6 + 9323336x^5 - 19725020x^4 + 24624288x^3 - 10862496x^2 - 19989504x + 10533888) - 21067776x + 19989504)\right) \cdot \sqrt{2x^2 - x + 3} \cdot \sqrt{247 \sqrt{2} + 1000} + 30107000 \sqrt{31} \sqrt{2} \cdot (28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2} \cdot (8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - \sqrt{5/119} \cdot (\sqrt{10}) \cdot (2 \cdot 24200^{3/4} \sqrt{31}) \cdot (46522x^7 - 71117x^6 + 257247x^5 - 273360x^4 + 484920x^3 - 269568x^2 - 16 \sqrt{2} \cdot (7714x^7 - 10881x^6 + 33771x^5 - 5576x^4 - 576x^3 + 32184x^2 - 32184x) + 269568x) + 5 \cdot 24200^{1/4} \sqrt{31} \cdot (309512x^7 - 4017952x^6 + 15741280x^5 - 22625280x^4 + 37693440x^3 - 13519872x^2 - \sqrt{2} \cdot (516957x^7 - 6676948x^6 + 25569820x^5 - 31522752x^4 + 34450848x^3 + 46199808x^2 - 46199808x) + 13519872x) \cdot \sqrt{2x^2 - x + 3} \cdot \sqrt{247 \sqrt{2} + 1000} + 130900 \sqrt{31} \sqrt{2} \cdot (123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2} \cdot (15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) + 5950 \sqrt{31} \cdot (254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488 \sqrt{2} \cdot (4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x) \cdot \sqrt{((24200^{1/4} \sqrt{10}) \sqrt{2x^2 - x + 3}) \cdot (\sqrt{2} \cdot (x - 75) + 74x - 76) \sqrt{247 \sqrt{2} + 1000} + 58310x^2 + 52360 \sqrt{2} \cdot (2x^2 - x + 3) - 179690x + 238000) / x^2} + 342125 \sqrt{31} \cdot (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744 \sqrt{2} \cdot (1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456) + 11/77500 \cdot 24200^{1/4} \sqrt{31} \sqrt{10} \sqrt{2} \sqrt{247 \sqrt{2} + 1000} \cdot \arctan\left(\frac{1}{10605875} \cdot (230 \sqrt{10}) \cdot (2 \cdot 24200^{3/4} \sqrt{31}) \cdot (20846x^7 - 109153x^6 + 215386x^5 - 427391x^4 + 234360x^3 - 156600x^2 - \sqrt{2} \cdot (28854x^7 - 90639x^6 + 200187x^5 - 262838x^4 + 117544x^3 - 23472x^2 - 186624x + 86400) - 172800x + 186624) + 5 \cdot 24200^{1/4} \sqrt{31} \cdot (112238x^7 - 1817988x^6 + 10351960x^5 - 25791248x^4 + 34522560x^3 - 28368000x^2 - \sqrt{2} \cdot (125839x^7 - 1864281x^6 + 9323336x^5 - 19725020x^4 + 24624288x^3 - 10862496x^2 - 19989504x + 10533888) - 21067776x + 19989504)\right) \cdot \sqrt{2x^2 - x + 3} \cdot \sqrt{247 \sqrt{2} + 1000} - 30107000 \sqrt{31} \sqrt{2} \cdot (28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2} \cdot (8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - \sqrt{5/119} \cdot (\sqrt{10}) \cdot (2 \cdot 24200^{3/4} \sqrt{31}) \cdot (46522x^7 - 71117x^6 + 257247x^5 - 273360x^4 + 484920x^3 - 269568x^2 - 16 \sqrt{2} \cdot (7714x^7 - 10881x^6 + 33771x^5 - 5576x^4 - 576x^3 + 32184x^2 - 32184x) + 269568x) + 5 \cdot 24200^{1/4} \sqrt{31} \cdot (309512x^7 - 4017952x^6 + 15741280x^5 - 22625280x^4 + 37693440x^3 - 13519872x^2 - \sqrt{2} \cdot (516957x^7 - 6676948x^6 + 25569820x^5 - 31522752x^4 + 34450848x^3 + 46199808x^2 - 46199808x) + 13519872x) \cdot \sqrt{2x^2 - x + 3} \cdot \sqrt{247 \sqrt{2} + 1000} - 130900 \sqrt{31} \sqrt{2} \cdot (123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2} \cdot (15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) - 5950 \sqrt{31} \cdot (254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488 \sqrt{2} \cdot (4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x) \cdot \sqrt{-(24200^{1/4} \sqrt{10}) \sqrt{2x^2 - x + 3}) \cdot (\sqrt{2} \cdot (x - 75) + 74x - 76) \sqrt{247 \sqrt{2} + 1000} - 58310x^2 + 52360 \sqrt{2} \cdot (2x^2 - x + 3) - 179690x + 238000) / x^2} + 342125 \sqrt{31} \cdot (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744 \sqrt{2} \cdot (1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456)$

$$x^2 - 52360\sqrt{2}(2x^2 - x + 3) + 179690x - 238000)/x^2) - 342125\sqrt{2}(31)(2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744\sqrt{2}(1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936))/(2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456)) + 11/36890000*24200^{(1/4)}\sqrt{10}\sqrt{247\sqrt{2} + 1000}*(247\sqrt{2} - 1000)*\log(1512500/119*(24200^{(1/4)}\sqrt{10}\sqrt{2x^2 - x + 3}*(\sqrt{2}*(x - 75) + 74x - 76)*\sqrt{247\sqrt{2} + 1000} + 58310x^2 + 52360\sqrt{2}*(2x^2 - x + 3) - 179690x + 238000)/x^2) - 11/36890000*24200^{(1/4)}\sqrt{10}\sqrt{247\sqrt{2} + 1000}*(247\sqrt{2} - 1000)*\log(-1512500/119*(24200^{(1/4)}\sqrt{10}\sqrt{2x^2 - x + 3}*(\sqrt{2}*(x - 75) + 74x - 76)*\sqrt{247\sqrt{2} + 1000} - 58310x^2 - 52360\sqrt{2}*(2x^2 - x + 3) + 179690x - 238000)/x^2) + 1/100\sqrt{2x^2 - x + 3}*(20x - 49) + 2203/4000\sqrt{2}*\log(-4\sqrt{2}*\sqrt{2x^2 - x + 3}*(4x - 1) - 32x^2 + 16x - 25)$$

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x);OUTPUT:Francis algorithm failure for[-1.0,infinity,  
 infinity,infinity,infinity]root error [1.0,infinity,infinity,infinity,infi  
 nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity  
 ]root error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai  
 lure for[-1.0,infinity,infinity,infinity,infinity]root error [1.0,infinity  
 ,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf  
 inity,infinity,infinity]root error [1.0,infinity,infinity,infinity,infinity  
 ]Evaluation time: 10.3Done

**maple [B]** time = 0.05, size = 3460, normalized size = 17.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2),x)

[Out]  $\frac{1}{5}x*(2x^2-x+3)^{(1/2)}-49/100*(2x^2-x+3)^{(1/2)}+2203/2000*2^{(1/2)}*\operatorname{arcsinh}\left(\frac{4/23*23^{(1/2)}*(x-1/4)}{-2/1321375*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(4245*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\operatorname{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})}\right)+6154*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\operatorname{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})}\right)+12325786*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})*2^{(1/2)}-359414*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})/(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})$

$$\begin{aligned}
& 2/(-x+2^{(1/2)+1})^2+8-3*2^{(1/2)})/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)+1}))^2)^{(1/2)}/( \\
& 1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)+1}))/((8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}-2/ \\
& 264275*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)} \\
& (1/2)+1)^2+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(2365*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)} \\
& )^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)} \\
& ))^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+24*2^{(1/2)} \\
& )-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+10368*(x+2^{(1/2)} \\
& )-1)^2/(-x+2^{(1/2)+1})^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)} \\
& )+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+23*(x+2^{(1/2)}-1)/(-x+2^{(1/2)+1}) \\
& *(8+3*2^{(1/2)})))+3338*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)} \\
& )^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-2 \\
& 3*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)} \\
& )-1)^2/(-x+2^{(1/2)+1})^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+22379*2 \\
& ^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)+1})^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)} \\
& )+1)^2+23*(x+2^{(1/2)}-1)/(-x+2^{(1/2)+1})*(8+3*2^{(1/2)})))+3192442*\operatorname{arctanh} \\
& (31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)} \\
& )+1)^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)}*2^{(1/2)}-5264358*\operatorname{ar} \\
& \operatorname{ctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(- \\
& x+2^{(1/2)+1})^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)}))/((8*(x+2^{(1/2)} \\
& )-1)^2/(-x+2^{(1/2)+1})^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+8-3*2^{(1/2)} \\
& (1/2))/((1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)+1}))^2)^{(1/2)}/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)} \\
& )+1))/((8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}-13/105710*(8*(x+2^{(1/2)}-1)^2 \\
& /(-x+2^{(1/2)+1})^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+8-3*2^{(1/2)})^{(1/2)} \\
& )^{(1/2)}*2^{(1/2)}*(285*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)} \\
& ))^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)}) \\
& )*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)} \\
& )*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+22 \\
& 379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)+1})^4+82*(x+2^{(1/2)}-1)^2/ \\
& (-x+2^{(1/2)+1})^2+23*(x+2^{(1/2)}-1)/(-x+2^{(1/2)+1})*(8+3*2^{(1/2)})))+386*(-8866 \\
& +6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775 \\
& 687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)} \\
& )+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2 \\
& +10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)} \\
& )-1)^4/(-x+2^{(1/2)+1})^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+23*(x+2^{(1/2)}- \\
& 1)/(-x+2^{(1/2)+1})*(8+3*2^{(1/2)})))-274846*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x \\
& +2^{(1/2)+1})^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+8-3*2^{(1/2)})^{(1/2)} \\
& /(-8866+6820*2^{(1/2)})^{(1/2)}*2^{(1/2)}-1543366*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2 \\
& /(-x+2^{(1/2)+1})^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+8-3*2^{(1/2)})^{(1/2)} \\
& /(-8866+6820*2^{(1/2)})^{(1/2)}))/((8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+3*2^{(1/2)} \\
& ^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+8-3*2^{(1/2)})^{(1/2)}/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)} \\
& )+1))^2)^{(1/2)}/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)+1}))/((8+3*2^{(1/2)})/(-8866+6 \\
& 820*2^{(1/2)})^{(1/2)}+3/10571*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+3*2^{(1/2)}*(x \\
& +2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(151*2^{(1/2)}*(-88 \\
& 66+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-7 \\
& 75687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)} \\
& )+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1}) \\
& )^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)} \\
& )-1)^4/(-x+2^{(1/2)+1})^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+23*(x+2^{(1/2)}- \\
& 1)/(-x+2^{(1/2)+1})*(8+3*2^{(1/2)})))+218*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+ \\
& 549362*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23 \\
& *(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+24*2^{(1/2)}-41))^{(1/2)}* \\
& (6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)} \\
& )+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)+1})^4+82*(x+ \\
& 2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+23*(x+2^{(1/2)}-1)/(-x+2^{(1/2)+1})*(8+3*2^{(1/2)} \\
& ))+401698*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+3*2^{(1/2)}*(x+2^{(1/2)} \\
& )-1)^2/(-x+2^{(1/2)+1})^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)}*2 \\
& ^{(1/2)}-63426*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+3*2^{(1/2)}*(x+ \\
& 2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)} \\
& ))/((8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)} \\
& )+1)^2/(-x+2^{(1/2)+1})^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)+1})^2+8-3*2^{(1/2)})^{(1/2)}
\end{aligned}$$

$$\begin{aligned} & 2)+1)^2+8-3*2^{(1/2)})/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1))^2)^{(1/2)}/(1+(x+2^{(1/2)} \\ & )-1)/(-x+2^{(1/2)}+1))/(8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}+9/21142*(8*(x \\ & +2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8 \\ & -3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(369*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+ \\ & 549362*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23 \\ & *(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}* \\ & (6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)} \\ & (1/2)+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+ \\ & 2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)} \\ & ))+520*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\arctan(1/1 \\ & 1692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1 \\ & )^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(- \\ & x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/ \\ & (23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23 \\ & )*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})))+465124*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)} \\ & (1/2)-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2 \\ & ^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})*2^{(1/2)}-866822*\operatorname{arctanh}(31/2*(8*(x \\ & +2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8 \\ & -3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})))/((8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}(1 \\ & /2)+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})/(1+(x+2^{(1 \\ & /2)}-1)/(-x+2^{(1/2)}+1))^2)^{(1/2)}/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1))/(8+3*2^{(1/ \\ & 2)})/(-8866+6820*2^{(1/2)})^{(1/2)} \end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] integrate((2\*x^2 - x + 3)^(3/2)/(5\*x^2 + 3\*x + 2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x^2 - x + 3)^{3/2}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(3/2)/(3\*x + 5\*x^2 + 2),x)

[Out] int((2\*x^2 - x + 3)^(3/2)/(3\*x + 5\*x^2 + 2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(3/2)/(5\*x\*\*2+3\*x+2),x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(3/2)/(5\*x\*\*2 + 3\*x + 2), x)

$$3.67 \quad \int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=232

$$\frac{(10x+3)(2x^2-x+3)^{3/2}}{31(5x^2+3x+2)} + \frac{4}{155}(4-5x)\sqrt{2x^2-x+3} + \frac{\sqrt{\frac{11}{31}(3169333+2265350\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{62(3169333+2265350\sqrt{2})}}}{\sqrt{2x^2-x+3}}\right)}{1550}$$

**Rubi [A]** time = 0.58, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {971, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)(2x^2-x+3)^{3/2}}{31(5x^2+3x+2)} + \frac{4}{155}(4-5x)\sqrt{2x^2-x+3} + \frac{\sqrt{\frac{11}{31}(3169333+2265350\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{62(3169333+2265350\sqrt{2})}}}{\sqrt{2x^2-x+3}}\right)}{1550} - \frac{\sqrt{\frac{11}{31}(2265350\sqrt{2}-3169333)} \tanh^{-1}\left(\frac{\sqrt{\frac{11}{62(2265350\sqrt{2}-3169333)}}}{\sqrt{2x^2-x+3}}\right)}{1550} - \frac{2}{25}\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(3/2)/(2 + 3\*x + 5\*x^2)^2, x]

[Out] (4\*(4 - 5\*x)\*Sqrt[3 - x + 2\*x^2])/155 + ((3 + 10\*x)\*(3 - x + 2\*x^2)^(3/2))/(31\*(2 + 3\*x + 5\*x^2)) - (2\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/25 + (Sqrt[(11\*(3169333 + 2265350\*Sqrt[2]))/31]\*ArcTan[(Sqrt[11/(62\*(3169333 + 2265350\*Sqrt[2]))])\*(3514 + 2963\*Sqrt[2] + (9440 + 6477\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/1550 - (Sqrt[(11\*(-3169333 + 2265350\*Sqrt[2]))/31]\*ArcTanh[(Sqrt[11/(62\*(-3169333 + 2265350\*Sqrt[2]))])\*(3514 - 2963\*Sqrt[2] + (9440 - 6477\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/1550

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 971

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/((b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[2\*c\*d\*(2\*p + 3) + b\*e\*q + (2\*b\*f\*q + 2\*c\*e\*(2\*p + q + 3))\*x + 2\*c\*f\*(2\*p + 2\*q + 3)\*x^2

, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

### Rule 1029

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

### Rule 1035

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 2]}, Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f - q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f + q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f + q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f - q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && NegQ[b^2 - 4\*a\*c]

### Rule 1066

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.)\*((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_.), x\_Symbol] := Simp[((B\*c\*f\*(2\*p + 2\*q + 3) + C\*(b\*f\*p - c\*e\*(2\*p + q + 2)) + 2\*c\*C\*f\*(p + q + 1)\*x)\*(a + b\*x + c\*x^2)^p\*(d + e\*x + f\*x^2)^(q + 1))/(2\*c\*f^2\*(p + q + 1)\*(2\*p + 2\*q + 3)), x] - Dist[1/(2\*c\*f^2\*(p + q + 1)\*(2\*p + 2\*q + 3)), Int[(a + b\*x + c\*x^2)^(p - 1)\*(d + e\*x + f\*x^2)^q\*Simp[p\*(b\*d - a\*e)\*(C\*(c\*e - b\*f)\*(q + 1) - c\*(C\*e - B\*f)\*(2\*p + 2\*q + 3)) + (p + q + 1)\*(b^2\*C\*d\*f\*p + a\*c\*(C\*(2\*d\*f - e^2\*(2\*p + q + 2)) + f\*(B\*e - 2\*A\*f)\*(2\*p + 2\*q + 3)) + (2\*p\*(c\*d - a\*f)\*(C\*(c\*e - b\*f)\*(q + 1) - c\*(C\*e - B\*f)\*(2\*p + 2\*q + 3)) + (p + q + 1)\*(C\*e\*f\*p\*(b^2 - 4\*a\*c) - b\*c\*(C\*(e^2 - 4\*d\*f)\*(2\*p + q + 2) + f\*(2\*C\*d - B\*e + 2\*A\*f)\*(2\*p + 2\*q + 3)))]\*x + (p\*(c\*e - b\*f)\*(C\*(c\*e - b\*f)\*(q + 1) - c\*(C\*e - B\*f)\*(2\*p + 2\*q + 3)) + (p + q + 1)\*(C\*f^2\*p\*(b^2 - 4\*a\*c) - c^2\*(C\*(e^2 - 4\*d\*f)\*(2\*p + q + 2) + f\*(2\*C\*d - B\*e + 2\*A\*f)\*(2\*p + 2\*q + 3)))]\*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2\*p + 2\*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

### Rule 1076

Int[((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Dist[C/c, Int[1/Sqrt[d + e\*x + f\*x^2], x], x] + Dist[1/c, Int[(A\*c - a\*C + (B\*c - b\*C)\*x)/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0]

### Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} dx = \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} - \frac{1}{31} \int \frac{\sqrt{3-x+2x^2} \left(-\frac{69}{2} + 13x + 40x^2\right)}{2+3x+5x^2} dx$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} + \frac{\int \frac{13070-5750x+2480x^2}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{3100}$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} + \frac{\int \frac{60390-36190x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{15500} + \dots$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} + \frac{1}{25} \left(2\sqrt{\frac{2}{23}}\right) \text{Subst} \left( \int \frac{1}{\sqrt{1-u^2}} du \right)$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} - \frac{2}{25} \sqrt{2} \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right) + \dots$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} - \frac{2}{25} \sqrt{2} \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right) + \dots$$

**Mathematica [C]** time = 2.56, size = 530, normalized size = 2.28

$$\frac{62000\sqrt{2-x+3x^2} + 62000\sqrt{2-x+3x^2} - 31000\sqrt{2-x+3x^2} - 31000\sqrt{2-x+3x^2} - 12400\sqrt{2-x+3x^2} + 93000\sqrt{2-x+3x^2} + 93000\sqrt{2-x+3x^2} + 9920\sqrt{2-x+3x^2} - \frac{\sqrt{286+22\sqrt{31}}(6477\sqrt{31}+10199)\operatorname{tanh}\left(\frac{4\sqrt{31}-22+2\sqrt{31}x}{2\sqrt{286+22\sqrt{31}}}\right) + 10199\sqrt{286+22\sqrt{31}}\operatorname{tanh}\left(\frac{4\sqrt{31}-22+2\sqrt{31}x}{2\sqrt{286+22\sqrt{31}}}\right) - 6477\sqrt{682(13+\sqrt{31})}\operatorname{tanh}\left(\frac{4\sqrt{31}-22+2\sqrt{31}x}{2\sqrt{286+22\sqrt{31}}}\right) - 7688\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{96100}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
[Out] (9920*Sqrt[3 - x + 2*x^2] - 12400*x*Sqrt[3 - x + 2*x^2] + (93000*Sqrt[3 - x + 2*x^2])/(3 - I*Sqrt[31] + 10*x) - (31000*x*Sqrt[3 - x + 2*x^2])/(3 - I*Sqrt[31] + 10*x) + (62000*x^2*Sqrt[3 - x + 2*x^2])/(3 - I*Sqrt[31] + 10*x) + (93000*Sqrt[3 - x + 2*x^2])/(3 + I*Sqrt[31] + 10*x) - (31000*x*Sqrt[3 - x + 2*x^2])/(3 + I*Sqrt[31] + 10*x) + (62000*x^2*Sqrt[3 - x + 2*x^2])/(3 + I*Sqrt[31] + 10*x) - 7688*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - (Sqrt[286 + (22*I)*Sqrt[31]]*(10199*I + 6477*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(-13*I + Sqrt[31]) - (6477*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/((13*I + Sqrt[31]) + ((10199*I)*Sqrt[286 - (22*I)*Sqrt[31]]*ArcTanh[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/((13*I + Sqrt[31]))/96100
```

**IntegrateAlgebraic [C]** time = 0.68, size = 422, normalized size = 1.82

$$\frac{11\operatorname{RootSum}\left[-5x^3 + 6\sqrt{2}x^2 + 17x - 26\sqrt{2} - 56, \frac{100x^2 \log(-41 + \sqrt{23-x+3x^2}) + 300\sqrt{2}x \log(-41 + \sqrt{23-x+3x^2}) - \sqrt{2} + 999 \log(-41 + \sqrt{23-x+3x^2})}{-10x^3 + 9\sqrt{2}x^2 + 17x - 13\sqrt{2}}\right] + \frac{11\operatorname{RootSum}\left[-5x^3 + 6\sqrt{2}x^2 + 17x - 26\sqrt{2} - 56, \frac{200x^2 \log(-41 + \sqrt{23-x+3x^2}) + 600\sqrt{2}x \log(-41 + \sqrt{23-x+3x^2}) - \sqrt{2} + 1998 \log(-41 + \sqrt{23-x+3x^2})}{-10x^3 + 9\sqrt{2}x^2 + 17x - 13\sqrt{2}}\right]}{38750} + \frac{11\sqrt{286+22\sqrt{31}}(10199\sqrt{286+22\sqrt{31}}+6477\sqrt{682(13+\sqrt{31})})\operatorname{ArcTanh}\left(\frac{4\sqrt{31}-22+2\sqrt{31}x}{2\sqrt{286+22\sqrt{31}}}\right) - 11\sqrt{286-22\sqrt{31}}(10199\sqrt{286-22\sqrt{31}}+6477\sqrt{682(13-\sqrt{31})})\operatorname{ArcTanh}\left(\frac{4\sqrt{31}-22+2\sqrt{31}x}{2\sqrt{286-22\sqrt{31}}}\right) - 7688\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{96100}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
[Out] (11*(7 + 13*x)*Sqrt[3 - x + 2*x^2])/(155*(2 + 3*x + 5*x^2)) - (2*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/25 + (11*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (999*Log[-(Sqrt[2]*x) + Sqrt[2]*Sqrt[3 - x + 2*x^2]] + 300*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[2]*Sqrt[3 - x + 2*x^2]] + 100*Log[-(Sqrt[2]*x) + Sqrt[2]*Sqrt[3 - x + 2*x^2]] - Sqrt[2])]/(-10*#1^3 + 9*Sqrt[2]*#1^2 + 17*#1 - 13*Sqrt[2]))/38750
```

$$3 - x + 2x^2] - \#1] + 310\sqrt{2}\log[-(\sqrt{2}x) + \sqrt{3 - x + 2x^2} - \#1]\#1 + 100\log[-(\sqrt{2}x) + \sqrt{3 - x + 2x^2} - \#1]\#1^2)/(-13\sqrt{2} + 17\#1 + 9\sqrt{2}\#1^2 - 10\#1^3) \& ])/625 + (11\text{RootSum}[-56 - 26\sqrt{2}\#1 + 17\#1^2 + 6\sqrt{2}\#1^3 - 5\#1^4 \& , (-72888\log[-(\sqrt{2}x) + \sqrt{3 - x + 2x^2} - \#1] + 8230\sqrt{2}\log[-(\sqrt{2}x) + \sqrt{3 - x + 2x^2} - \#1]\#1 + 2025\log[-(\sqrt{2}x) + \sqrt{3 - x + 2x^2} - \#1]\#1^2)/(-13\sqrt{2} + 17\#1 + 9\sqrt{2}\#1^2 - 10\#1^3) \& ])/38750$$

**fricas [B]** time = 1.29, size = 2150, normalized size = 9.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out]  $\frac{1}{90746855745853600} \cdot (10421084 \cdot 1987037073032^{1/4} \cdot \sqrt{45307} \cdot \sqrt{62} \cdot \sqrt{(2)(5x^2 + 3x + 2) \cdot \sqrt{3169333 \cdot \sqrt{2} + 4530700}} \cdot \arctan(1/172758074198807633719789 \cdot (64607782 \cdot \sqrt{45307}) \cdot (2 \cdot 1987037073032^{3/4} \cdot \sqrt{62}) \cdot (2433118x^7 - 9616349x^6 + 20077988x^5 - 32895253x^4 + 16664280x^3 - 8289000x^2 - \sqrt{2} \cdot (1842432x^7 - 6916062x^6 + 14611071x^5 - 22920229x^4 + 11367152x^3 - 5107176x^2 - 12897792x + 8726400) - 17452800x + 12897792) + 1404517 \cdot 1987037073032^{1/4} \cdot \sqrt{62} \cdot (373384x^7 - 5757834x^6 + 30631880x^5 - 70476664x^4 + 91370880x^3 - 59457600x^2 - \sqrt{2} \cdot (276977x^7 - 4232733x^6 + 22218448x^5 - 50249260x^4 + 64668384x^3 - 39479328x^2 - 46697472x + 32016384) - 64032768x + 46697472)) \cdot \sqrt{2x^2 - x + 3} \cdot \sqrt{3169333 \cdot \sqrt{2} + 4530700} + 490410017080486186043272 \cdot \sqrt{31} \cdot \sqrt{2} \cdot (28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2} \cdot (8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - \sqrt{45307/2711} \cdot (\sqrt{45307} \cdot (2 \cdot 1987037073032^{3/4} \cdot \sqrt{62}) \cdot (8480726x^7 - 12210811x^6 + 39548601x^5 - 16962480x^4 + 21434760x^3 + 14432256x^2 - \sqrt{2} \cdot (6779042x^7 - 9704193x^6 + 31062363x^5 - 11094928x^4 + 12114072x^3 + 16301952x^2 - 16301952x) - 14432256x) + 1404517 \cdot 1987037073032^{1/4} \cdot \sqrt{62} \cdot (1312966x^7 - 16987736x^6 + 65572040x^5 - 85530240x^4 + 112374720x^3 + 57314304x^2 - \sqrt{2} \cdot (1011501x^7 - 13081364x^6 + 50391260x^5 - 64806336x^4 + 81634464x^3 + 56070144x^2 - 56070144x) - 57314304x)) \cdot \sqrt{2x^2 - x + 3} \cdot \sqrt{3169333 \cdot \sqrt{2} + 4530700} + 7590571938849196 \cdot \sqrt{31} \cdot \sqrt{2} \cdot (123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2} \cdot (15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) + 345025997220418 \cdot \sqrt{31} \cdot (254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488 \cdot \sqrt{2} \cdot (4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x) \cdot \sqrt{-(1987037073032^{1/4} \cdot \sqrt{45307} \cdot \sqrt{62} \cdot \sqrt{31} \cdot \sqrt{2x^2 - x + 3} \cdot (\sqrt{2} \cdot (1867x + 1425) - 3292x - 442) \cdot \sqrt{3169333 \cdot \sqrt{2} + 4530700} - 11567627293306x^2 - 10387257161336 \cdot \sqrt{2}) \cdot (2x^2 - x + 3) + 35647177985494x - 47214805278800)/x^2) + 5572841103187343023219 \cdot \sqrt{31} \cdot (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744 \cdot \sqrt{2} \cdot (1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936))/ (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456) + 10421084 \cdot 1987037073032^{1/4} \cdot \sqrt{45307} \cdot \sqrt{62} \cdot \sqrt{2} \cdot (5x^2 + 3x + 2) \cdot \sqrt{3169333 \cdot \sqrt{2} + 4530700} \cdot \arctan(1/172758074198807633719789 \cdot (64607782 \cdot \sqrt{45307}) \cdot (2 \cdot 1987037073032^{3/4} \cdot \sqrt{62}) \cdot (2433118x^7 - 9616349x^6 + 20077988x^5 - 32895253x^4 + 16664280x^3 - 8289000x^2 - \sqrt{2} \cdot (1842432x^7 - 6916062x^6 + 14611071x^5 - 22920229x^4 + 11367152x^3 - 5107176x^2 - 12897792x + 8726400) - 17452800x + 12897792) + 1404517 \cdot 1987037073032^{1/4} \cdot \sqrt{62} \cdot (373384x^7 - 5757834x^6 + 30631880x^5 - 70476664x^4 + 91370880x^3 - 59457600x^2 - \sqrt{2} \cdot (276977x^7 - 4232733x^$



```

6 + 22218448*x^5 - 50249260*x^4 + 64668384*x^3 - 39479328*x^2 - 46697472*x
+ 32016384) - 64032768*x + 46697472))*sqrt(2*x^2 - x + 3)*sqrt(3169333*sqrt
(2) + 4530700) - 490410017080486186043272*sqrt(31)*sqrt(2)*(28180*x^8 - 254
666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 -
sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 7
52088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(45
307/2711)*(sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(8480726*x^7 - 12210
811*x^6 + 39548601*x^5 - 16962480*x^4 + 21434760*x^3 + 14432256*x^2 - sqrt(
2)*(6779042*x^7 - 9704193*x^6 + 31062363*x^5 - 11094928*x^4 + 12114072*x^3
+ 16301952*x^2 - 16301952*x) - 14432256*x) + 1404517*1987037073032^(1/4)*sq
rt(62)*(1312966*x^7 - 16987736*x^6 + 65572040*x^5 - 85530240*x^4 + 11237472
0*x^3 + 57314304*x^2 - sqrt(2)*(1011501*x^7 - 13081364*x^6 + 50391260*x^5 -
64806336*x^4 + 81634464*x^3 + 56070144*x^2 - 56070144*x) - 57314304*x))*sq
rt(2*x^2 - x + 3)*sqrt(3169333*sqrt(2) + 4530700) - 7590571938849196*sqrt(3
1)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^
4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
- 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276
288*x) - 345025997220418*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6
- 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(
2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 -
1944*x) + 144820224*x))*sqrt((1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt
(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*sqrt(3169
333*sqrt(2) + 4530700) + 11567627293306*x^2 + 10387257161336*sqrt(2)*(2*x^2
- x + 3) - 35647177985494*x + 47214805278800)/x^2) - 557284110318734302321
9*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 2541
46592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^
7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184)
+ 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490
880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 1857945
6)) + 1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(31)*(22653500*x^2 - 316
9333*sqrt(2)*(5*x^2 + 3*x + 2) + 13592100*x + 9061400)*sqrt(3169333*sqrt(2)
+ 4530700)*log(113267500/2711*(1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sq
rt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*sqrt(31
69333*sqrt(2) + 4530700) + 11567627293306*x^2 + 10387257161336*sqrt(2)*(2*x
^2 - x + 3) - 35647177985494*x + 47214805278800)/x^2) - 1987037073032^(1/4)
*sqrt(45307)*sqrt(62)*sqrt(31)*(22653500*x^2 - 3169333*sqrt(2)*(5*x^2 + 3*x
+ 2) + 13592100*x + 9061400)*sqrt(3169333*sqrt(2) + 4530700)*log(-11326750
0/2711*(1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x +
3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*sqrt(3169333*sqrt(2) + 4530700)
- 11567627293306*x^2 - 10387257161336*sqrt(2)*(2*x^2 - x + 3) + 3564717798
5494*x - 47214805278800)/x^2) + 3629874229834144*sqrt(2)*(5*x^2 + 3*x + 2)*
log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 644009
9440028320*sqrt(2*x^2 - x + 3)*(13*x + 7))/(5*x^2 + 3*x + 2)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er  
ror%{174900625, [8]%%}+%{[-419761500,0]: [1,0,-2]%%}, [7]%%}+%{-685  
610450, [6]%%}+%{[3246155600,0]: [1,0,-2]%%}, [5]%%}+%{1574105625, [4]  
%%}+%{-10885814900,0]: [1,0,-2]%%}, [3]%%}+%{-3861805800, [2]%%}+%  
%{[20372424800,0]: [1,0,-2]%%}, [1]%%}+%{21939534400, [0]%%} / %%{50, [8]  
%%}+%{poly1[-120,0]: [1,0,-2]%%}, [7]%%}+%{-196, [6]%%}+%{poly  
y1[928,0]: [1,0,-2]%%}, [5]%%}+%{450, [4]%%}+%{poly1[-3112,0]: [1,0,-2

```
]%%}, [3]%%}+%%{-1104, [2]%%}+%%{poly1 [5824, 0] : [1, 0, -2]%%}, [1]%%}+%%
{6272, [0]%%} Error: Bad Argument Value
```

**maple** [B] time = 0.16, size = 28185, normalized size = 121.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^2, x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(2x^2 - x + 3)^{3/2}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**2, x)
```

$$3.68 \quad \int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3} dx$$

**Optimal.** Leaf size=223

$$\frac{(10x+3)(2x^2-x+3)^{3/2}}{62(5x^2+3x+2)^2} + \frac{3(696x+277)\sqrt{2x^2-x+3}}{3844(5x^2+3x+2)} + \frac{3\sqrt{\frac{1}{682}(366990269+259509026\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{31(366990269+259509026\sqrt{2})}}{\sqrt{2x^2-x+3}}\right)}{7688}$$

**Rubi [A]** time = 0.43, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {971, 1013, 1035, 1029, 206, 204}

$$\frac{(10x+3)(2x^2-x+3)^{3/2}}{62(5x^2+3x+2)^2} + \frac{3(696x+277)\sqrt{2x^2-x+3}}{3844(5x^2+3x+2)} + \frac{3\sqrt{\frac{1}{682}(366990269+259509026\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{31(366990269+259509026\sqrt{2})}}{\sqrt{2x^2-x+3}}\right)}{7688} - \frac{3\sqrt{\frac{1}{682}(259509026\sqrt{2}-366990269)} \tanh^{-1}\left(\frac{\sqrt{31(259509026\sqrt{2}-366990269)}}{\sqrt{2x^2-x+3}}\right)}{7688}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(3/2)/(2 + 3\*x + 5\*x^2)^3, x]

[Out] ((3 + 10\*x)\*(3 - x + 2\*x^2)^(3/2)/(62\*(2 + 3\*x + 5\*x^2)^2) + (3\*(277 + 696\*x)\*Sqrt[3 - x + 2\*x^2])/(3844\*(2 + 3\*x + 5\*x^2)) + (3\*Sqrt[(366990269 + 259509026\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(366990269 + 259509026\*Sqrt[2]))])\*(29367 + 20575\*Sqrt[2] + (70517 + 49942\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]])/7688 - (3\*Sqrt[(-366990269 + 259509026\*Sqrt[2])/682]\*ArcTanh[(Sqrt[11/(31\*(-366990269 + 259509026\*Sqrt[2]))])\*(29367 - 20575\*Sqrt[2] + (70517 - 49942\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]])/7688

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 971

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/((b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[2\*c\*d\*(2\*p + 3) + b\*e\*q + (2\*b\*f\*q + 2\*c\*e\*(2\*p + q + 3))\*x + 2\*c\*f\*(2\*p + 2\*q + 3)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

#### Rule 1013

Int[((g\_.) + (h\_.)\*(x\_))\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[(g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/((b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[e\*q\*(g\*b - 2\*a\*h) - d\*(b\*h - 2\*g\*c)\*(2\*p + 3) + (2\*f\*q\*(g\*b - 2\*a\*h) - e\*(b\*h - 2\*g\*c)\*(2\*p + q + 3))\*x - f\*(b\*h - 2\*g\*c)\*(

$2*p + 2*q + 3)*x^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[q, 0]$

### Rule 1029

$\text{Int}[\frac{(g_. + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*\text{Sqrt}[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]}{x\_Symbol}, x] := \text{Dist}[-2*g*(g*b - 2*a*h), \text{Subst}[\text{Int}[1/\text{Simp}[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, \text{Simp}[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/\text{Sqrt}[d + e*x + f*x^2]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{NeQ}[b*d - a*e, 0] \ \&\& \ \text{EqQ}[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]$

### Rule 1035

$\text{Int}[\frac{(g_. + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*\text{Sqrt}[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]}{x\_Symbol}, x] := \text{With}[\{q = \text{Rt}[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)], 2\}, \text{Dist}[1/(2*q), \text{Int}[\text{Simp}[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x] - \text{Dist}[1/(2*q), \text{Int}[\text{Simp}[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{NeQ}[b*d - a*e, 0] \ \&\& \ \text{NegQ}[b^2 - 4*a*c]$

### Rubi steps

$$\begin{aligned} \int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3} dx &= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} - \frac{1}{62} \int \frac{\left(-\frac{189}{2} + 33x\right) \sqrt{3-x+2x^2}}{(2+3x+5x^2)^2} dx \\ &= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{\int \frac{\frac{13359}{4} - 1353x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{1922} \\ &= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{\int \frac{-\frac{33}{4}(6257-4453\sqrt{2}) + \frac{33}{4}(2649-1353x)}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{42284\sqrt{2}} \\ &= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{(99(519018052 - 366990269 + 25953\sqrt{2}))\sqrt{3-x+2x^2}}{42284\sqrt{2}} \\ &= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{3\sqrt{\frac{1}{682}(366990269 + 25953\sqrt{2})}\sqrt{3-x+2x^2}}{42284\sqrt{2}} \end{aligned}$$

**Mathematica [C]** time = 5.34, size = 1262, normalized size = 5.66

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(3/2)/(2 + 3\*x + 5\*x^2)^3,x]

```
[Out] (((248000*I)*Sqrt[31]*(3 - x + 2*x^2)^(3/2))/(3*I + Sqrt[31] + (10*I)*x)^2
+ (744000*(3 - x + 2*x^2)^(3/2))/(3 - I*Sqrt[31] + 10*x) + ((248000*I)*Sqrt
[31]*(3 - x + 2*x^2)^(3/2))/(3 + I*Sqrt[31] + 10*x)^2 + (744000*(3 - x + 2*
x^2)^(3/2))/(3 + I*Sqrt[31] + 10*x) + (3*I)*Sqrt[31]*(20*(1199 + (98*I)*Sqr
t[31] - 20*(11 + (2*I)*Sqrt[31])*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(13453 +
(4406*I)*Sqrt[31])*ArcSinh[(1 - 4*x)/Sqrt[23]] - (352*Sqrt[286 + (22*I)*Sqr
t[31]]*(-69*I + 13*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[3
1])*x]/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])))/(-13*I + Sqrt[
31])) + 558*(20*(27 + (4*I)*Sqrt[31] - 20*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*
(569 + (88*I)*Sqrt[31])*ArcSinh[(1 - 4*x)/Sqrt[23]] - (4*Sqrt[286 + (22*I)*
Sqrt[31]]*(-81*I + 37*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqr
t[31])*x]/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])))/(-13*I + Sq
rt[31])) + (744*Sqrt[31]*(220*(-439 + (497*I)*Sqrt[31] + 20*(69 + (13*I)*Sq
rt[31])*x)*Sqrt[3 - x + 2*x^2] + 88*Sqrt[2]*(4426 - (398*I)*Sqrt[31] + 5*(4
7 - (281*I)*Sqrt[31])*x)*ArcSinh[(-1 + 4*x)/Sqrt[23]] + Sqrt[286 + (22*I)*S
qrt[31]]*(19548 - (4904*I)*Sqrt[31] + (-23345 - (8565*I)*Sqrt[31])*x)*ArcTa
nh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x]/(2*Sqrt[286 + (22*I)*Sqrt[3
1]]*Sqrt[3 - x + 2*x^2])))/(11*(-13*I + Sqrt[31])^2*(-3*I + Sqrt[31] - (10
*I)*x)) + (744*Sqrt[31]*(220*(-439 - (497*I)*Sqrt[31] + 20*(69 - (13*I)*Sqr
t[31])*x)*Sqrt[3 - x + 2*x^2] + 88*Sqrt[2]*(4426 + (398*I)*Sqrt[31] + 5*(47
+ (281*I)*Sqrt[31])*x)*ArcSinh[(-1 + 4*x)/Sqrt[23]] + Sqrt[286 - (22*I)*Sq
rt[31]]*(-19548 - (4904*I)*Sqrt[31] + 5*(4669 - (1713*I)*Sqrt[31])*x)*ArcTa
nh[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x]/(2*Sqrt[286 - (22*I)*Sqrt[3
1]]*Sqrt[3 - x + 2*x^2])))/(11*(13*I + Sqrt[31])^2*(3*I + Sqrt[31] + (10*I
)*x)) + (3*Sqrt[31]*(20*(12549 - (2473*I)*Sqrt[31] + (20*I)*(81*I + 37*Sqrt
[31])*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(38303 - (70731*I)*Sqrt[31])*ArcSinh
[(1 - 4*x)/Sqrt[23]] + (352*I)*Sqrt[286 - (22*I)*Sqrt[31]]*(69*I + 13*Sqrt[
31])*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x]/(2*Sqrt[286 - (22
*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])))/(13*I + Sqrt[31]) + 558*(-20*(-27 + (
4*I)*Sqrt[31] + 20*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(569 - (88*I)*Sqrt[31])
*ArcSinh[(1 - 4*x)/Sqrt[23]] - (4*Sqrt[286 - (22*I)*Sqrt[31]]*(81*I + 37*Sq
rt[31])*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x]/(2*Sqrt[286 -
(22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])))/(13*I + Sqrt[31])))/4766560
```

**IntegrateAlgebraic [C]** time = 1.06, size = 578, normalized size = 2.59

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(2220 + 8343*x + 10171*x^2 + 11680*x^3))/(3844*(2 + 3*
x + 5*x^2)^2) - (3219561*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*
#1^3 - 5*#1^4 & , Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]/(-13*Sqrt[2]
+ 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/961000 + (4*RootSum[-56 - 26*Sqrt
[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (93*Log[-(Sqrt[2]*x) + Sqrt[
3 - x + 2*x^2] - #1] + 10*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
#1]*#1)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/125 + (7*Root
Sum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (4926449381
*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 2660991465*Log[-
(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt
[2]*#1^2 - 10*#1^3) & ])/6354612500 - (3*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#
1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (155209944*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqr
t[3 - x + 2*x^2] - #1]*#1 - 248390285*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2
] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/2049875
00
```

**fricas [B]** time = 1.27, size = 2183, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 
$$-1/85773071417697924109696*(189113268*134689869150937352^{(1/4)}*\sqrt{129754513}*\sqrt{341}*\sqrt{2}*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*\sqrt{366990269*\sqrt{2} + 519018052}*\arctan(1/1067259092343193675559267622545473*(16089559612*\sqrt{129754513}*(11*134689869150937352^{(3/4)}*\sqrt{341}*(38305160*x^7 - 147261352*x^6 + 309398878*x^5 - 495410374*x^4 + 248212864*x^3 - 117285552*x^2 - \sqrt{2}*(26988622*x^7 - 104036813*x^6 + 218448200*x^5 - 350579241*x^4 + 175844824*x^3 - 83534472*x^2 - 191303424*x + 135585792) - 271171584*x + 191303424) + 4022389903*134689869150937352^{(1/4)}*\sqrt{341}*(2906601*x^7 - 44604657*x^6 + 235604928*x^5 - 537156764*x^4 + 693706464*x^3 - 436717728*x^2 - \sqrt{2}*(2050114*x^7 - 31475955*x^6 + 166375268*x^5 - 379661892*x^4 + 490500864*x^3 - 309827808*x^2 - 348696576*x + 246965760) - 493931520*x + 348696576))*\sqrt{2*x^2 - x + 3}*\sqrt{366990269*\sqrt{2} + 519018052} + 3029638713748420756426308089806504*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*\sqrt{259509026/713}*(\sqrt{129754513}*(11*134689869150937352^{(3/4)}*\sqrt{341}*(5980372*x^7 - 8582986*x^6 + 27618126*x^5 - 10751392*x^4 + 12649968*x^3 + 12517632*x^2 - \sqrt{2}*(4201650*x^7 - 6032009*x^6 + 19421619*x^5 - 7633552*x^4 + 9050328*x^3 + 8640000*x^2 - 8640000*x) - 12517632*x) + 4022389903*134689869150937352^{(1/4)}*\sqrt{341}*(453599*x^7 - 5867420*x^6 + 22622900*x^5 - 29282112*x^4 + 37610208*x^3 + 22726656*x^2 - \sqrt{2}*(319303*x^7 - 4130364*x^6 + 15927060*x^5 - 20630592*x^4 + 26556768*x^3 + 15800832*x^2 - 15800832*x) - 22726656*x))*\sqrt{2*x^2 - x + 3}*\sqrt{366990269*\sqrt{2} + 519018052} + 8186887989068712800954*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 372131272230396036407*\sqrt{31}*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\sqrt{-(134689869150937352^{(1/4)}*\sqrt{129754513}*\sqrt{341}*\sqrt{31}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(696*x + 277) - 973*x - 419)*\sqrt{366990269*\sqrt{2} + 519018052} - 4356437317274441*x^2 - 3911902897144396*\sqrt{2}*(2*x^2 - x + 3) + 13424939487927359*x - 17781376805201800)/x^2} + 34427712656232054050298955565983*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 189113268*134689869150937352^{(1/4)}*\sqrt{129754513}*\sqrt{341}*\sqrt{2}*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*\sqrt{366990269*\sqrt{2} + 519018052}*\arctan(1/1067259092343193675559267622545473*(16089559612*\sqrt{129754513}*(11*134689869150937352^{(3/4)}*\sqrt{341}*(38305160*x^7 - 147261352*x^6 + 309398878*x^5 - 495410374*x^4 + 248212864*x^3 - 117285552*x^2 - \sqrt{2}*(26988622*x^7 - 104036813*x^6 + 218448200*x^5 - 350579241*x^4 + 175844824*x^3 - 83534472*x^2 - 191303424*x + 135585792) - 271171584*x + 191303424) + 4022389903*134689869150937352^{(1/4)}*\sqrt{341}*(2906601*x^7 - 44604657*x^6 + 235604928*x^5 - 537156764*x^4 + 693706464*x^3 - 436717728*x^2 - \sqrt{2}*(2050114*x^7 - 31475955*x^6 + 166375268*x^5 - 379661892*x^4 + 490500864*x^3 - 309827808*x^2 - 348696576*x + 246965760) - 493931520*x + 348696576))*\sqrt{2*x^2 - x + 3}*\sqrt{366990269*\sqrt{2} + 519018052} - 3029638713748420756426308089806504*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*\sqrt{259509026/713}*(\sqrt{129754513}*(11*134689869150937352^{(3/4)}*\sqrt{341}*$$

```

*(5980372*x^7 - 8582986*x^6 + 27618126*x^5 - 10751392*x^4 + 12649968*x^3 +
12517632*x^2 - sqrt(2)*(4201650*x^7 - 6032009*x^6 + 19421619*x^5 - 7633552*
x^4 + 9050328*x^3 + 8640000*x^2 - 8640000*x) - 12517632*x) + 4022389903*134
689869150937352^(1/4)*sqrt(341)*(453599*x^7 - 5867420*x^6 + 22622900*x^5 -
29282112*x^4 + 37610208*x^3 + 22726656*x^2 - sqrt(2)*(319303*x^7 - 4130364*
x^6 + 15927060*x^5 - 20630592*x^4 + 26556768*x^3 + 15800832*x^2 - 15800832*
x) - 22726656*x))*sqrt(2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 519018052) -
8186887989068712800954*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888
*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550
*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1
209600*x^2 - 1036800*x) + 3276288*x) - 372131272230396036407*sqrt(31)*(2545
91*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 742193
28*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5
+ 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((134689869
150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sq
rt(2)*(696*x + 277) - 973*x - 419)*sqrt(366990269*sqrt(2) + 519018052) + 435
6437317274441*x^2 + 3911902897144396*sqrt(2)*(2*x^2 - x + 3) - 134249394879
27359*x + 17781376805201800)/x^2) - 34427712656232054050298955565983*sqrt(3
1)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^
4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789
*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306
4064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5
- 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 3*
134689869150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*(12975451300*x^
4 + 15570541560*x^3 + 15051523508*x^2 - 366990269*sqrt(2)*(25*x^4 + 30*x^3
+ 29*x^2 + 12*x + 4) + 6228216624*x + 2076072208)*sqrt(366990269*sqrt(2) +
519018052)*log(9342324936/713*(134689869150937352^(1/4)*sqrt(129754513)*sq
rt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(696*x + 277) - 973*x - 419)*s
qrt(366990269*sqrt(2) + 519018052) + 4356437317274441*x^2 + 391190289714439
6*sqrt(2)*(2*x^2 - x + 3) - 13424939487927359*x + 17781376805201800)/x^2) +
3*134689869150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*(12975451300
*x^4 + 15570541560*x^3 + 15051523508*x^2 - 366990269*sqrt(2)*(25*x^4 + 30*x^
3 + 29*x^2 + 12*x + 4) + 6228216624*x + 2076072208)*sqrt(366990269*sqrt(2)
+ 519018052)*log(-9342324936/713*(134689869150937352^(1/4)*sqrt(129754513)
*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(696*x + 277) - 973*x - 41
9)*sqrt(366990269*sqrt(2) + 519018052) - 4356437317274441*x^2 - 39119028971
44396*sqrt(2)*(2*x^2 - x + 3) + 13424939487927359*x - 17781376805201800)/x^
2) - 22313494125311634784*(11680*x^3 + 10171*x^2 + 8343*x + 2220)*sqrt(2*x^
2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,inf
inity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity,infinity]Francis algori
th failure for[-1.0,infinity,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity
]Evaluation time: 36.71Done
```

**maple** [B] time = 0.35, size = 81552, normalized size = 365.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x)`

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")`

[Out] `integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^3, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^3,x)`

[Out] `int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)`

[Out] `Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**3, x)`



$$3.69 \quad \int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2)^4 dx$$

**Optimal.** Leaf size=254

$$\frac{122595067 (2x^2 - x + 3)^{7/2} x^2}{19169280} + \frac{112244125 (2x^2 - x + 3)^{7/2} x}{122683392} + \frac{25250178739 (2x^2 - x + 3)^{7/2}}{5725224960} - \frac{401135647(1 - 4x) \sqrt{3 - x + 2x^2}}{2147483648} - \frac{9226119881 (1 - 4x)^2 \sqrt{3 - x + 2x^2}}{335544320} + \frac{25250178739 (3 - x + 2x^2)^{7/2}}{5725224960} + \frac{12244125 x (3 - x + 2x^2)^{7/2}}{122683392} + \frac{122595067 x^2 (3 - x + 2x^2)^{7/2}}{19169280} + \frac{23460839 x^3 (3 - x + 2x^2)^{7/2}}{532480} + \frac{3684995 x^4 (3 - x + 2x^2)^{7/2}}{39936} + \frac{1046225 x^5 (3 - x + 2x^2)^{7/2}}{9984} + \frac{13875 x^6 (3 - x + 2x^2)^{7/2}}{208} + \frac{625 x^7 (3 - x + 2x^2)^{7/2}}{28} - \frac{14641852251147 \operatorname{ArcSinh}[(1 - 4x)/\sqrt{23}]}{68719476736 \sqrt{2}}$$

**Rubi [A]** time = 0.37, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 27, number of rules / integrand size = 0.185, Rules used = {1661, 640, 612, 619, 215}

$\frac{625}{28} (2x^2 - x + 3)^{7/2} x^7 + \frac{13875}{208} (2x^2 - x + 3)^{7/2} x^6 + \frac{1046225}{9984} (2x^2 - x + 3)^{7/2} x^5 + \frac{3684995}{39936} (2x^2 - x + 3)^{7/2} x^4 + \frac{23460839}{532480} (2x^2 - x + 3)^{7/2} x^3 + \frac{122595067}{19169280} (2x^2 - x + 3)^{7/2} x^2 + \frac{112244125}{122683392} (2x^2 - x + 3)^{7/2} x + \frac{25250178739}{5725224960} (2x^2 - x + 3)^{7/2} + \frac{401135647(1 - 4x) \sqrt{3 - x + 2x^2}}{2147483648} + \frac{9226119881(1 - 4x)^2 \sqrt{3 - x + 2x^2}}{335544320} + \frac{14641852251147 \operatorname{ArcSinh}[(1 - 4x)/\sqrt{23}]}{68719476736 \sqrt{2}}$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] (-636602271789\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/34359738368 - (9226119881\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(3/2))/2147483648 - (401135647\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(5/2))/335544320 + (25250178739\*(3 - x + 2\*x^2)^(7/2))/5725224960 + (12244125\*x\*(3 - x + 2\*x^2)^(7/2))/122683392 + (122595067\*x^2\*(3 - x + 2\*x^2)^(7/2))/19169280 + (23460839\*x^3\*(3 - x + 2\*x^2)^(7/2))/532480 + (3684995\*x^4\*(3 - x + 2\*x^2)^(7/2))/39936 + (1046225\*x^5\*(3 - x + 2\*x^2)^(7/2))/9984 + (13875\*x^6\*(3 - x + 2\*x^2)^(7/2))/208 + (625\*x^7\*(3 - x + 2\*x^2)^(7/2))/28 - (14641852251147\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(68719476736\*Sqrt[2])

**Rule 215**

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

**Rule 612**

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

**Rule 619**

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

**Rule 640**

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

**Rule 1661**

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x]] /; FreeQ[{a, b, c,

p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2)^4 dx &= \frac{625}{28}x^7 (3 - x + 2x^2)^{7/2} + \frac{1}{28} \int (3 - x + 2x^2)^{5/2} (448 + 2688x + 10528x^2 \\ &= \frac{13875}{208}x^6 (3 - x + 2x^2)^{7/2} + \frac{625}{28}x^7 (3 - x + 2x^2)^{7/2} + \frac{1}{728} \int (3 - x + 2x^2)^{5/2} (448 + 2688x + 10528x^2 \\ &= \frac{1046225x^5 (3 - x + 2x^2)^{7/2}}{9984} + \frac{13875}{208}x^6 (3 - x + 2x^2)^{7/2} + \frac{625}{28}x^7 (3 - x + 2x^2)^{7/2} \\ &= \frac{3684995x^4 (3 - x + 2x^2)^{7/2}}{39936} + \frac{1046225x^5 (3 - x + 2x^2)^{7/2}}{9984} + \frac{13875}{208}x^6 (3 - x + 2x^2)^{7/2} \\ &= \frac{23460839x^3 (3 - x + 2x^2)^{7/2}}{532480} + \frac{3684995x^4 (3 - x + 2x^2)^{7/2}}{39936} + \frac{1046225x^5 (3 - x + 2x^2)^{7/2}}{9984} \\ &= \frac{122595067x^2 (3 - x + 2x^2)^{7/2}}{19169280} + \frac{23460839x^3 (3 - x + 2x^2)^{7/2}}{532480} + \frac{3684995x^4 (3 - x + 2x^2)^{7/2}}{39936} \\ &= \frac{112244125x (3 - x + 2x^2)^{7/2}}{122683392} + \frac{122595067x^2 (3 - x + 2x^2)^{7/2}}{19169280} + \frac{23460839x^3 (3 - x + 2x^2)^{7/2}}{532480} \\ &= \frac{25250178739 (3 - x + 2x^2)^{7/2}}{5725224960} + \frac{112244125x (3 - x + 2x^2)^{7/2}}{122683392} + \frac{122595067x^2 (3 - x + 2x^2)^{7/2}}{19169280} \\ &= -\frac{401135647(1 - 4x) (3 - x + 2x^2)^{5/2}}{335544320} + \frac{25250178739 (3 - x + 2x^2)^{7/2}}{5725224960} \\ &= -\frac{9226119881(1 - 4x) (3 - x + 2x^2)^{3/2}}{2147483648} - \frac{401135647(1 - 4x) (3 - x + 2x^2)^{5/2}}{335544320} \\ &= -\frac{636602271789(1 - 4x)\sqrt{3 - x + 2x^2}}{34359738368} - \frac{9226119881(1 - 4x) (3 - x + 2x^2)^{3/2}}{2147483648} \\ &= -\frac{636602271789(1 - 4x)\sqrt{3 - x + 2x^2}}{34359738368} - \frac{9226119881(1 - 4x) (3 - x + 2x^2)^{3/2}}{2147483648} \\ &= -\frac{636602271789(1 - 4x)\sqrt{3 - x + 2x^2}}{34359738368} - \frac{9226119881(1 - 4x) (3 - x + 2x^2)^{3/2}}{2147483648} \end{aligned}$$

**Mathematica [A]** time = 0.45, size = 105, normalized size = 0.41

$4\sqrt{23} \sqrt{3} \left( 25125558681600000x^{13} + 37398427729920000x^{12} + 137233466130432000x^{11} + 204932411660697600x^{10} + 363646430503501824x^9 + 4390645588463216x^8 + 530502956133122048x^7 + 485091164642279424x^6 + 4054683828461034x^5 + 257786732552566784x^4 + 14249093153377856x^3 + 5008474038215008x^2 + 12071614275862524x + 10821650749858669 \right) - 5995838498448015\sqrt{23} \operatorname{ArcSinh}\left(\frac{1-4x}{\sqrt{23}}\right)$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]
[Out] (4*Sqrt[3 - x + 2*x^2]*(10820567498568669 + 12071614275862524*x + 500641740
38215008*x^2 + 142490931553577856*x^3 + 257786732552566784*x^4 + 405468382
84161024*x^5 + 485091164642279424*x^6 + 530502956133122048*x^7 + 4390645588
46345216*x^8 + 363646430503501824*x^9 + 204932411660697600*x^10 + 137233466
130432000*x^11 + 37398427729920000*x^12 + 25125558681600000*x^13) - 5995838
49684446965*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/562812514467840
```

**IntegrateAlgebraic [A]** time = 1.83, size = 120, normalized size = 0.47

$\frac{\sqrt{2x^2-1}}{1407031286} (5125588640000x^{12} + 37984277920000x^{11} + 1372346613420000x^{10} + 2049241666070000x^9 + 303644335151824x^8 + 439645788464232x^7 + 5305029561322048x^6 + 48509114642279424x^5 + 40546838284161024x^4 + 257786732552566784x^3 + 14248093515377856x^2 + 500647410215008x + 10820567498568669) \sqrt{2x^2-1} + 14641852251147 \log(\sqrt{2x^2-1} \sqrt{3-x+2x^2}) - 1407031286 \sqrt{2x^2-1} \log(\sqrt{2x^2-1} \sqrt{3-x+2x^2})$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^4,x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(10820567498568669 + 12071614275862524\*x + 50064174038215008\*x^2 + 142490931553577856\*x^3 + 257786732552566784\*x^4 + 405468382284161024\*x^5 + 485091164642279424\*x^6 + 530502956133122048\*x^7 + 439064558846345216\*x^8 + 363646430503501824\*x^9 + 204932411660697600\*x^10 + 137233466130432000\*x^11 + 37398427729920000\*x^12 + 25125558681600000\*x^13))/140703128616960 - (14641852251147\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(68719476736\*Sqrt[2])

**fricas [A]** time = 0.42, size = 118, normalized size = 0.46

$\frac{1}{1407031286} (25125558681600000x^{13} + 37398427729920000x^{12} + 137233466130432000x^{11} + 204932411660697600x^{10} + 363646430503501824x^9 + 439064558846345216x^8 + 530502956133122048x^7 + 485091164642279424x^6 + 405468382284161024x^5 + 257786732552566784x^4 + 14248093515377856x^3 + 500647410215008x^2 + 10820567498568669) \sqrt{2x^2-1} + 14641852251147 \log(\sqrt{2x^2-1} \sqrt{3-x+2x^2}) - 1407031286 \sqrt{2x^2-1} \log(\sqrt{2x^2-1} \sqrt{3-x+2x^2})$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^4,x, algorithm="fricas")

[Out] 1/140703128616960\*(25125558681600000\*x^13 + 37398427729920000\*x^12 + 137233466130432000\*x^11 + 204932411660697600\*x^10 + 363646430503501824\*x^9 + 439064558846345216\*x^8 + 530502956133122048\*x^7 + 485091164642279424\*x^6 + 405468382284161024\*x^5 + 257786732552566784\*x^4 + 142490931553577856\*x^3 + 50064174038215008\*x^2 + 12071614275862524\*x + 10820567498568669)\*sqrt(2\*x^2 - x + 3) + 14641852251147/274877906944\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac [A]** time = 0.54, size = 113, normalized size = 0.44

$\frac{1}{1407031286} (4(814(16(4(18(4(32(12(2(20(260x + 387)x + 340823)x + 1017903)x + 3612502719)x + 52340574127)x + 202370817617x + 7401903757359)x + 49495652134297)x + 125872428004183)x + 1113210402762327)x + 1564505438694219)x + 3017903568965631)x + 10820567498568669) \sqrt{2x^2-1} + 14641852251147 \log(\sqrt{2x^2-1} \sqrt{3-x+2x^2}) - 1407031286 \sqrt{2x^2-1} \log(\sqrt{2x^2-1} \sqrt{3-x+2x^2}))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^4,x, algorithm="giac")

[Out] 1/140703128616960\*(4\*(8\*(4\*(16\*(4\*(8\*(4\*(32\*(12\*(200\*(20\*(240\*(260\*x + 387)\*x + 340823)\*x + 10179103)\*x + 3612502719)\*x + 52340574127)\*x + 2023708176167)\*x + 7401903757359)\*x + 49495652134297)\*x + 125872428004183)\*x + 1113210402762327)\*x + 1564505438694219)\*x + 3017903568965631)\*x + 10820567498568669)\*sqrt(2\*x^2 - x + 3) - 14641852251147/137438953472\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple [A]** time = 0.04, size = 204, normalized size = 0.80

$\frac{625}{28} (2x^2-x+3)^{\frac{7}{2}} + \frac{13025}{208} (2x^2-x+3)^{\frac{5}{2}} + \frac{1046225}{9984} (2x^2-x+3)^{\frac{3}{2}} + \frac{3684995}{39936} (2x^2-x+3)^{\frac{1}{2}} + \frac{23460839}{532480} (2x^2-x+3)^{\frac{1}{2}} + \frac{122959007}{19169280} (2x^2-x+3)^{\frac{1}{2}} + \frac{112244125}{122683392} (2x^2-x+3)^{\frac{1}{2}} + \frac{14641852251147 \sqrt{2} \operatorname{arcsinh}\left(\frac{\sqrt{2}(x-1)}{2}\right)}{137438953472} + \frac{2520078739}{975224960} (2x^2-x+3)^{\frac{1}{2}} + \frac{636602271789}{3439973856} (4x-1) \sqrt{2x^2-x+3} + \frac{401136647}{335044320} (4x-1) (2x^2-x+3)^{\frac{1}{2}} + \frac{9238119881}{2147483648} (4x-1) (2x^2-x+3)^{\frac{3}{2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^4,x)

[Out] 625/28\*x^7\*(2\*x^2-x+3)^(7/2)+13875/208\*x^6\*(2\*x^2-x+3)^(7/2)+25250178739/5725224960\*(2\*x^2-x+3)^(7/2)+1046225/9984\*x^5\*(2\*x^2-x+3)^(7/2)+3684995/39936\*x^4\*(2\*x^2-x+3)^(7/2)+23460839/532480\*x^3\*(2\*x^2-x+3)^(7/2)+122595067/19169280\*x^2\*(2\*x^2-x+3)^(7/2)+112244125/122683392\*x\*(2\*x^2-x+3)^(7/2)+14641852251147/137438953472\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+636602271789/34359738368\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)+401135647/335544320\*(4\*x-1)\*(2\*x^2-x+3)^(5/2)+9226119881/2147483648\*(4\*x-1)\*(2\*x^2-x+3)^(3/2)

**maxima** [A] time = 1.02, size = 235, normalized size = 0.93

$\frac{625}{28} (2x^2 - x + 3)^{\frac{7}{2}} + \frac{13875}{208} (2x^2 - x + 3)^{\frac{7}{2}} x + \frac{46225}{9984} (2x^2 - x + 3)^{\frac{7}{2}} x^2 + \frac{3684995}{39936} (2x^2 - x + 3)^{\frac{7}{2}} x^3 + \frac{23460839}{532480} (2x^2 - x + 3)^{\frac{7}{2}} x^4 + \frac{122595067}{19169280} (2x^2 - x + 3)^{\frac{7}{2}} x^5 + \frac{112244125}{122683392} (2x^2 - x + 3)^{\frac{7}{2}} x^6 + \frac{25250178739}{5725224960} (2x^2 - x + 3)^{\frac{7}{2}} x^7 + \frac{401135647}{83886080} (2x^2 - x + 3)^{\frac{5}{2}} x - \frac{401135647}{335544320} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{9226119881}{536870912} (2x^2 - x + 3)^{\frac{3}{2}} x - \frac{9226119881}{2147483648} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{636602271789}{8589934592} \sqrt{2x^2 - x + 3} x + \frac{14641852251147}{137438953472} \sqrt{2} \operatorname{arcsinh}\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{636602271789}{34359738368} \sqrt{2x^2 - x + 3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^4,x, algorithm="maxima")

[Out] 625/28\*(2\*x^2 - x + 3)^(7/2)\*x^7 + 13875/208\*(2\*x^2 - x + 3)^(7/2)\*x^6 + 1046225/9984\*(2\*x^2 - x + 3)^(7/2)\*x^5 + 3684995/39936\*(2\*x^2 - x + 3)^(7/2)\*x^4 + 23460839/532480\*(2\*x^2 - x + 3)^(7/2)\*x^3 + 122595067/19169280\*(2\*x^2 - x + 3)^(7/2)\*x^2 + 112244125/122683392\*(2\*x^2 - x + 3)^(7/2)\*x + 25250178739/5725224960\*(2\*x^2 - x + 3)^(7/2) + 401135647/83886080\*(2\*x^2 - x + 3)^(5/2)\*x - 401135647/335544320\*(2\*x^2 - x + 3)^(5/2) + 9226119881/536870912\*(2\*x^2 - x + 3)^(3/2)\*x - 9226119881/2147483648\*(2\*x^2 - x + 3)^(3/2) + 636602271789/8589934592\*sqrt(2\*x^2 - x + 3)\*x + 14641852251147/137438953472\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 636602271789/34359738368\*sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (2x^2 - x + 3)^{5/2} (5x^2 + 3x + 2)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(5/2)\*(3\*x + 5\*x^2 + 2)^4,x)

[Out] int((2\*x^2 - x + 3)^(5/2)\*(3\*x + 5\*x^2 + 2)^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(5/2)\*(5\*x\*\*2+3\*x+2)\*\*4,x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(5/2)\*(5\*x\*\*2 + 3\*x + 2)\*\*4, x)

$$3.70 \quad \int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2)^3 dx$$

**Optimal.** Leaf size=212

$$\frac{80483(2x^2 - x + 3)^{7/2} x^2}{9216} + \frac{509257(2x^2 - x + 3)^{7/2} x}{294912} - \frac{1696165(2x^2 - x + 3)^{7/2}}{2752512} - \frac{57915(1 - 4x)(2x^2 - x + 3)^{5/2}}{2097152}$$

**Rubi [A]** time = 0.22, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 27, number of rules / integrand size = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{24}(2x^2 - x + 3)^{7/2} x^2 + \frac{1175}{96}(2x^2 - x + 3)^{7/2} x + \frac{3823}{256}(2x^2 - x + 3)^{7/2} x^2 + \frac{80483(2x^2 - x + 3)^{7/2} x^2}{9216} + \frac{509257(2x^2 - x + 3)^{7/2} x}{294912} - \frac{1696165(2x^2 - x + 3)^{7/2}}{2752512} - \frac{57915(1 - 4x)(2x^2 - x + 3)^{5/2}}{2097152} - \frac{6660225(1 - 4x)(2x^2 - x + 3)^{3/2}}{67108864} - \frac{459555525(1 - 4x)\sqrt{2x^2 - x + 3}}{1073741824} - \frac{10569777075 \operatorname{arcsinh}\left(\frac{1 - 4x}{\sqrt{23}}\right)}{2147483648\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] (-459555525\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/1073741824 - (6660225\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(3/2))/67108864 - (57915\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(5/2))/2097152 - (1696165\*(3 - x + 2\*x^2)^(7/2))/2752512 + (509257\*x\*(3 - x + 2\*x^2)^(7/2))/294912 + (80483\*x^2\*(3 - x + 2\*x^2)^(7/2))/9216 + (3823\*x^3\*(3 - x + 2\*x^2)^(7/2))/256 + (1175\*x^4\*(3 - x + 2\*x^2)^(7/2))/96 + (125\*x^5\*(3 - x + 2\*x^2)^(7/2))/24 - (10569777075\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(2147483648\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^3 dx &= \frac{125}{24}x^5(3-x+2x^2)^{7/2} + \frac{1}{24} \int (3-x+2x^2)^{5/2} (192+864x+2736x^2- \\
&= \frac{1175}{96}x^4(3-x+2x^2)^{7/2} + \frac{125}{24}x^5(3-x+2x^2)^{7/2} + \frac{1}{528} \int (3-x+2x^2)^{5/2} \\
&= \frac{3823}{256}x^3(3-x+2x^2)^{7/2} + \frac{1175}{96}x^4(3-x+2x^2)^{7/2} + \frac{125}{24}x^5(3-x+2x^2)^{7/2} \\
&= \frac{80483x^2(3-x+2x^2)^{7/2}}{9216} + \frac{3823}{256}x^3(3-x+2x^2)^{7/2} + \frac{1175}{96}x^4(3-x+2x^2)^{7/2} \\
&= \frac{509257x(3-x+2x^2)^{7/2}}{294912} + \frac{80483x^2(3-x+2x^2)^{7/2}}{9216} + \frac{3823}{256}x^3(3-x+2x^2)^{7/2} \\
&= -\frac{1696165(3-x+2x^2)^{7/2}}{2752512} + \frac{509257x(3-x+2x^2)^{7/2}}{294912} + \frac{80483x^2(3-x+2x^2)^{7/2}}{9216} \\
&= -\frac{57915(1-4x)(3-x+2x^2)^{5/2}}{2097152} - \frac{1696165(3-x+2x^2)^{7/2}}{2752512} + \frac{509257x(3-x+2x^2)^{7/2}}{294912} \\
&= -\frac{6660225(1-4x)(3-x+2x^2)^{3/2}}{67108864} - \frac{57915(1-4x)(3-x+2x^2)^{5/2}}{2097152} + \frac{509257x(3-x+2x^2)^{7/2}}{294912} \\
&= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x)(3-x+2x^2)^{3/2}}{67108864} + \frac{509257x(3-x+2x^2)^{7/2}}{294912} \\
&= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x)(3-x+2x^2)^{3/2}}{67108864} + \frac{509257x(3-x+2x^2)^{7/2}}{294912} \\
&= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x)(3-x+2x^2)^{3/2}}{67108864} + \frac{509257x(3-x+2x^2)^{7/2}}{294912}
\end{aligned}$$

**Mathematica [A]** time = 0.30, size = 95, normalized size = 0.45

$$\frac{4\sqrt{2x^2-x+3} (2818572288000x^{11} + 2395786444800x^{10} + 12943588589568x^9 + 14341894045696x^8 + 27835561148416x^7 + 28347538538496x^6 + 34378613923840x^5 + 26186527209472x^4 + 20384824684416x^3 + 10060731582048x^2 + 4560943728924x - 1191399152715) - 665895955725\sqrt{2} \operatorname{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]}{270582939648}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] (4\*sqrt[3 - x + 2\*x^2]\*(-1191399152715 + 4560943728924\*x + 10060731582048\*x^2 + 20384824684416\*x^3 + 26186527209472\*x^4 + 34378613923840\*x^5 + 28347538538496\*x^6 + 27835561148416\*x^7 + 14341894045696\*x^8 + 12943588589568\*x^9 + 2395786444800\*x^10 + 2818572288000\*x^11) - 665895955725\*sqrt[2]\*ArcSinh[(1 - 4\*x)/sqrt[23]])/270582939648

**IntegrateAlgebraic [A]** time = 1.34, size = 110, normalized size = 0.52

$$\frac{\sqrt{2x^2-x+3} (2818572288000x^{11} + 2395786444800x^{10} + 12943588589568x^9 + 14341894045696x^8 + 27835561148416x^7 + 28347538538496x^6 + 34378613923840x^5 + 26186527209472x^4 + 20384824684416x^3 + 10060731582048x^2 + 4560943728924x - 1191399152715) \log\left[2\sqrt{2}\sqrt{2x^2-x+3} - 4x+1\right]}{2147483648\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^3,x]

[Out] (sqrt[3 - x + 2\*x^2]\*(-1191399152715 + 4560943728924\*x + 10060731582048\*x^2 + 20384824684416\*x^3 + 26186527209472\*x^4 + 34378613923840\*x^5 + 28347538538496\*x^6 + 27835561148416\*x^7 + 14341894045696\*x^8 + 12943588589568\*x^9 + 2395786444800\*x^10 + 2818572288000\*x^11) \log[2\sqrt{2}\sqrt{2x^2-x+3} - 4x+1])/2147483648\sqrt{2}

38496\*x^6 + 27835561148416\*x^7 + 14341894045696\*x^8 + 12943588589568\*x^9 + 2395786444800\*x^10 + 2818572288000\*x^11)/67645734912 - (10569777075\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(2147483648\*Sqrt[2])

**fricas** [A] time = 0.42, size = 108, normalized size = 0.51

$\frac{1}{67645734912} (2818572288000x^{11} + 2395786444800x^{10} + 12943588589568x^9 + 14341894045696x^8 + 27835561148416x^7 + 28347538538496x^6 + 34378613923840x^5 + 26186527209472x^4 + 20384824684416x^3 + 10060731582048x^2 + 4560943728924x - 1191399152715)\sqrt{2x^2 - x + 3} - \frac{1056977075}{858993452}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/67645734912\*(2818572288000\*x^11 + 2395786444800\*x^10 + 12943588589568\*x^9 + 14341894045696\*x^8 + 27835561148416\*x^7 + 28347538538496\*x^6 + 34378613923840\*x^5 + 26186527209472\*x^4 + 20384824684416\*x^3 + 10060731582048\*x^2 + 4560943728924\*x - 1191399152715)\*sqrt(2\*x^2 - x + 3) + 10569777075/8589934592\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.55, size = 103, normalized size = 0.49

$\frac{1}{67645734912} (4(8(4(16(4(8(28(32(12(200(20x + 17)x + 18369)x + 244241)x + 15169177)x + 432549111)x + 4196608145)x + 12786390239)x + 159256442847)x + 314397861939)x + 1140235932231)x - 1191399152715)\sqrt{2x^2 - x + 3} - \frac{1056977075}{4294967296}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2x^2 - x + 3}\right) + 1\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] 1/67645734912\*(4\*(8\*(4\*(16\*(4\*(8\*(28\*(32\*(12\*(200\*(20\*x + 17)\*x + 18369)\*x + 244241)\*x + 15169177)\*x + 432549111)\*x + 4196608145)\*x + 12786390239)\*x + 159256442847)\*x + 314397861939)\*x + 1140235932231)\*x - 1191399152715)\*sqrt(2\*x^2 - x + 3) - 10569777075/4294967296\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 170, normalized size = 0.80

$\frac{125(2x^2 - x + 3)^{5/2}}{24} + \frac{1175(2x^2 - x + 3)^{3/2}}{96} + \frac{3823(2x^2 - x + 3)^{1/2}}{256} + \frac{80483(2x^2 - x + 3)^{3/2}}{9216} + \frac{509257(2x^2 - x + 3)^{5/2}}{294912} + \frac{1056977075\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{2}(x-1)}{23}\right)}{4294967296} - \frac{1696165(2x^2 - x + 3)^{7/2}}{2752512} + \frac{459555525(4x-1)\sqrt{2x^2 - x + 3}}{1073741824} + \frac{57915(4x-1)(2x^2 - x + 3)^{3/2}}{2097152} + \frac{6660225(4x-1)(2x^2 - x + 3)^{5/2}}{67108864}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^3,x)

[Out] -1696165/2752512\*(2\*x^2-x+3)^(7/2)+125/24\*(2\*x^2-x+3)^(7/2)\*x^5+1175/96\*(2\*x^2-x+3)^(7/2)\*x^4+3823/256\*(2\*x^2-x+3)^(7/2)\*x^3+80483/9216\*(2\*x^2-x+3)^(7/2)\*x^2+509257/294912\*(2\*x^2-x+3)^(7/2)\*x+10569777075/4294967296\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+459555525/1073741824\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)+57915/2097152\*(4\*x-1)\*(2\*x^2-x+3)^(5/2)+6660225/67108864\*(4\*x-1)\*(2\*x^2-x+3)^(3/2)

**maxima** [A] time = 1.00, size = 201, normalized size = 0.95

$\frac{125}{24}(2x^2 - x + 3)^{5/2} + \frac{1175}{96}(2x^2 - x + 3)^{3/2} + \frac{3823}{256}(2x^2 - x + 3)^{1/2} + \frac{80483}{9216}(2x^2 - x + 3)^{3/2} + \frac{509257}{294912}(2x^2 - x + 3)^{5/2} - \frac{1696165}{2752512}(2x^2 - x + 3)^{7/2} + \frac{57915}{1073741824}(2x^2 - x + 3)^{1/2} - \frac{57915}{2097152}(2x^2 - x + 3)^{5/2} + \frac{6660225}{67108864}(2x^2 - x + 3)^{3/2} + \frac{459555525}{268435456}\sqrt{2x^2 - x + 3} + \frac{1056977075}{4294967296}\sqrt{2}\operatorname{arcsinh}\left(\frac{4}{23}\sqrt{23}(4x-1)\right) - \frac{459555525}{1073741824}\sqrt{2x^2 - x + 3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] 125/24\*(2\*x^2 - x + 3)^(7/2)\*x^5 + 1175/96\*(2\*x^2 - x + 3)^(7/2)\*x^4 + 3823/256\*(2\*x^2 - x + 3)^(7/2)\*x^3 + 80483/9216\*(2\*x^2 - x + 3)^(7/2)\*x^2 + 509257/294912\*(2\*x^2 - x + 3)^(7/2)\*x - 1696165/2752512\*(2\*x^2 - x + 3)^(7/2) + 57915/524288\*(2\*x^2 - x + 3)^(5/2)\*x - 57915/2097152\*(2\*x^2 - x + 3)^(5/2) + 6660225/16777216\*(2\*x^2 - x + 3)^(3/2)\*x - 6660225/67108864\*(2\*x^2 - x + 3)^(3/2) + 459555525/268435456\*sqrt(2\*x^2 - x + 3)\*x + 10569777075/429496

$7296\sqrt{2}\operatorname{arcsinh}(1/23\sqrt{23}(4x - 1)) - 45955525/1073741824\sqrt{2x^2 - x + 3}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (2x^2 - x + 3)^{5/2} (5x^2 + 3x + 2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3,x)`

[Out] `int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**3,x)`

[Out] `Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3, x)`



$$3.71 \quad \int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2)^2 dx$$

Optimal. Leaf size=170

$$\frac{305}{144}x^2(2x^2 - x + 3)^{7/2} + \frac{8467x(2x^2 - x + 3)^{7/2}}{4608} + \frac{23225(2x^2 - x + 3)^{7/2}}{43008} - \frac{1547(1 - 4x)(2x^2 - x + 3)^{5/2}}{98304} - \frac{177905(1 - 4x)^2(2x^2 - x + 3)^{3/2}}{3145728} - \frac{4091815(1 - 4x)\sqrt{2x^2 - x + 3}}{16777216} - \frac{9411745 \operatorname{arcsinh}\left(\frac{1 - 4x}{\sqrt{23}}\right)}{33554432\sqrt{2}}$$

Rubi [A] time = 0.13, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{4}x^3(2x^2 - x + 3)^{7/2} + \frac{305}{144}x^2(2x^2 - x + 3)^{7/2} + \frac{8467x(2x^2 - x + 3)^{7/2}}{4608} + \frac{23225(2x^2 - x + 3)^{7/2}}{43008} - \frac{1547(1 - 4x)(2x^2 - x + 3)^{5/2}}{98304} - \frac{177905(1 - 4x)^2(2x^2 - x + 3)^{3/2}}{3145728} - \frac{4091815(1 - 4x)\sqrt{2x^2 - x + 3}}{16777216} - \frac{9411745 \operatorname{arcsinh}\left(\frac{1 - 4x}{\sqrt{23}}\right)}{33554432\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] (-4091815\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/16777216 - (177905\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(3/2))/3145728 - (1547\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(5/2))/98304 + (23225\*(3 - x + 2\*x^2)^(7/2))/43008 + (8467\*x\*(3 - x + 2\*x^2)^(7/2))/4608 + (305\*x^2\*(3 - x + 2\*x^2)^(7/2))/144 + (5\*x^3\*(3 - x + 2\*x^2)^(7/2))/4 - (9411745\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(33554432\*Sqrt[2])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^2 dx &= \frac{5}{4}x^3(3-x+2x^2)^{7/2} + \frac{1}{20} \int (3-x+2x^2)^{5/2} (80+240x+355x^2+15x^3) dx \\
&= \frac{305}{144}x^2(3-x+2x^2)^{7/2} + \frac{5}{4}x^3(3-x+2x^2)^{7/2} + \frac{1}{360} \int (3-x+2x^2)^{5/2} (80+240x+355x^2+15x^3) dx \\
&= \frac{8467x(3-x+2x^2)^{7/2}}{4608} + \frac{305}{144}x^2(3-x+2x^2)^{7/2} + \frac{5}{4}x^3(3-x+2x^2)^{7/2} \\
&= \frac{23225(3-x+2x^2)^{7/2}}{43008} + \frac{8467x(3-x+2x^2)^{7/2}}{4608} + \frac{305}{144}x^2(3-x+2x^2)^{7/2} \\
&= -\frac{1547(1-4x)(3-x+2x^2)^{5/2}}{98304} + \frac{23225(3-x+2x^2)^{7/2}}{43008} + \frac{8467x(3-x+2x^2)^{7/2}}{4608} \\
&= -\frac{177905(1-4x)(3-x+2x^2)^{3/2}}{3145728} - \frac{1547(1-4x)(3-x+2x^2)^{5/2}}{98304} + \frac{23225(3-x+2x^2)^{7/2}}{43008} \\
&= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216} - \frac{177905(1-4x)(3-x+2x^2)^{3/2}}{3145728} - \frac{1547(1-4x)(3-x+2x^2)^{5/2}}{98304} + \frac{23225(3-x+2x^2)^{7/2}}{43008} \\
&= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216} - \frac{177905(1-4x)(3-x+2x^2)^{3/2}}{3145728} - \frac{1547(1-4x)(3-x+2x^2)^{5/2}}{98304} + \frac{23225(3-x+2x^2)^{7/2}}{43008} \\
&= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216} - \frac{177905(1-4x)(3-x+2x^2)^{3/2}}{3145728} - \frac{1547(1-4x)(3-x+2x^2)^{5/2}}{98304} + \frac{23225(3-x+2x^2)^{7/2}}{43008}
\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 85, normalized size = 0.50

$$\frac{4\sqrt{2x^2-x+3}(10569646080x^9+2055208960x^8+44163137536x^7+26401898496x^6+75389820928x^5+57147467776x^4+77872272000x^3+42992644128x^2+39533249652x+14824182519)-5929039935\sqrt{2}\operatorname{ArcSinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4227858432}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] (4\*sqrt[3 - x + 2\*x^2]\*(14824182519 + 39533249652\*x + 42992644128\*x^2 + 77872272000\*x^3 + 57147467776\*x^4 + 75389820928\*x^5 + 26401898496\*x^6 + 44163137536\*x^7 + 2055208960\*x^8 + 10569646080\*x^9) - 5929039935\*sqrt[2]\*ArcSinh[(1 - 4\*x)/sqrt[23]])/4227858432

**IntegrateAlgebraic [A]** time = 1.05, size = 100, normalized size = 0.59

$$\frac{\sqrt{2x^2-x+3}(10569646080x^9+2055208960x^8+44163137536x^7+26401898496x^6+75389820928x^5+57147467776x^4+77872272000x^3+42992644128x^2+39533249652x+14824182519)-94111745\log\left(\frac{2\sqrt{2x^2-x+3}-4x+1}{\sqrt{23}}\right)}{1056964608 \cdot 33554432\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^2,x]

[Out] (sqrt[3 - x + 2\*x^2]\*(14824182519 + 39533249652\*x + 42992644128\*x^2 + 77872272000\*x^3 + 57147467776\*x^4 + 75389820928\*x^5 + 26401898496\*x^6 + 44163137536\*x^7 + 2055208960\*x^8 + 10569646080\*x^9))/1056964608 - (94111745\*Log[1 - 4\*x + 2\*sqrt[2]\*sqrt[3 - x + 2\*x^2]])/(33554432\*sqrt[2])

**fricas [A]** time = 0.41, size = 98, normalized size = 0.58

$$\frac{1}{1056964608}(10569646080x^9+2055208960x^8+44163137536x^7+26401898496x^6+75389820928x^5+57147467776x^4+77872272000x^3+42992644128x^2+39533249652x+14824182519)\sqrt{2x^2-x+3}+\frac{94111745}{134217728}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^2,x, algorithm="fricas")

[Out] 1/1056964608\*(10569646080\*x^9 + 2055208960\*x^8 + 44163137536\*x^7 + 26401898496\*x^6 + 75389820928\*x^5 + 57147467776\*x^4 + 77872272000\*x^3 + 42992644128\*x^2 + 39533249652\*x + 14824182519)\*sqrt(2\*x^2 - x + 3) + 94111745/134217728\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.52, size = 93, normalized size = 0.55

$$\frac{1}{1056964608} (4 (8 (4 (16 (4 (8 (28 (160 (36 x + 7) x + 24067) x + 402861) x + 9202859) x + 27904037) x + 608377125) x + 1343520129) x + 9883312413) x + 14824182519) \sqrt{2x^2 - x + 3} - \frac{94111745}{67108864} \sqrt{2} \log(-2\sqrt{2}(\sqrt{2x - \sqrt{2x^2 - x + 3}}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] 1/1056964608\*(4\*(8\*(4\*(16\*(4\*(8\*(28\*(160\*(36\*x + 7)\*x + 24067)\*x + 402861)\*x + 9202859)\*x + 27904037)\*x + 608377125)\*x + 1343520129)\*x + 9883312413)\*x + 14824182519)\*sqrt(2\*x^2 - x + 3) - 94111745/67108864\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 136, normalized size = 0.80

$$\frac{5(2x^2-x+3)^{\frac{7}{2}}x^3}{4} + \frac{305(2x^2-x+3)^{\frac{7}{2}}x^2}{144} + \frac{8467(2x^2-x+3)^{\frac{7}{2}}x}{4608} + \frac{94111745\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}(x-1)}{23}\right)}{67108864} + \frac{23225(2x^2-x+3)^{\frac{7}{2}}}{43008} + \frac{4091815(4x-1)\sqrt{2x^2-x+3}}{16777216} + \frac{1547(4x-1)(2x^2-x+3)^{\frac{5}{2}}}{98304} + \frac{177905(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{3145728}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^2,x)

[Out] 23225/43008\*(2\*x^2-x+3)^(7/2)+5/4\*(2\*x^2-x+3)^(7/2)\*x^3+305/144\*(2\*x^2-x+3)^(7/2)\*x^2+8467/4608\*(2\*x^2-x+3)^(7/2)\*x+94111745/67108864\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+4091815/16777216\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)+1547/98304\*(4\*x-1)\*(2\*x^2-x+3)^(5/2)+177905/3145728\*(4\*x-1)\*(2\*x^2-x+3)^(3/2)

**maxima** [A] time = 1.03, size = 167, normalized size = 0.98

$$\frac{5}{4}(2x^2-x+3)^{\frac{7}{2}}x^3 + \frac{305}{144}(2x^2-x+3)^{\frac{7}{2}}x^2 + \frac{8467}{4608}(2x^2-x+3)^{\frac{7}{2}}x + \frac{23225}{43008}(2x^2-x+3)^{\frac{7}{2}} + \frac{1547}{24576}(2x^2-x+3)^{\frac{5}{2}} + \frac{1547}{98304}(2x^2-x+3)^{\frac{5}{2}} + \frac{177905}{786432}(2x^2-x+3)^{\frac{5}{2}} - \frac{177905}{3145728}(2x^2-x+3)^{\frac{3}{2}} + \frac{4091815}{4194304}\sqrt{2x^2-x+3} + \frac{94111745}{67108864}\sqrt{2} \operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{4091815}{16777216}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] 5/4\*(2\*x^2 - x + 3)^(7/2)\*x^3 + 305/144\*(2\*x^2 - x + 3)^(7/2)\*x^2 + 8467/4608\*(2\*x^2 - x + 3)^(7/2)\*x + 23225/43008\*(2\*x^2 - x + 3)^(7/2) + 1547/24576\*(2\*x^2 - x + 3)^(5/2)\*x - 1547/98304\*(2\*x^2 - x + 3)^(5/2) + 177905/786432\*(2\*x^2 - x + 3)^(3/2)\*x - 177905/3145728\*(2\*x^2 - x + 3)^(3/2) + 4091815/4194304\*sqrt(2\*x^2 - x + 3)\*x + 94111745/67108864\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 4091815/16777216\*sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x^2 - x + 3)^{5/2} (5x^2 + 3x + 2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(5/2)\*(3\*x + 5\*x^2 + 2)^2,x)

[Out] int((2\*x^2 - x + 3)^(5/2)\*(3\*x + 5\*x^2 + 2)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{5/2} (5x^2 + 3x + 2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2, x)
```

$$3.72 \quad \int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2) dx$$

Optimal. Leaf size=128

$$\frac{5}{16}x(2x^2 - x + 3)^{7/2} + \frac{141}{448}(2x^2 - x + 3)^{7/2} - \frac{277(1 - 4x)(2x^2 - x + 3)^{5/2}}{3072} - \frac{31855(1 - 4x)(2x^2 - x + 3)^{3/2}}{98304} - \frac{732665(1 - 4x)\sqrt{2x^2 - x + 3}}{524288} - \frac{16851295 \operatorname{arcsinh}\left(\frac{1 - 4x}{\sqrt{23}}\right)}{1048576\sqrt{2}}$$

**Rubi [A]** time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{16}x(2x^2 - x + 3)^{7/2} + \frac{141}{448}(2x^2 - x + 3)^{7/2} - \frac{277(1 - 4x)(2x^2 - x + 3)^{5/2}}{3072} - \frac{31855(1 - 4x)(2x^2 - x + 3)^{3/2}}{98304} - \frac{732665(1 - 4x)\sqrt{2x^2 - x + 3}}{524288} - \frac{16851295 \operatorname{arcsinh}\left(\frac{1 - 4x}{\sqrt{23}}\right)}{1048576\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2), x]

[Out] (-732665\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/524288 - (31855\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(3/2))/98304 - (277\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(5/2))/3072 + (141\*(3 - x + 2\*x^2)^(7/2))/448 + (5\*x\*(3 - x + 2\*x^2)^(7/2))/16 - (16851295\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(1048576\*Sqrt[2])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{5/2} (2+3x+5x^2) dx &= \frac{5}{16}x(3-x+2x^2)^{7/2} + \frac{1}{16} \int \left(17 + \frac{141x}{2}\right) (3-x+2x^2)^{5/2} dx \\
&= \frac{141}{448} (3-x+2x^2)^{7/2} + \frac{5}{16}x(3-x+2x^2)^{7/2} + \frac{277}{128} \int (3-x+2x^2)^{5/2} dx \\
&= -\frac{277(1-4x)(3-x+2x^2)^{5/2}}{3072} + \frac{141}{448} (3-x+2x^2)^{7/2} + \frac{5}{16}x(3-x+2x^2)^{7/2} \\
&= -\frac{31855(1-4x)(3-x+2x^2)^{3/2}}{98304} - \frac{277(1-4x)(3-x+2x^2)^{5/2}}{3072} + \frac{141}{448} (3-x+2x^2)^{7/2} \\
&= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288} - \frac{31855(1-4x)(3-x+2x^2)^{3/2}}{98304} - \frac{277(1-4x)(3-x+2x^2)^{5/2}}{3072} \\
&= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288} - \frac{31855(1-4x)(3-x+2x^2)^{3/2}}{98304} - \frac{277(1-4x)(3-x+2x^2)^{5/2}}{3072} \\
&= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288} - \frac{31855(1-4x)(3-x+2x^2)^{3/2}}{98304} - \frac{277(1-4x)(3-x+2x^2)^{5/2}}{3072}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 75, normalized size = 0.59

$$\frac{4\sqrt{2x^2-x+3} (27525120x^7 - 13565952x^6 + 118808576x^5 - 1619968x^4 + 172684416x^3 + 67272352x^2 + 148957444x + 58536675) - 353877195\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{44040192}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2), x]

[Out] (4\*sqrt(3 - x + 2\*x^2)\*(58536675 + 148957444\*x + 67272352\*x^2 + 172684416\*x^3 - 1619968\*x^4 + 118808576\*x^5 - 13565952\*x^6 + 27525120\*x^7) - 353877195\*sqrt(2)\*ArcSinh[(1 - 4\*x)/sqrt(23)])/44040192

**IntegrateAlgebraic [A]** time = 0.84, size = 90, normalized size = 0.70

$$\frac{\sqrt{2x^2-x+3} (27525120x^7 - 13565952x^6 + 118808576x^5 - 1619968x^4 + 172684416x^3 + 67272352x^2 + 148957444x + 58536675)}{11010048} - \frac{16851295 \log(2\sqrt{2}\sqrt{2x^2-x+3} - 4x + 1)}{1048576\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2), x]

[Out] (sqrt(3 - x + 2\*x^2)\*(58536675 + 148957444\*x + 67272352\*x^2 + 172684416\*x^3 - 1619968\*x^4 + 118808576\*x^5 - 13565952\*x^6 + 27525120\*x^7))/11010048 - (16851295\*Log[1 - 4\*x + 2\*sqrt(2)\*sqrt(3 - x + 2\*x^2)])/(1048576\*sqrt(2))

**fricas [A]** time = 0.41, size = 88, normalized size = 0.69

$$\frac{1}{11010048} (27525120x^7 - 13565952x^6 + 118808576x^5 - 1619968x^4 + 172684416x^3 + 67272352x^2 + 148957444x + 58536675) \sqrt{2x^2-x+3} + \frac{16851295}{4194304} \sqrt{2} \log(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2), x, algorithm="fricas")

[Out] 1/11010048\*(27525120\*x^7 - 13565952\*x^6 + 118808576\*x^5 - 1619968\*x^4 + 172684416\*x^3 + 67272352\*x^2 + 148957444\*x + 58536675)\*sqrt(2\*x^2 - x + 3) + 16851295/4194304\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.39, size = 83, normalized size = 0.65

$$\frac{1}{11010048} (4 (8 (4 (16 (4 (24 (140 x - 69) x + 14503) x - 791) x + 1349097) x + 2102261) x + 37239361) x + 58536675) \sqrt{2x^2 - x + 3} - \frac{16851295}{2097152} \sqrt{2} \log(-2\sqrt{2}(\sqrt{2x - \sqrt{2x^2 - x + 3}}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] 1/11010048\*(4\*(8\*(4\*(16\*(4\*(24\*(140\*x - 69)\*x + 14503)\*x - 791)\*x + 1349097)\*x + 2102261)\*x + 37239361)\*x + 58536675)\*sqrt(2\*x^2 - x + 3) - 16851295/2097152\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 102, normalized size = 0.80

$$\frac{5(2x^2 - x + 3)^{\frac{7}{2}} x}{16} + \frac{16851295\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2097152} + \frac{141(2x^2 - x + 3)^{\frac{7}{2}}}{448} + \frac{277(4x - 1)(2x^2 - x + 3)^{\frac{5}{2}}}{3072} + \frac{31855(4x - 1)(2x^2 - x + 3)^{\frac{3}{2}}}{98304} + \frac{732665(4x - 1)\sqrt{2x^2 - x + 3}}{524288}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2),x)

[Out] 5/16\*(2\*x^2-x+3)^(7/2)\*x+141/448\*(2\*x^2-x+3)^(7/2)+277/3072\*(4\*x-1)\*(2\*x^2-x+3)^(5/2)+31855/98304\*(4\*x-1)\*(2\*x^2-x+3)^(3/2)+732665/524288\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)+16851295/2097152\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))

**maxima** [A] time = 0.97, size = 133, normalized size = 1.04

$$\frac{5}{16}(2x^2 - x + 3)^{\frac{7}{2}} x + \frac{141}{448}(2x^2 - x + 3)^{\frac{7}{2}} + \frac{277}{768}(2x^2 - x + 3)^{\frac{5}{2}} x - \frac{277}{3072}(2x^2 - x + 3)^{\frac{5}{2}} + \frac{31855}{24576}(2x^2 - x + 3)^{\frac{3}{2}} x - \frac{31855}{98304}(2x^2 - x + 3)^{\frac{3}{2}} + \frac{732665}{131072}\sqrt{2x^2 - x + 3} x + \frac{16851295}{2097152}\sqrt{2} \operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - \frac{732665}{524288}\sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)\*(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] 5/16\*(2\*x^2 - x + 3)^(7/2)\*x + 141/448\*(2\*x^2 - x + 3)^(7/2) + 277/768\*(2\*x^2 - x + 3)^(5/2)\*x - 277/3072\*(2\*x^2 - x + 3)^(5/2) + 31855/24576\*(2\*x^2 - x + 3)^(3/2)\*x - 31855/98304\*(2\*x^2 - x + 3)^(3/2) + 732665/131072\*sqrt(2\*x^2 - x + 3)\*x + 16851295/2097152\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 732665/524288\*sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x^2 - x + 3)^{5/2} (5x^2 + 3x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(5/2)\*(3\*x + 5\*x^2 + 2),x)

[Out] int((2\*x^2 - x + 3)^(5/2)\*(3\*x + 5\*x^2 + 2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(5/2)\*(5\*x\*\*2+3\*x+2),x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(5/2)\*(5\*x\*\*2 + 3\*x + 2), x)

$$3.73 \quad \int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2} dx$$

**Optimal.** Leaf size=222

$$-\frac{1}{600}(103-60x)(2x^2-x+3)^{3/2} - \frac{(226249-99620x)\sqrt{2x^2-x+3}}{80000} - \frac{121\sqrt{\frac{11}{31}(25000\sqrt{2}-15457)}}{3125} \tan^{-1}\left(\frac{\sqrt{\frac{62(25000\sqrt{2}-15457)}{31}}}{\sqrt{2x^2-x+3}}\right)$$

**Rubi [A]** time = 0.54, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {977, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{1}{600}(103-60x)(2x^2-x+3)^{3/2} - \frac{(226249-99620x)\sqrt{2x^2-x+3}}{80000} - \frac{121\sqrt{\frac{11}{31}(25000\sqrt{2}-15457)}}{3125} \tan^{-1}\left(\frac{\sqrt{\frac{62(25000\sqrt{2}-15457)}{31}}}{\sqrt{2x^2-x+3}}\right) + \frac{121\sqrt{\frac{11}{31}(15457+25000\sqrt{2})}}{3125} \tanh^{-1}\left(\frac{\sqrt{\frac{62(15457+25000\sqrt{2})}{31}}}{\sqrt{2x^2-x+3}}\right) - \frac{7216203 \operatorname{sinh}^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{80000\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(5/2)/(2 + 3\*x + 5\*x^2), x]

[Out] -((226249 - 99620\*x)\*Sqrt[3 - x + 2\*x^2])/80000 - ((103 - 60\*x)\*(3 - x + 2\*x^2)^(3/2))/600 - (7216203\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(80000\*Sqrt[2]) - (121\*Sqrt[(11\*(-15457 + 25000\*Sqrt[2]))/31]\*ArcTan[(Sqrt[11/(62\*(-15457 + 25000\*Sqrt[2]))])\*(196 - 443\*Sqrt[2] - (690 + 247\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/3125 + (121\*Sqrt[(11\*(15457 + 25000\*Sqrt[2]))/31]\*ArcTanh[(Sqrt[11/(62\*(15457 + 25000\*Sqrt[2]))])\*(196 + 443\*Sqrt[2] - (690 - 247\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2])/3125

#### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 977

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b\*f\*(3\*p + 2\*q) - c\*e\*(2\*p + q) + 2\*c\*f\*(p + q)\*x)\*(a + b\*x + c\*x^2)^(p - 1)\*(d + e\*x + f\*x^2)^(q + 1))/(2\*f^2\*(p + q)\*(2\*p + 2\*q + 1)), x] - Dist[1/(2\*f^2\*(p + q)\*(2\*p + 2\*q + 1)), Int[(a + b\*x + c\*x^2)^(p - 2)\*(d + e\*x + f\*x^2)^q\*Simp[(b\*d - a\*e)\*(c\*e - b\*f)\*(1 - p)\*(2\*p + q) - (p + q)\*(b^2\*d\*f\*(1 - p) - a\*(f\*(b\*e - 2\*a\*f)\*(2\*p + 2\*q + 1))



+ c\*(2\*d\*f - e^2\*(2\*p + q)))) + (2\*(c\*d - a\*f)\*(c\*e - b\*f)\*(1 - p)\*(2\*p + q) - (p + q)\*((b^2 - 4\*a\*c)\*e\*f\*(1 - p) + b\*(c\*(e^2 - 4\*d\*f)\*(2\*p + q) + f\*(2\*c\*d - b\*e + 2\*a\*f)\*(2\*p + 2\*q + 1))))\*x + ((c\*e - b\*f)^2\*(1 - p)\*p + c\*(p + q)\*(f\*(b\*e - 2\*a\*f)\*(4\*p + 2\*q - 1) - c\*(2\*d\*f\*(1 - 2\*p) + e^2\*(3\*p + q - 1))))\*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2\*p + 2\*q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

### Rule 1029

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

### Rule 1035

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 2]}, Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f - q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f + q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f + q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f - q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && NegQ[b^2 - 4\*a\*c]

### Rule 1066

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((B\*c\*f\*(2\*p + 2\*q + 3) + C\*(b\*f\*p - c\*e\*(2\*p + q + 2)) + 2\*c\*C\*f\*(p + q + 1)\*x)\*(a + b\*x + c\*x^2)^p\*(d + e\*x + f\*x^2)^(q + 1))/(2\*c\*f^2\*(p + q + 1)\*(2\*p + 2\*q + 3)), x] - Dist[1/(2\*c\*f^2\*(p + q + 1)\*(2\*p + 2\*q + 3)), Int[(a + b\*x + c\*x^2)^(p - 1)\*(d + e\*x + f\*x^2)^q\*Simp[p\*(b\*d - a\*e)\*(C\*(c\*e - b\*f)\*(q + 1) - c\*(C\*e - B\*f)\*(2\*p + 2\*q + 3)) + (p + q + 1)\*(b^2\*C\*d\*f\*p + a\*c\*(C\*(2\*d\*f - e^2\*(2\*p + q + 2)) + f\*(B\*e - 2\*A\*f)\*(2\*p + 2\*q + 3)))] + (2\*p\*(c\*d - a\*f)\*(C\*(c\*e - b\*f)\*(q + 1) - c\*(C\*e - B\*f)\*(2\*p + 2\*q + 3)) + (p + q + 1)\*(C\*e\*f\*p\*(b^2 - 4\*a\*c) - b\*c\*(C\*(e^2 - 4\*d\*f)\*(2\*p + q + 2) + f\*(2\*C\*d - B\*e + 2\*A\*f)\*(2\*p + 2\*q + 3)))]\*x + (p\*(c\*e - b\*f)\*(C\*(c\*e - b\*f)\*(q + 1) - c\*(C\*e - B\*f)\*(2\*p + 2\*q + 3)) + (p + q + 1)\*(C\*f^2\*p\*(b^2 - 4\*a\*c) - c^2\*(C\*(e^2 - 4\*d\*f)\*(2\*p + q + 2) + f\*(2\*C\*d - B\*e + 2\*A\*f)\*(2\*p + 2\*q + 3)))]\*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2\*p + 2\*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

### Rule 1076

Int[((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Dist[C/c, Int[1/Sqrt[d + e\*x + f\*x^2], x], x] + Dist[1/c, Int[(A\*c - a\*C + (B\*c - b\*C)\*x)/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0]

### Rubi steps

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2} dx = -\frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} - \frac{1}{300} \int \frac{\left(-\frac{4731}{2} + \frac{6135x}{4} - \frac{14943x^2}{4}\right) \sqrt{3-x+2x^2}}{2+3x+5x^2} dx$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} + \frac{\int \frac{\frac{3205293}{8} - \frac{1133938x}{16}}{\sqrt{3-x+2x^2}} dx}{300}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} + \frac{\int \frac{-702768-7602x}{\sqrt{3-x+2x^2}} dx}{150000}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} - \frac{\int \frac{-702768(108-11x)}{\sqrt{3-x}} dx}{3}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} - \frac{7216203 \sinh^{-1}\left(\frac{4x-3}{\sqrt{3}}\right)}{800000\sqrt{3}}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} - \frac{7216203 \sinh^{-1}\left(\frac{4x-3}{\sqrt{3}}\right)}{800000\sqrt{3}}$$

**Mathematica [C]** time = 1.05, size = 229, normalized size = 1.03

$$\frac{46464\sqrt{286+22i\sqrt{31}}(403-69i\sqrt{31})\tanh^{-1}\left(\frac{(-22-4i\sqrt{31})\sqrt{3-x+2x^2}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)-46464i\sqrt{286-22i\sqrt{31}}(69\sqrt{31}-403i)\tanh^{-1}\left(\frac{(22-4i\sqrt{31})\sqrt{3-x+2x^2}-63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)+620\sqrt{2x^2-x+3}(48000x^3-106400x^2+412060x-802347)+671106879\sqrt{2}\sinh^{-1}\left(\frac{4x-3}{\sqrt{3}}\right)}{148800000}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(5/2)/(2 + 3\*x + 5\*x^2), x]

[Out] (620\*Sqrt[3 - x + 2\*x^2]\*(-802347 + 412060\*x - 106400\*x^2 + 48000\*x^3) + 671106879\*Sqrt[2]\*ArcSinh[(-1 + 4\*x)/Sqrt[23]] + 46464\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*(403 - (69\*I)\*Sqrt[31])\*ArcTanh[(63 + I\*Sqrt[31] + (-22 - (4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])] - (46464\*I)\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*(-403\*I + 69\*Sqrt[31])\*ArcTanh[(-63 + I\*Sqrt[31] + (22 - (4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])])/148800000

**IntegrateAlgebraic [C]** time = 0.76, size = 245, normalized size = 1.10

$$\frac{1331\text{RootSum}\left[-5\#1^4 + 6\sqrt{2}\#1^3 + 17\#1^2 - 26\sqrt{2}\#1 - 56\&, \frac{-119\#1^2 \log(-\#1 + \sqrt{2x^2-x+3} - \sqrt{2}) + 22\sqrt{2}\#1 \log(-\#1 + \sqrt{2x^2-x+3} - \sqrt{2}) + 368 \log(-\#1 + \sqrt{2x^2-x+3} - \sqrt{2})}{-10\#1^3 + 9\sqrt{2}\#1^2 + 17\#1 - 13\sqrt{2}}\right] - 7216203 \log(2\sqrt{2}\sqrt{2x^2-x+3} - 4x + 1) + \frac{\sqrt{2x^2-x+3}(48000x^3 - 106400x^2 + 412060x - 802347)}{240000}}{3125}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^(5/2)/(2 + 3\*x + 5\*x^2), x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(-802347 + 412060\*x - 106400\*x^2 + 48000\*x^3))/240000 - (7216203\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(800000\*Sqrt[2]) - (1331\*RootSum[-56 - 26\*Sqrt[2]\*#1 + 17\*#1^2 + 6\*Sqrt[2]\*#1^3 - 5\*#1^4 &, (368\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1] + 22\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1 - 119\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1^2)/(-13\*Sqrt[2] + 17\*#1 + 9\*Sqrt[2]\*#1^2 - 10\*#1^3) & ])/3125

**fricas [B]** time = 1.04, size = 2010, normalized size = 9.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out] 121/96875000\*6050^(1/4)\*sqrt(31)\*sqrt(2)\*sqrt(-772850000\*sqrt(2) + 2500000000)\*arctan(1/254496437500\*(722441500000\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x^8 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752088\*x^3 + 396144\*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) + 2300\*(4\*6050^(3/4)\*sqrt(31)\*(35898\*x^7 - 441939\*x^6 + 782418\*x^5 - 2117233\*x^4 + 1272680\*x^3 - 1081800\*x^2 - sqrt(2)\*(173702\*x^7 - 453907\*x^6 + 1056481\*x^5 - 1083344\*x^4 + 393672\*x^3 + 152064\*x^2 - 1043712\*x + 259200) - 518400\*x + 1043712) + 5\*6050^(1/4)\*sqrt(31)\*(317294\*x^7 - 5870544\*x^6 + 38857480\*x^5 - 111531424\*x^4 + 156761280\*x^3 - 168192000\*x^2 - sqrt(2)\*(712757\*x^7 - 10233303\*x^6 + 48529768\*x^5 - 94500260\*x^4 + 113086944\*x^3 - 22282848\*x^2 - 106417152\*x + 37407744) - 74815488\*x + 106417152))\*sqrt(2\*x^2 - x + 3)\*sqrt(-772850000\*sqrt(2) + 2500000000) - sqrt(10/5711)\*(314105000\*sqrt(31)\*sqrt(2)\*(123408\*x^8 - 914152\*x^7 + 1578888\*x^6 - 3293072\*x^5 + 396480\*x^4 + 798336\*x^3 - 3822336\*x^2 - sqrt(2)\*(15550\*x^8 - 118051\*x^7 + 244047\*x^6 - 707374\*x^5 + 1053960\*x^4 - 1667952\*x^3 + 1209600\*x^2 - 1036800\*x) + 3276288\*x) - (4\*6050^(3/4)\*sqrt(31)\*(167914\*x^7 - 195429\*x^6 + 331239\*x^5 + 1685680\*x^4 - 3693960\*x^3 + 4195584\*x^2 + 22\*sqrt(2)\*(37846\*x^7 - 52859\*x^6 + 160569\*x^5 - 4464\*x^4 - 49464\*x^3 + 202176\*x^2 - 202176\*x) - 4195584\*x) - 5\*6050^(1/4)\*sqrt(31)\*(160956\*x^7 - 2232176\*x^6 + 11218640\*x^5 - 38096640\*x^4 + 139374720\*x^3 - 296027136\*x^2 - sqrt(2)\*(3246491\*x^7 - 41888524\*x^6 + 159670660\*x^5 - 190080576\*x^4 + 180496224\*x^3 + 376648704\*x^2 - 376648704\*x) + 296027136\*x))\*sqrt(2\*x^2 - x + 3)\*sqrt(-772850000\*sqrt(2) + 2500000000) + 14277500\*sqrt(31)\*(254591\*x^8 - 4815126\*x^7 + 32303580\*x^6 - 90866808\*x^5 + 108781920\*x^4 - 74219328\*x^3 - 168956928\*x^2 - 15488\*sqrt(2)\*(4\*x^8 - 76\*x^7 + 517\*x^6 - 1536\*x^5 + 2385\*x^4 - 3618\*x^3 + 2268\*x^2 - 1944\*x) + 144820224\*x))\*sqrt((6050^(1/4)\*sqrt(2\*x^2 - x + 3)\*(sqrt(2)\*(163\*x - 725) + 562\*x - 888)\*sqrt(-772850000\*sqrt(2) + 2500000000) + 139919500\*x^2 + 125642000\*sqrt(2)\*(2\*x^2 - x + 3) - 431180500\*x + 571100000)/x^2) + 8209562500\*sqrt(31)\*(2828123\*x^8 - 9696916\*x^7 + 53385560\*x^6 - 142835344\*x^5 + 254146592\*x^4 - 249300096\*x^3 + 37981440\*x^2 - 7744\*sqrt(2)\*(1348\*x^8 - 2692\*x^7 + 9789\*x^6 - 10070\*x^5 + 15569\*x^4 - 5568\*x^3 + 1080\*x^2 + 4320\*x - 5184) + 223064064\*x - 94887936))/(2585191\*x^8 - 4661200\*x^7 + 14191920\*x^6 + 490880\*x^5 - 13562944\*x^4 + 44249088\*x^3 - 34615296\*x^2 - 24772608\*x + 18579456) + 121/96875000\*6050^(1/4)\*sqrt(31)\*sqrt(2)\*sqrt(-772850000\*sqrt(2) + 2500000000)\*arctan(-1/254496437500\*(722441500000\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x^8 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752088\*x^3 + 396144\*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) - 2300\*(4\*6050^(3/4)\*sqrt(31)\*(35898\*x^7 - 441939\*x^6 + 782418\*x^5 - 2117233\*x^4 + 1272680\*x^3 - 1081800\*x^2 - sqrt(2)\*(173702\*x^7 - 453907\*x^6 + 1056481\*x^5 - 1083344\*x^4 + 393672\*x^3 + 152064\*x^2 - 1043712\*x + 259200) - 518400\*x + 1043712) + 5\*6050^(1/4)\*sqrt(31)\*(317294\*x^7 - 5870544\*x^6 + 38857480\*x^5 - 111531424\*x^4 + 156761280\*x^3 - 168192000\*x^2 - sqrt(2)\*(712757\*x^7 - 10233303\*x^6 + 48529768\*x^5 - 94500260\*x^4 + 113086944\*x^3 - 22282848\*x^2 - 106417152\*x + 37407744) - 74815488\*x + 106417152))\*sqrt(2\*x^2 - x + 3)\*sqrt(-772850000\*sqrt(2) + 2500000000) - sqrt(10/5711)\*(314105000\*sqrt(31)\*sqrt(2)\*(123408\*x^8 - 914152\*x^7 + 1578888\*x^6 - 3293072\*x^5 + 396480\*x^4 + 798336\*x^3 - 3822336\*x^2 - sqrt(2)\*(15550\*x^8 - 118051\*x^7 + 244047\*x^6 - 707374\*x^5 + 1053960\*x^4 - 1667952\*x^3 + 1209600\*x^2 - 1036800\*x) + 3276288\*x) + (4\*6050^(3/4)\*sqrt(31)\*(167914\*x^7 - 195429\*x^6 + 331239\*x^5 + 1685680\*x^4 - 3693960\*x^3 + 4195584\*x^2 + 22\*sqrt(2)\*(37846\*x^7 - 52859\*x^6 + 160569\*x^5 - 4464\*x^4 - 49464\*x^3 + 202176\*x^2 - 202176\*x) - 4195584\*x) - 5\*6050^(1/4)\*sqrt(31)\*(160956\*x^7 - 2232176\*x^6 + 11218640\*x^5 - 38096640\*x^4 + 139374720\*x^3 - 296027136\*x^2 - sqrt(2)\*(3246491\*x^7 - 41888524\*x^6 + 159670660\*x^5 - 190080576\*x^4 + 180496224\*x^3 + 376648704\*x^2 - 376648704\*x) + 296027136\*x))\*sqrt(2\*x^2

```

- x + 3)*sqrt(-772850000*sqrt(2) + 2500000000) + 14277500*sqrt(31)*(254591*
x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*
x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 +
2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(6050^(1/4)*
sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 888)*sqrt(-772850000*
sqrt(2) + 2500000000) - 139919500*x^2 - 125642000*sqrt(2)*(2*x^2 - x + 3) +
431180500*x - 571100000)/x^2) + 8209562500*sqrt(31)*(2828123*x^8 - 9696916*
x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 379814
40*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x
^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(25851
91*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*
x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 121/2213012500000*6050^(1/4)
*(15457*sqrt(2) + 50000)*sqrt(-772850000*sqrt(2) + 2500000000)*log(91506250
00/5711*(6050^(1/4)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 88
8)*sqrt(-772850000*sqrt(2) + 2500000000) + 139919500*x^2 + 125642000*sqrt(2)
)*(2*x^2 - x + 3) - 431180500*x + 571100000)/x^2) + 121/2213012500000*6050^
(1/4)*(15457*sqrt(2) + 50000)*sqrt(-772850000*sqrt(2) + 2500000000)*log(-91
50625000/5711*(6050^(1/4)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*
x - 888)*sqrt(-772850000*sqrt(2) + 2500000000) - 139919500*x^2 - 125642000*
sqrt(2)*(2*x^2 - x + 3) + 431180500*x - 571100000)/x^2) + 1/240000*(48000*x
^3 - 106400*x^2 + 412060*x - 802347)*sqrt(2*x^2 - x + 3) + 7216203/3200000*
sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)

```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]root error [1.0,infinity,infinity,infinity,infini
ty]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity]
]root error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]root error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]root error [1.0,infinity,infinity,infinity,infinity]
]Evaluation time: 15.78Done
```

**maple [B]** time = 0.05, size = 4860, normalized size = 21.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/5*x^3*(2*x^2-x+3)^(1/2)-133/300*x^2*(2*x^2-x+3)^(1/2)+20603/12000*x*(2*x^
2-x+3)^(1/2)-267449/80000*(2*x^2-x+3)^(1/2)+7216203/1600000*2^(1/2)*arcsinh
(4/23*23^(1/2)*(x-1/4))+4/33034375*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^
(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^2^(1/2)*(75195*2^
(1/2)*(-8866+6820*2^(1/2))^2^(1/2)*(-775687+549362*2^(1/2))^2^(1/2)*arctan(1/11
692487*(-775687+549362*2^(1/2))^2^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)
^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^2^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x
+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(
23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)
*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+106294*(-8866+6820*2^(1/2))^2^(
1/2)*(-775687+549362*2^(1/2))^2^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2)
))^2^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)

```



$$\begin{aligned}
& 4*2^{(1/2)-41})^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})))+3192442*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})*2^{(1/2)}-5264358*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})))/((8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)))^2)^{(1/2)}/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)))/(8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}-63/105710*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(285*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\operatorname{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})))+386*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\operatorname{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})))-274846*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})*2^{(1/2)}-1543366*\operatorname{arctanh}(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})))/((8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)))^2)^{(1/2)}/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)))/(8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}+27/21142*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(151*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\operatorname{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})))+218*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\operatorname{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})))+520*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\operatorname{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41)
\end{aligned}$$

$$\left. \begin{aligned} & \right)^{1/2} * (6485 * 2^{1/2} * (x+2^{1/2}-1)^2 / (-x+2^{1/2}+1)^2 + 10368 * (x+2^{1/2}-1)^2 / (-x+2^{1/2}+1)^2 + 22379 * 2^{1/2} + 32016) / (23 * (x+2^{1/2}-1)^4 / (-x+2^{1/2}+1)^4 + 82 * (x+2^{1/2}-1)^2 / (-x+2^{1/2}+1)^2 + 23) * (x+2^{1/2}-1) / (-x+2^{1/2}+1) * (8 + 3 * 2^{1/2}) + 465124 * \operatorname{arctanh}(31/2 * (8 * (x+2^{1/2}-1)^2 / (-x+2^{1/2}+1)^2 + 3 * 2^{1/2}) * (x+2^{1/2}-1)^2 / (-x+2^{1/2}+1)^2 + 8 - 3 * 2^{1/2}))^{1/2} / (-8866 + 6820 * 2^{1/2})^{1/2} * 2^{1/2} - 866822 * \operatorname{arctanh}(31/2 * (8 * (x+2^{1/2}-1)^2 / (-x+2^{1/2}+1)^2 + 3 * 2^{1/2}) * (x+2^{1/2}-1)^2 / (-x+2^{1/2}+1)^2 + 8 - 3 * 2^{1/2}))^{1/2} / (-8866 + 6820 * 2^{1/2})^{1/2} / ((8 * (x+2^{1/2}-1)^2 / (-x+2^{1/2}+1)^2 + 3 * 2^{1/2}) * (x+2^{1/2}-1)^2 / (-x+2^{1/2}+1)^2 + 8 - 3 * 2^{1/2})) / (1 + (x+2^{1/2}-1) / (-x+2^{1/2}+1))^2)^{1/2} / (1 + (x+2^{1/2}-1) / (-x+2^{1/2}+1)) / (8 + 3 * 2^{1/2}) / (-8866 + 6820 * 2^{1/2})^{1/2} \end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{5/2}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] integrate((2\*x^2 - x + 3)^(5/2)/(5\*x^2 + 3\*x + 2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(2x^2 - x + 3)^{5/2}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(5/2)/(3\*x + 5\*x^2 + 2),x)

[Out] int((2\*x^2 - x + 3)^(5/2)/(3\*x + 5\*x^2 + 2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{5/2}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(5/2)/(5\*x\*\*2+3\*x+2),x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(5/2)/(5\*x\*\*2 + 3\*x + 2), x)

**3.74**  $\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2} dx$

**Optimal.** Leaf size=255

$$\frac{(10x + 3)(2x^2 - x + 3)^{5/2}}{31(5x^2 + 3x + 2)} + \frac{4}{155}(4-5x)(2x^2 - x + 3)^{3/2} - \frac{(2240x + 1277)\sqrt{2x^2 - x + 3}}{7750} + \frac{11\sqrt{\frac{11}{31}(224510383 + 194487500\sqrt{2})}}{2500\sqrt{2}}$$

**Rubi [A]** time = 0.66, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {971, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x + 3)(2x^2 - x + 3)^{5/2}}{31(5x^2 + 3x + 2)} + \frac{4}{155}(4-5x)(2x^2 - x + 3)^{3/2} - \frac{(2240x + 1277)\sqrt{2x^2 - x + 3}}{7750} + \frac{11\sqrt{\frac{11}{31}(224510383 + 194487500\sqrt{2})} \operatorname{atan}^{-1}\left(\frac{\sqrt{2x^2 - x + 3}}{\sqrt{2x^2 + 3}}\right) + \frac{11\sqrt{\frac{11}{31}(194487500\sqrt{2} - 224510383)} \operatorname{atanh}^{-1}\left(\frac{\sqrt{2x^2 - x + 3}}{\sqrt{2x^2 + 3}}\right) + \frac{4799 \operatorname{sinh}^{-1}\left(\frac{1-x}{\sqrt{23}}\right)}{2500\sqrt{2}}}{38750}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2, x]
```

```
[Out] -((1277 + 2240*x)*Sqrt[3 - x + 2*x^2])/7750 + (4*(4 - 5*x)*(3 - x + 2*x^2)^(3/2))/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(5/2))/(31*(2 + 3*x + 5*x^2)) - (4799*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2500*Sqrt[2]) + (11*Sqrt[(11*(224510383 + 194487500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(224510383 + 194487500*Sqrt[2]))])*(21136 + 33287*Sqrt[2] + (87710 + 54423*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/38750 - (11*Sqrt[(11*(-224510383 + 194487500*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/(62*(-224510383 + 194487500*Sqrt[2]))])*(21136 - 33287*Sqrt[2] + (87710 - 54423*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/38750
```

**Rule 204**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

**Rule 206**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

**Rule 215**

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

**Rule 619**

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

**Rule 971**

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p + 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
```



, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

#### Rule 1029

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

#### Rule 1035

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 2]}, Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f - q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f + q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f + q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f - q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && NegQ[b^2 - 4\*a\*c]

#### Rule 1066

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((B\*c\*f\*(2\*p + 2\*q + 3) + C\*(b\*f\*p - c\*e\*(2\*p + q + 2)) + 2\*c\*C\*f\*(p + q + 1)\*x)\*(a + b\*x + c\*x^2)^p\*(d + e\*x + f\*x^2)^(q + 1))/(2\*c\*f^2\*(p + q + 1)\*(2\*p + 2\*q + 3)), x] - Dist[1/(2\*c\*f^2\*(p + q + 1)\*(2\*p + 2\*q + 3)), Int[(a + b\*x + c\*x^2)^(p - 1)\*(d + e\*x + f\*x^2)^q\*Simp[p\*(b\*d - a\*e)\*(C\*(c\*e - b\*f)\*(q + 1) - c\*(C\*e - B\*f)\*(2\*p + 2\*q + 3)) + (p + q + 1)\*(b^2\*C\*d\*f\*p + a\*c\*(C\*(2\*d\*f - e^2\*(2\*p + q + 2)) + f\*(B\*e - 2\*A\*f)\*(2\*p + 2\*q + 3))) + (2\*p\*(c\*d - a\*f)\*(C\*(c\*e - b\*f)\*(q + 1) - c\*(C\*e - B\*f)\*(2\*p + 2\*q + 3)) + (p + q + 1)\*(C\*e\*f\*p\*(b^2 - 4\*a\*c) - b\*c\*(C\*(e^2 - 4\*d\*f)\*(2\*p + q + 2) + f\*(2\*C\*d - B\*e + 2\*A\*f)\*(2\*p + 2\*q + 3)))]\*x + (p\*(c\*e - b\*f)\*(C\*(c\*e - b\*f)\*(q + 1) - c\*(C\*e - B\*f)\*(2\*p + 2\*q + 3)) + (p + q + 1)\*(C\*f^2\*p\*(b^2 - 4\*a\*c) - c^2\*(C\*(e^2 - 4\*d\*f)\*(2\*p + q + 2) + f\*(2\*C\*d - B\*e + 2\*A\*f)\*(2\*p + 2\*q + 3)))]\*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2\*p + 2\*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

#### Rule 1076

Int[((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Dist[C/c, Int[1/Sqrt[d + e\*x + f\*x^2], x], x] + Dist[1/c, Int[(A\*c - a\*C + (B\*c - b\*C)\*x)/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2} dx &= \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} - \frac{1}{31} \int \frac{(3-x+2x^2)^{3/2} \left(-\frac{75}{2} + 15x + 80x^2\right)}{2+3x+5x^2} dx \\
&= \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} + \frac{\int \frac{(87660-54300x-53760x^2)\sqrt{3-x+2x^2}}{2+3x+5x^2}}{18600} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)}
\end{aligned}$$

**Mathematica [C]** time = 1.65, size = 685, normalized size = 2.69

RootSum[541 + 6\*sqrt(2)\*x^2 + 1791\*x^2 - 26\*sqrt(2)\*x - 566, Integrate[(3 - x + 2\*x^2)^(5/2)/(2 + 3\*x + 5\*x^2)^2, x]] + RootSum[541 + 6\*sqrt(2)\*x^2 + 1791\*x^2 - 26\*sqrt(2)\*x - 566, Integrate[(3 - x + 2\*x^2)^(3/2)\*(-75/2 + 15\*x + 80\*x^2)/(2 + 3\*x + 5\*x^2), x]] + RootSum[541 + 6\*sqrt(2)\*x^2 + 1791\*x^2 - 26\*sqrt(2)\*x - 566, Integrate[(87660 - 54300\*x - 53760\*x^2)\*sqrt(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2), x]]

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(5/2)/(2 + 3\*x + 5\*x^2)^2, x]

[Out] -1/4805000\*(-5577520\*sqrt[3 - x + 2\*x^2] - 5759180\*x\*sqrt[3 - x + 2\*x^2] + 7784100\*x^2\*sqrt[3 - x + 2\*x^2] - 1922000\*x^3\*sqrt[3 - x + 2\*x^2] - 4611839\*sqrt[2]\*(2 + 3\*x + 5\*x^2)\*ArcSinh[(-1 + 4\*x)/sqrt[23]] + (11\*I)\*sqrt[286 + (22\*I)\*sqrt[31]]\*(5177\*I + 8771\*sqrt[31])\*(2 + 3\*x + 5\*x^2)\*ArcTanh[(63 + I\*sqrt[31] - 22\*x - (4\*I)\*sqrt[31]\*x)/(2\*sqrt[286 + (22\*I)\*sqrt[31]]\*sqrt[3 - x + 2\*x^2])] + (192962\*I)\*sqrt[682\*(13 - I\*sqrt[31])]\*ArcTanh[(-63 + I\*sqrt[31] + 22\*x - (4\*I)\*sqrt[31]\*x)/(2\*sqrt[286 - (22\*I)\*sqrt[31]]\*sqrt[3 - x + 2\*x^2])] + 113894\*sqrt[286 - (22\*I)\*sqrt[31]]\*ArcTanh[(-63 + I\*sqrt[31] + 22\*x - (4\*I)\*sqrt[31]\*x)/(2\*sqrt[286 - (22\*I)\*sqrt[31]]\*sqrt[3 - x + 2\*x^2])] + (289443\*I)\*sqrt[682\*(13 - I\*sqrt[31])]\*x\*ArcTanh[(-63 + I\*sqrt[31] + 22\*x - (4\*I)\*sqrt[31]\*x)/(2\*sqrt[286 - (22\*I)\*sqrt[31]]\*sqrt[3 - x + 2\*x^2])] + 170841\*sqrt[286 - (22\*I)\*sqrt[31]]\*x\*ArcTanh[(-63 + I\*sqrt[31] + 22\*x - (4\*I)\*sqrt[31]\*x)/(2\*sqrt[286 - (22\*I)\*sqrt[31]]\*sqrt[3 - x + 2\*x^2])] + (482405\*I)\*sqrt[682\*(13 - I\*sqrt[31])]\*x^2\*ArcTanh[(-63 + I\*sqrt[31] + 22\*x - (4\*I)\*sqrt[31]\*x)/(2\*sqrt[286 - (22\*I)\*sqrt[31]]\*sqrt[3 - x + 2\*x^2])] + 284735\*sqrt[286 - (22\*I)\*sqrt[31]]\*x^2\*ArcTanh[(-63 + I\*sqrt[31] + 22\*x - (4\*I)\*sqrt[31]\*x)/(2\*sqrt[286 - (22\*I)\*sqrt[31]]\*sqrt[3 - x + 2\*x^2])]/(2 + 3\*x + 5\*x^2)

**IntegrateAlgebraic [C]** time = 0.97, size = 442, normalized size = 1.73

RootSum[541 + 6\*sqrt(2)\*x^2 + 1791\*x^2 - 26\*sqrt(2)\*x - 566, Integrate[(3 - x + 2\*x^2)^(5/2)/(2 + 3\*x + 5\*x^2)^2, x]] + RootSum[541 + 6\*sqrt(2)\*x^2 + 1791\*x^2 - 26\*sqrt(2)\*x - 566, Integrate[(3 - x + 2\*x^2)^(3/2)\*(-75/2 + 15\*x + 80\*x^2)/(2 + 3\*x + 5\*x^2), x]] + RootSum[541 + 6\*sqrt(2)\*x^2 + 1791\*x^2 - 26\*sqrt(2)\*x - 566, Integrate[(87660 - 54300\*x - 53760\*x^2)\*sqrt(3 - x + 2\*x^2)/(2 + 3\*x + 5\*x^2), x]]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(8996 + 9289*x - 12555*x^2 + 3100*x^3))/(7750*(2 + 3*x
+ 5*x^2)) - (4799*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(2500*Sqrt
[2]) + (121*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4
& , (5237*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 2880*Sqrt[2]*Log[
-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 2225*Log[-(Sqrt[2]*x) + Sqrt[
3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3)
& ])/15625 - (121*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 -
5*#1^4 & , (639994*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] -
22980*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 1175*Sqrt[2]*Log[-(
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[
2]*#1^2 - 10*#1^3) & ])/(968750*Sqrt[2])
```

**fricas** [B] time = 1.32, size = 2161, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/1322759922435707900000*(38925001324*1464599010050^(1/4)*sqrt(155590)*sqrt
(62)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(224510383*sqrt(2) + 388975000)*arctan(1
/296975447063866363819995875*(110935670*sqrt(155590)*(4*1464599010050^(3/4)
*sqrt(62)*(18997882*x^7 - 82713851*x^6 + 169131062*x^5 - 298338397*x^4 + 15
6222120*x^3 - 89116200*x^2 - sqrt(2)*(18111018*x^7 - 62947113*x^6 + 1354639
29*x^5 - 197908246*x^4 + 94500248*x^3 - 34095024*x^2 - 122404608*x + 714528
00) - 142905600*x + 122404608) + 2411645*1464599010050^(1/4)*sqrt(62)*(3035
566*x^7 - 47612316*x^6 + 259553720*x^5 - 615321136*x^4 + 807721920*x^3 - 57
9888000*x^2 - sqrt(2)*(2643323*x^7 - 39854517*x^6 + 204950152*x^5 - 4510041
40*x^4 + 573424416*x^3 - 311722272*x^2 - 434377728*x + 268655616) - 5373112
32*x + 434377728))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 388975000)
+ 843027075536136774714827000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 70
4270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(87
46*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 +
396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(77795/920561)*
(sqrt(155590)*(4*1464599010050^(3/4)*sqrt(62)*(58767374*x^7 - 85793239*x^6
+ 285539949*x^5 - 168939120*x^4 + 253241640*x^3 + 601344*x^2 - 4*sqrt(2)*(1
7889302*x^7 - 25424283*x^6 + 80174553*x^5 - 21241168*x^4 + 15593832*x^3 + 5
8564512*x^2 - 58564512*x) - 601344*x) + 2411645*1464599010050^(1/4)*sqrt(62)
)*(9891184*x^7 - 128099264*x^6 + 496592960*x^5 - 666984960*x^4 + 949582080*
x^3 + 183223296*x^2 - sqrt(2)*(10181049*x^7 - 131588036*x^6 + 505509740*x^5
- 637596864*x^4 + 754818336*x^3 + 725677056*x^2 - 725677056*x) - 183223296
*x))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 388975000) + 759924265600
1778100*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x
^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^
7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 103
6800*x) + 3276288*x) + 345420120727353550*sqrt(31)*(254591*x^8 - 4815126*x^
7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*
x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*
x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(1464599010050^(1/4)*sqrt(15
5590)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(9733*x + 29025) - 387
58*x + 19292)*sqrt(224510383*sqrt(2) + 388975000) - 6744561519183110*x^2 -
6056340956001160*sqrt(2)*(2*x^2 - x + 3) + 20784261008094890*x - 2752882252
7278000)/x^2) + 9579853131092463349032125*sqrt(31)*(2828123*x^8 - 9696916*x
^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3798144
0*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^
4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(258519
1*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x
```

$$\begin{aligned}
&^3 - 34615296x^2 - 24772608x + 18579456)) + 38925001324*1464599010050^{(1/4)} \\
&*\sqrt{155590}*\sqrt{62}*\sqrt{2}*(5x^2 + 3x + 2)*\sqrt{224510383*\sqrt{2} + 388975000} \\
&*\arctan(1/296975447063866363819995875*(110935670*\sqrt{155590}*(4 \\
&*1464599010050^{(3/4)}*\sqrt{62}*(18997882*x^7 - 82713851*x^6 + 169131062*x^5 \\
&- 298338397*x^4 + 156222120*x^3 - 89116200*x^2 - \sqrt{2}*(18111018*x^7 - 62 \\
&947113*x^6 + 135463929*x^5 - 197908246*x^4 + 94500248*x^3 - 34095024*x^2 - \\
&122404608*x + 71452800) - 142905600*x + 122404608) + 2411645*1464599010050^{(1/4)} \\
&*\sqrt{62}*(3035566*x^7 - 47612316*x^6 + 259553720*x^5 - 615321136*x^4 \\
&+ 807721920*x^3 - 579888000*x^2 - \sqrt{2}*(2643323*x^7 - 39854517*x^6 + 204 \\
&950152*x^5 - 451004140*x^4 + 573424416*x^3 - 311722272*x^2 - 434377728*x + \\
&268655616) - 537311232*x + 434377728))*\sqrt{2*x^2 - x + 3}*\sqrt{224510383*\sqrt{2} + 388975000} \\
&- 843027075536136774714827000*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 984 \\
&96*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710 \\
&*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - \\
&\sqrt{77795/920561}*(\sqrt{155590}*(4*1464599010050^{(3/4)}*\sqrt{62}*(58767374 \\
&*x^7 - 85793239*x^6 + 285539949*x^5 - 168939120*x^4 + 253241640*x^3 + 60134 \\
&4*x^2 - 4*\sqrt{2}*(17889302*x^7 - 25424283*x^6 + 80174553*x^5 - 21241168*x^4 \\
&+ 15593832*x^3 + 58564512*x^2 - 58564512*x) - 601344*x) + 2411645*1464599 \\
&010050^{(1/4)}*\sqrt{62}*(9891184*x^7 - 128099264*x^6 + 496592960*x^5 - 666984 \\
&960*x^4 + 949582080*x^3 + 183223296*x^2 - \sqrt{2}*(10181049*x^7 - 131588036 \\
&*x^6 + 505509740*x^5 - 637596864*x^4 + 754818336*x^3 + 725677056*x^2 - 7256 \\
&77056*x) - 183223296*x))*\sqrt{2*x^2 - x + 3}*\sqrt{224510383*\sqrt{2} + 388975000} \\
&- 7599242656001778100*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914152*x^7 + 157 \\
&8888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - \sqrt{2}*(1 \\
&5550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 \\
&+ 1209600*x^2 - 1036800*x) + 3276288*x) - 345420120727353550*\sqrt{31}*(254 \\
&591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219 \\
&328*x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 \\
&+ 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\sqrt{(14645990 \\
&10050^{(1/4)}*\sqrt{155590}*\sqrt{62}*\sqrt{31}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(97 \\
&33*x + 29025) - 38758*x + 19292)*\sqrt{224510383*\sqrt{2} + 388975000} + 6744 \\
&561519183110*x^2 + 6056340956001160*\sqrt{2}*(2*x^2 - x + 3) - 2078426100809 \\
&4890*x + 27528822527278000)/x^2) - 9579853131092463349032125*\sqrt{31}*(2828 \\
&123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 2493 \\
&00096*x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348*x^8 - 2692*x^7 + 9789*x^6 - 1 \\
&0070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - \\
&94887936))/((2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 135629 \\
&44*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 11*1464599 \\
&010050^{(1/4)}*\sqrt{155590}*\sqrt{62}*\sqrt{31}*(1944875000*x^2 - 224510383*\sqrt{2} \\
&*\sqrt{2}*(5*x^2 + 3*x + 2) + 1166925000*x + 777950000)*\sqrt{224510383*\sqrt{2} + 388975000} \\
&*\log(14708117187500/920561*(1464599010050^{(1/4)}*\sqrt{155590}*\sqrt{62} \\
&*\sqrt{31}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(9733*x + 29025) - 38758*x + 19 \\
&292)*\sqrt{224510383*\sqrt{2} + 388975000} + 6744561519183110*x^2 + 605634095 \\
&6001160*\sqrt{2}*(2*x^2 - x + 3) - 20784261008094890*x + 27528822527278000)/ \\
&x^2) - 11*1464599010050^{(1/4)}*\sqrt{155590}*\sqrt{62}*\sqrt{31}*(1944875000*x^2 \\
&- 224510383*\sqrt{2}*(5*x^2 + 3*x + 2) + 1166925000*x + 777950000)*\sqrt{224510383*\sqrt{2} + 388975000} \\
&*\log(-14708117187500/920561*(1464599010050^{(1/4)}*\sqrt{155590}*\sqrt{62}*\sqrt{31}*\sqrt{2*x^2 - x + 3} \\
&*(\sqrt{2}*(9733*x + 29025) - 38758*x + 19292)*\sqrt{224510383*\sqrt{2} + 388975000} - 67445615191831 \\
&10*x^2 - 6056340956001160*\sqrt{2}*(2*x^2 - x + 3) + 20784261008094890*x - 2 \\
&7528822527278000)/x^2) + 634792486776896221210*\sqrt{2}*(5*x^2 + 3*x + 2)*\log(-4*\sqrt{2}*\sqrt{2*x^2 - x + 3} \\
&*(4*x - 1) - 32*x^2 + 16*x - 25) + 170678699669123600*(3100*x^3 - 12555*x^2 + 9289*x + 8996)*\sqrt{2*x^2 - x + 3} \\
&)/(5*x^2 + 3*x + 2)
\end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error  
 %%%{15625, [8]%%}%+%%{%%{-37500,0]: [1,0,-2]%%}, [7]%%}%+%%{-61250, [6]%%}%  
 %%%{290000,0]: [1,0,-2]%%}, [5]%%}%+%%{140625, [4]%%}%+%%{%%{-972500,  
 ,0]: [1,0,-2]%%}, [3]%%}%+%%{-345000, [2]%%}%+%%{%%{1820000,0]: [1,0,-2]%%},  
 [1]%%}%+%%{1960000, [0]%%}% / %%{50, [8]%%}%+%%{%%{poly1[-120,0]: [1,0,-2]%%},  
 [7]%%}%+%%{-196, [6]%%}%+%%{%%{poly1[928,0]: [1,0,-2]%%}, [5]%%}%+%%{450,  
 , [4]%%}%+%%{%%{poly1[-3112,0]: [1,0,-2]%%}, [3]%%}%+%%{-1104, [2]%%}%+%%{%%  
 {poly1[5824,0]: [1,0,-2]%%}, [1]%%}%+%%{6272, [0]%%}% Error: Bad Argument Value

**maple** [B] time = 0.16, size = 40028, normalized size = 156.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^2,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] integrate((2\*x^2 - x + 3)^(5/2)/(5\*x^2 + 3\*x + 2)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(2x^2 - x + 3)^{5/2}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2 - x + 3)^(5/2)/(3\*x + 5\*x^2 + 2)^2,x)

[Out] int((2\*x^2 - x + 3)^(5/2)/(3\*x + 5\*x^2 + 2)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(5/2)/(5\*x\*\*2+3\*x+2)\*\*2,x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(5/2)/(5\*x\*\*2 + 3\*x + 2)\*\*2, x)

$$3.75 \quad \int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3} dx$$

**Optimal.** Leaf size=281

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{62(5x^2+3x+2)^2} + \frac{(2336x+769)(2x^2-x+3)^{3/2}}{3844(5x^2+3x+2)} + \frac{(11359-12920x)\sqrt{2x^2-x+3}}{48050} + \frac{\sqrt{11(1+4\sqrt{2})}}{2}$$

**Rubi [A]** time = 0.65, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$ , Rules used = {971, 1054, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{62(5x^2+3x+2)^2} + \frac{(2336x+769)(2x^2-x+3)^{3/2}}{3844(5x^2+3x+2)} + \frac{(11359-12920x)\sqrt{2x^2-x+3}}{48050} + \frac{\sqrt{11(1+4\sqrt{2})}}{2} \tan^{-1} \left( \frac{\sqrt{11(1+4\sqrt{2})} \sqrt{2x^2-x+3}}{\sqrt{2x^2-x+3}} \right) - \frac{(2937349-1978861\sqrt{2}) \sqrt{11(1+4\sqrt{2})} \tanh^{-1} \left( \frac{\sqrt{11(1+4\sqrt{2})} \sqrt{2x^2-x+3}}{\sqrt{2x^2-x+3}} \right)}{29791000} - \frac{(2937349-1978861\sqrt{2}) \sqrt{11(1+4\sqrt{2})} \tanh^{-1} \left( \frac{\sqrt{11(1+4\sqrt{2})} \sqrt{2x^2-x+3}}{\sqrt{2x^2-x+3}} \right)}{29791000} - \frac{4}{125} \sqrt{2} \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(5/2)/(2 + 3\*x + 5\*x^2)^3,x]

[Out] ((11359 - 12920\*x)\*Sqrt[3 - x + 2\*x^2])/48050 + ((3 + 10\*x)\*(3 - x + 2\*x^2)^(5/2))/(62\*(2 + 3\*x + 5\*x^2)^2) + ((769 + 2336\*x)\*(3 - x + 2\*x^2)^(3/2))/(3844\*(2 + 3\*x + 5\*x^2)) - (4\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/125 + (Sqrt[11\*(1 + 4\*Sqrt[2])]\*(2937349 + 1978861\*Sqrt[2])\*ArcTan[(Sqrt[11/(62\*(3531015707557 + 2498852071250\*Sqrt[2]))]\*(3957722 + 2937349\*Sqrt[2] + (9832420 + 6895071\*Sqrt[2])\*x))/Sqrt[3 - x + 2\*x^2]])/29791000 - ((2937349 - 1978861\*Sqrt[2])\*Sqrt[11\*(-1 + 4\*Sqrt[2])]\*ArcTanh[(Sqrt[11/(62\*(-3531015707557 + 2498852071250\*Sqrt[2]))]\*(3957722 - 2937349\*Sqrt[2] + (9832420 - 6895071\*Sqrt[2])\*x))/Sqrt[3 - x + 2\*x^2]])/29791000

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 215**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

**Rule 619**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

**Rule 971**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/((b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[2\*c\*d\*(2\*p +

3) + b\*e\*q + (2\*b\*f\*q + 2\*c\*e\*(2\*p + q + 3))\*x + 2\*c\*f\*(2\*p + 2\*q + 3)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

#### Rule 1029

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

#### Rule 1035

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 2]}, Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f - q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f + q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[1/(2\*q), Int[Simp[h\*(b\*d - a\*e) - g\*(c\*d - a\*f + q) - (g\*(c\*e - b\*f) - h\*(c\*d - a\*f - q))\*x, x]/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && NegQ[b^2 - 4\*a\*c]

#### Rule 1054

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)\*((d\_) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((A\*b\*c - 2\*a\*B\*c + a\*b\*C - (c\*(b\*B - 2\*A\*c) - C\*(b^2 - 2\*a\*c))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/(c\*(b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/(c\*(b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[e\*q\*(A\*b\*c - 2\*a\*B\*c + a\*b\*C) - d\*(c\*(b\*B - 2\*A\*c)\*(2\*p + 3) + C\*(2\*a\*c - b^2\*(p + 2))) + (2\*f\*q\*(A\*b\*c - 2\*a\*B\*c + a\*b\*C) - e\*(c\*(b\*B - 2\*A\*c)\*(2\*p + q + 3) + C\*(2\*a\*c\*(q + 1) - b^2\*(p + q + 2)))]\*x - f\*(c\*(b\*B - 2\*A\*c)\*(2\*p + 2\*q + 3) + C\*(2\*a\*c\*(2\*q + 1) - b^2\*(p + 2\*q + 2)))\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

#### Rule 1066

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)\*((d\_) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((B\*c\*f\*(2\*p + 2\*q + 3) + C\*(b\*f\*p - c\*e\*(2\*p + q + 2)) + 2\*c\*C\*f\*(p + q + 1)\*x)\*(a + b\*x + c\*x^2)^p\*(d + e\*x + f\*x^2)^(q + 1))/(2\*c\*f^2\*(p + q + 1)\*(2\*p + 2\*q + 3)), x] - Dist[1/(2\*c\*f^2\*(p + q + 1)\*(2\*p + 2\*q + 3)), Int[(a + b\*x + c\*x^2)^(p - 1)\*(d + e\*x + f\*x^2)^q\*Simp[p\*(b\*d - a\*e)\*(C\*(c\*e - b\*f)\*(q + 1) - c\*(C\*e - B\*f)\*(2\*p + 2\*q + 3)) + (p + q + 1)\*(b^2\*C\*d\*f\*p + a\*c\*(C\*(2\*d\*f - e^2\*(2\*p + q + 2)) + f\*(B\*e - 2\*A\*f)\*(2\*p + 2\*q + 3)))] + (2\*p\*(c\*d - a\*f)\*(C\*(c\*e - b\*f)\*(q + 1) - c\*(C\*e - B\*f)\*(2\*p + 2\*q + 3)) + (p + q + 1)\*(C\*e\*f\*p\*(b^2 - 4\*a\*c) - b\*c\*(C\*(e^2 - 4\*d\*f)\*(2\*p + q + 2) + f\*(2\*C\*d - B\*e + 2\*A\*f)\*(2\*p + 2\*q + 3)))]\*x + (p\*(c\*e - b\*f)\*(C\*(c\*e - b\*f)\*(q + 1) - c\*(C\*e - B\*f)\*(2\*p + 2\*q + 3)) + (p + q + 1)\*(C\*f^2\*p\*(b^2 - 4\*a\*c) - c^2\*(C\*(e^2 - 4\*d\*f)\*(2\*p + q + 2) + f\*(2\*C\*d - B\*e + 2\*A\*f)\*(2\*p + 2\*q + 3)))]\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2\*p + 2\*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

## Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

## Rubi steps

$$\begin{aligned} \int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3} dx &= \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} - \frac{1}{62} \int \frac{(3-x+2x^2)^{3/2} \left(-\frac{195}{2} + 35x + 40x^2\right)}{(2+3x+5x^2)^2} dx \\ &= \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} + \frac{\int \frac{\left(\frac{66735}{4} - 7375x - 25840x^2\right)\sqrt{3-x+2x^2}}{2+3x+5x^2} dx}{9610} \\ &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} \\ &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} \\ &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} \\ &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} \\ &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} \end{aligned}$$

**Mathematica** [C] time = 2.00, size = 1009, normalized size = 3.59

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(5/2)/(2 + 3\*x + 5\*x^2)^3, x]

[Out] (153804640\*sqrt[3 - x + 2\*x^2] + 474815220\*x\*sqrt[3 - x + 2\*x^2] + 640207040\*x^2\*sqrt[3 - x + 2\*x^2] + 662597100\*x^3\*sqrt[3 - x + 2\*x^2] + 1906624\*sqrt[2]\*(2 + 3\*x + 5\*x^2)^2\*ArcSinh[(-1 + 4\*x)/sqrt[23]] - I\*sqrt[286 + (22\*I)\*sqrt[31]]\*(-503998\*I + 491621\*sqrt[31])\*(2 + 3\*x + 5\*x^2)^2\*ArcTanh[(63 + I\*sqrt[31] - 22\*x - (4\*I)\*sqrt[31]\*x)/(2\*sqrt[286 + (22\*I)\*sqrt[31]]\*sqrt[3 - x + 2\*x^2])] - (1966484\*I)\*sqrt[682\*(13 - I\*sqrt[31])]\*ArcTanh[(-63 + I\*sqrt[31] + 22\*x - (4\*I)\*sqrt[31]\*x)/(2\*sqrt[286 - (22\*I)\*sqrt[31]]\*sqrt[3 - x + 2\*x^2])] + 2015992\*sqrt[286 - (22\*I)\*sqrt[31]]\*ArcTanh[(-63 + I\*sqrt[31] + 22\*x - (4\*I)\*sqrt[31]\*x)/(2\*sqrt[286 - (22\*I)\*sqrt[31]]\*sqrt[3 - x + 2\*x^2])] - (5899452\*I)\*sqrt[682\*(13 - I\*sqrt[31])]\*x\*ArcTanh[(-63 + I\*sqrt[31] + 22\*x - (4\*I)\*sqrt[31]\*x)/(2\*sqrt[286 - (22\*I)\*sqrt[31]]\*sqrt[3 - x + 2\*x^2])])



$$\begin{aligned} & 1] + 22*x - (4*I)*\text{Sqrt}[31]*x)/(2*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2 \\ & *x^2]) + 6047976*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*x*\text{ArcTanh}[(-63 + I*\text{Sqrt}[31] + \\ & 22*x - (4*I)*\text{Sqrt}[31]*x)/(2*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2 \\ & ])] - (14257009*I)*\text{Sqrt}[682*(13 - I*\text{Sqrt}[31])] *x^2*\text{ArcTanh}[(-63 + I*\text{Sqrt}[31] \\ & ] + 22*x - (4*I)*\text{Sqrt}[31]*x)/(2*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2* \\ & x^2])] + 14615942*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*x^2*\text{ArcTanh}[(-63 + I*\text{Sqrt}[31] \\ & + 22*x - (4*I)*\text{Sqrt}[31]*x)/(2*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x \\ & ^2])] - (14748630*I)*\text{Sqrt}[682*(13 - I*\text{Sqrt}[31])] *x^3*\text{ArcTanh}[(-63 + I*\text{Sqrt}[ \\ & 31] + 22*x - (4*I)*\text{Sqrt}[31]*x)/(2*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + \\ & 2*x^2])] + 15119940*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*x^3*\text{ArcTanh}[(-63 + I*\text{Sqrt}[3 \\ & 1] + 22*x - (4*I)*\text{Sqrt}[31]*x)/(2*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2 \\ & *x^2])] - (12290525*I)*\text{Sqrt}[682*(13 - I*\text{Sqrt}[31])] *x^4*\text{ArcTanh}[(-63 + I*\text{Sqr \\ & t}[31] + 22*x - (4*I)*\text{Sqrt}[31]*x)/(2*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x \\ & + 2*x^2])] + 12599950*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*x^4*\text{ArcTanh}[(-63 + I*\text{Sqrt} \\ & [31] + 22*x - (4*I)*\text{Sqrt}[31]*x)/(2*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + \\ & 2*x^2)])]/(59582000*(2 + 3*x + 5*x^2)^2) \end{aligned}$$

**IntegrateAlgebraic [C]** time = 1.09, size = 627, normalized size = 2.23

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Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^(5/2)/(2 + 3\*x + 5\*x^2)^3,x]

[Out] (11\*Sqrt[3 - x + 2\*x^2]\*(22552 + 69621\*x + 93872\*x^2 + 97155\*x^3))/(96100\*(2 + 3\*x + 5\*x^2)^2) - (4\*Sqrt[2]\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/125 + (22\*RootSum[-56 - 26\*Sqrt[2]\*#1 + 17\*#1^2 + 6\*Sqrt[2]\*#1^3 - 5\*#1^4 & , (3781\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1] + 630\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1 + 150\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1^2)/(-13\*Sqrt[2] + 17\*#1 + 9\*Sqrt[2]\*#1^2 - 10\*#1^3) & ])/3125 + (11\*RootSum[-56 - 26\*Sqrt[2]\*#1 + 17\*#1^2 + 6\*Sqrt[2]\*#1^3 - 5\*#1^4 & , (4978708507\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1] - 165870920\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1 + 1110955025\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1^2)/(-13\*Sqrt[2] + 17\*#1 + 9\*Sqrt[2]\*#1^2 - 10\*#1^3) & ])/(1114062500\*Sqrt[2]) - (11\*RootSum[-56 - 26\*Sqrt[2]\*#1 + 17\*#1^2 + 6\*Sqrt[2]\*#1^3 - 5\*#1^4 & , (492740319684\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1] - 128644699540\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1 + 55365920925\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1^2)/(-13\*Sqrt[2] + 17\*#1 + 9\*Sqrt[2]\*#1^2 - 10\*#1^3) & ])/(69071875000\*Sqrt[2])

**fricas [B]** time = 1.48, size = 2240, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/758714159921174808909075728000\*(3184949732636\*3868444992270541948232^(1/4)\*sqrt(1999081657)\*sqrt(62)\*sqrt(2)\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*sqrt(3531015707557\*sqrt(2) + 4997704142500)\*arctan(1/4535484880629403103991789624695893204150231\*(2850690442882\*sqrt(1999081657)\*(2\*3868444992270541948232^(3/4)\*sqrt(62)\*(2627559914\*x^7 - 10187615527\*x^6 + 21362956024\*x^5 - 34451465819\*x^4 + 17321103240\*x^3 - 8320757400\*x^2 - sqrt(2)\*(1893366636\*x^7 - 7237484076\*x^6 + 15226003533\*x^5 - 24262105817\*x^4 + 12127036096\*x^3 - 5664787848\*x^2 - 13367586816\*x + 9338025600) - 18676051200\*x + 13367586816) + 61971531367\*3868444992270541948232^(1/4)\*sqrt(62)\*(400116332\*x^7 - 6149336082\*x^6 + 32552996440\*x^5 - 74427496472\*x^4 + 96235107840\*x^3 - 61219656000\*x^2 - sqrt(2)\*(286685371\*x^7 - 4395067059\*x^6 + 23180544704\*x^5 - 52748573780\*x^4 + 68065744032\*x^3 - 42544702944\*x^2 - 48625837056\*x + 34092306432)

$$\begin{aligned}
& - 68184612864*x + 48625837056) * \sqrt{2*x^2 - x + 3} * \sqrt{3531015707557 * \sqrt{2} + 4997704142500} + 12874924822431853972621854418491567805329688 * \sqrt{31} \\
& ) * \sqrt{2} * (28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2} * (8746*x^8 - 102335*x^7 + 396104*x^6 - 78 \\
& 3113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - \sqrt{1999081657/828550919} * (\sqrt{1999081657}) * (2*38684449 \\
& 92270541948232^{(3/4)} * \sqrt{62} * (9351066298*x^7 - 13433496653*x^6 + 43310345823*x^5 - 17374572240*x^4 + 20927636280*x^3 + 18483199488*x^2 - \sqrt{2} * (683 \\
& 9273266*x^7 - 9809465289*x^6 + 31524099699*x^5 - 12024617744*x^4 + 13914887256*x^3 + 14839341696*x^2 - 14839341696*x) - 18483199488*x) + 61971531367*3 \\
& 868444992270541948232^{(1/4)} * \sqrt{62} * (1427210918*x^7 - 18462714328*x^6 + 71210222920*x^5 - 92387041920*x^4 + 119489780160*x^3 + 68726817792*x^2 - \sqrt{2} * (1033310523*x^7 - 13365477772*x^6 + 51521534980*x^5 - 66583614528*x^4 + \\
& 85122955872*x^3 + 53108877312*x^2 - 53108877312*x) - 68726817792*x) * \sqrt{2*x^2 - x + 3} * \sqrt{3531015707557 * \sqrt{2} + 4997704142500} + 45164233298567 \\
& 21284677540671884 * \sqrt{31} * \sqrt{2} * (123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - \sqrt{2} * (15550*x^8 - \\
& 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 205291969538941876576251848722 * \sqrt{31} * (2 \\
& 54591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488 * \sqrt{2} * (4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x) * \sqrt{-(38684 \\
& 44992270541948232^{(1/4)} * \sqrt{1999081657} * \sqrt{62} * \sqrt{31} * \sqrt{2*x^2 - x + 3} * (\sqrt{2} * (2141441*x + 1076175) - 3217616*x - 1065266) * \sqrt{353101570755 \\
& 7 * \sqrt{2} + 4997704142500) - 155990877430002205517374*x^2 - 140073440957553000872744 * \sqrt{2} * (2*x^2 - x + 3) + 480706581467965980267826*x - 6366974588 \\
& 97968185785200)/x^2) + 146305963891271067870702891119222361424201 * \sqrt{31} * (2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - \\
& 249300096*x^3 + 37981440*x^2 - 7744 * \sqrt{2} * (1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406 \\
& 4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 31849 \\
& 49732636*3868444992270541948232^{(1/4)} * \sqrt{1999081657} * \sqrt{62} * \sqrt{2} * (25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) * \sqrt{3531015707557 * \sqrt{2} + 49977041425 \\
& 00) * \arctan(1/4535484880629403103991789624695893204150231 * (2850690442882 * \sqrt{1999081657}) * (2*3868444992270541948232^{(3/4)} * \sqrt{62} * (2627559914*x^7 - 10 \\
& 187615527*x^6 + 21362956024*x^5 - 34451465819*x^4 + 17321103240*x^3 - 8320757400*x^2 - \sqrt{2} * (1893366636*x^7 - 7237484076*x^6 + 15226003533*x^5 - 24 \\
& 262105817*x^4 + 12127036096*x^3 - 5664787848*x^2 - 13367586816*x + 9338025600) - 18676051200*x + 13367586816) + 61971531367*3868444992270541948232^{(1/ \\
& 4)} * \sqrt{62} * (400116332*x^7 - 6149336082*x^6 + 32552996440*x^5 - 74427496472*x^4 + 96235107840*x^3 - 61219656000*x^2 - \sqrt{2} * (286685371*x^7 - 4395067 \\
& 059*x^6 + 23180544704*x^5 - 52748573780*x^4 + 68065744032*x^3 - 42544702944*x^2 - 48625837056*x + 34092306432) - 68184612864*x + 48625837056) * \sqrt{2*x^2 - x + 3} * \sqrt{3531015707557 * \sqrt{2} + 4997704142500} - 1287492482243185 \\
& 3972621854418491567805329688 * \sqrt{31} * \sqrt{2} * (28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2} * (874 \\
& 6*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - \sqrt{1999081657/8285 \\
& 50919} * (\sqrt{1999081657}) * (2*3868444992270541948232^{(3/4)} * \sqrt{62} * (9351066298*x^7 - 13433496653*x^6 + 43310345823*x^5 - 17374572240*x^4 + 20927636280*x^3 + 18483199488*x^2 - \sqrt{2} * (6839273266*x^7 - 9809465289*x^6 + 31524099 \\
& 699*x^5 - 12024617744*x^4 + 13914887256*x^3 + 14839341696*x^2 - 14839341696*x) - 18483199488*x) + 61971531367*3868444992270541948232^{(1/4)} * \sqrt{62} * (1 \\
& 427210918*x^7 - 18462714328*x^6 + 71210222920*x^5 - 92387041920*x^4 + 119489780160*x^3 + 68726817792*x^2 - \sqrt{2} * (1033310523*x^7 - 13365477772*x^6 + \\
& 51521534980*x^5 - 66583614528*x^4 + 85122955872*x^3 + 53108877312*x^2 - 53108877312*x) - 68726817792*x) * \sqrt{2*x^2 - x + 3} * \sqrt{3531015707557 * \sqrt{2} + 4997704142500} - 4516423329856721284677540671884 * \sqrt{31} * \sqrt{2} * (123
\end{aligned}$$

$$\begin{aligned}
& 408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 \\
& - 3822336x^2 - \sqrt{2}(15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + \\
& 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) - 205291 \\
& 969538941876576251848722\sqrt{31}(254591x^8 - 4815126x^7 + 32303580x^6 \\
& - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488\sqrt{(2)} \\
& (4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - \\
& 1944x) + 144820224x)\sqrt{(3868444992270541948232^{1/4}\sqrt{1999081657})} \\
& \sqrt{62}\sqrt{31}\sqrt{2x^2 - x + 3}(\sqrt{2}(2141441x + 1076175) - 321 \\
& 7616x - 1065266)\sqrt{3531015707557\sqrt{2} + 4997704142500} + 15599087743 \\
& 0002205517374x^2 + 140073440957553000872744\sqrt{2}(2x^2 - x + 3) - 4807 \\
& 06581467965980267826x + 636697458897968185785200)/x^2) - 14630596389127106 \\
& 7870702891119222361424201\sqrt{31}(2828123x^8 - 9696916x^7 + 53385560x^6 \\
& - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744\sqrt{(2)} \\
& (1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1 \\
& 080x^2 + 4320x - 5184) + 223064064x - 94887936)/(2585191x^8 - 4661200x \\
& x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 \\
& - 24772608x + 18579456) + 3868444992270541948232^{1/4}\sqrt{1999081657} \\
& \sqrt{62}\sqrt{31}(124942603562500x^4 + 149931124275000x^3 + 14493342013 \\
& 2500x^2 - 3531015707557\sqrt{2}(25x^4 + 30x^3 + 29x^2 + 12x + 4) + 59 \\
& 972449710000x + 19990816570000)\sqrt{3531015707557\sqrt{2} + 4997704142500} \\
& )\log(3123565089062500/828550919*(3868444992270541948232^{1/4}\sqrt{1999081} \\
& 657)\sqrt{62}\sqrt{31}\sqrt{2x^2 - x + 3}(\sqrt{2}(2141441x + 1076175) - \\
& 3217616x - 1065266)\sqrt{3531015707557\sqrt{2} + 4997704142500} + 1559908 \\
& 77430002205517374x^2 + 140073440957553000872744\sqrt{2}(2x^2 - x + 3) - \\
& 480706581467965980267826x + 636697458897968185785200)/x^2) - 3868444992270 \\
& 541948232^{1/4}\sqrt{1999081657}\sqrt{62}\sqrt{31}(124942603562500x^4 + 1 \\
& 49931124275000x^3 + 144933420132500x^2 - 3531015707557\sqrt{2}(25x^4 + \\
& 30x^3 + 29x^2 + 12x + 4) + 59972449710000x + 19990816570000)\sqrt{35310} \\
& 15707557\sqrt{2} + 4997704142500)\log(-3123565089062500/828550919*(38684449 \\
& 92270541948232^{1/4}\sqrt{1999081657}\sqrt{62}\sqrt{31}\sqrt{2x^2 - x + 3} \\
& (\sqrt{2}(2141441x + 1076175) - 3217616x - 1065266)\sqrt{3531015707557\sqrt{2}} \\
& \sqrt{2} + 4997704142500) - 155990877430002205517374x^2 - 140073440957553000 \\
& 872744\sqrt{2}(2x^2 - x + 3) + 480706581467965980267826x - 6366974588979 \\
& 68185785200)/x^2) + 12139426558738796942545211648\sqrt{2}(25x^4 + 30x^3 \\
& + 29x^2 + 12x + 4)\log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 \\
& + 16x - 25) + 86845533393682860541101280(97155x^3 + 93872x^2 + 69621x \\
& + 22552)\sqrt{2x^2 - x + 3})/(25x^4 + 30x^3 + 29x^2 + 12x + 4)
\end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP  
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,  
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infini  
ty]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity  
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai  
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity  
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf  
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit  
y]Evaluation time: 59.56Done

**maple** [B] time = 0.38, size = 119458, normalized size = 425.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x)`

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")`

[Out] `integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^3, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(2x^2 - x + 3)^{5/2}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^3,x)`

[Out] `int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)`

[Out] `Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**3, x)`

$$3.76 \quad \int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}} dx$$

Optimal. Leaf size=185

$$-\frac{15428243\sqrt{2x^2-x+3}x^2}{131072} + \frac{1572007407\sqrt{2x^2-x+3}x}{7340032} + \frac{16493087661\sqrt{2x^2-x+3}}{29360128} + \frac{625}{16}\sqrt{2x^2-x+3}x^7 +$$

**Rubi [A]** time = 0.31, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1661, 640, 619, 215}

$$\frac{625}{16}\sqrt{2x^2-x+3}x^7 + \frac{57375}{448}\sqrt{2x^2-x+3}x^6 + \frac{2116475\sqrt{2x^2-x+3}x^5}{10752} + \frac{686531\sqrt{2x^2-x+3}x^4}{6144} - \frac{19750457\sqrt{2x^2-x+3}x^3}{229376} - \frac{15428243\sqrt{2x^2-x+3}x^2}{131072} + \frac{1572007407\sqrt{2x^2-x+3}x}{7340032} + \frac{16493087661\sqrt{2x^2-x+3}}{29360128} + \frac{2899366573\sinh^{-1}\left(\frac{1-4x}{\sqrt{20}}\right)}{8388608\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^4/Sqrt[3 - x + 2\*x^2], x]

[Out] (16493087661\*Sqrt[3 - x + 2\*x^2])/29360128 + (1572007407\*x\*Sqrt[3 - x + 2\*x^2])/7340032 - (15428243\*x^2\*Sqrt[3 - x + 2\*x^2])/131072 - (19750457\*x^3\*Sqrt[3 - x + 2\*x^2])/229376 + (686531\*x^4\*Sqrt[3 - x + 2\*x^2])/6144 + (2116475\*x^5\*Sqrt[3 - x + 2\*x^2])/10752 + (57375\*x^6\*Sqrt[3 - x + 2\*x^2])/448 + (625\*x^7\*Sqrt[3 - x + 2\*x^2])/16 + (2899366573\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(8388608\*Sqrt[2])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^4}{\sqrt{3 - x + 2x^2}} dx &= \frac{625}{16} x^7 \sqrt{3 - x + 2x^2} + \frac{1}{16} \int \frac{256 + 1536x + 6016x^2 + 14976x^3 + 28176x^4 + 37440x^5 + \dots}{\sqrt{3 - x + 2x^2}} \\
&= \frac{57375}{448} x^6 \sqrt{3 - x + 2x^2} + \frac{625}{16} x^7 \sqrt{3 - x + 2x^2} + \frac{1}{224} \int \frac{3584 + 21504x + 84224x^2 + 20\dots}{\sqrt{3 - x + 2x^2}} \\
&= \frac{2116475x^5 \sqrt{3 - x + 2x^2}}{10752} + \frac{57375}{448} x^6 \sqrt{3 - x + 2x^2} + \frac{625}{16} x^7 \sqrt{3 - x + 2x^2} + \int \frac{43008 + 258\dots}{\sqrt{3 - x + 2x^2}} \\
&= \frac{686531x^4 \sqrt{3 - x + 2x^2}}{6144} + \frac{2116475x^5 \sqrt{3 - x + 2x^2}}{10752} + \frac{57375}{448} x^6 \sqrt{3 - x + 2x^2} + \frac{625}{16} x^7 \sqrt{3 - x + 2x^2} \\
&= -\frac{19750457x^3 \sqrt{3 - x + 2x^2}}{229376} + \frac{686531x^4 \sqrt{3 - x + 2x^2}}{6144} + \frac{2116475x^5 \sqrt{3 - x + 2x^2}}{10752} + \frac{57375}{448} x^6 \sqrt{3 - x + 2x^2} \\
&= -\frac{15428243x^2 \sqrt{3 - x + 2x^2}}{131072} - \frac{19750457x^3 \sqrt{3 - x + 2x^2}}{229376} + \frac{686531x^4 \sqrt{3 - x + 2x^2}}{6144} + \frac{2116475x^5 \sqrt{3 - x + 2x^2}}{10752} \\
&= \frac{1572007407x \sqrt{3 - x + 2x^2}}{7340032} - \frac{15428243x^2 \sqrt{3 - x + 2x^2}}{131072} - \frac{19750457x^3 \sqrt{3 - x + 2x^2}}{229376} + \frac{686531x^4 \sqrt{3 - x + 2x^2}}{6144} \\
&= \frac{16493087661 \sqrt{3 - x + 2x^2}}{29360128} + \frac{1572007407x \sqrt{3 - x + 2x^2}}{7340032} - \frac{15428243x^2 \sqrt{3 - x + 2x^2}}{131072} \\
&= \frac{16493087661 \sqrt{3 - x + 2x^2}}{29360128} + \frac{1572007407x \sqrt{3 - x + 2x^2}}{7340032} - \frac{15428243x^2 \sqrt{3 - x + 2x^2}}{131072} \\
&= \frac{16493087661 \sqrt{3 - x + 2x^2}}{29360128} + \frac{1572007407x \sqrt{3 - x + 2x^2}}{7340032} - \frac{15428243x^2 \sqrt{3 - x + 2x^2}}{131072}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 75, normalized size = 0.41

$$\frac{4\sqrt{2x^2 - x + 3} (3440640000x^7 + 11280384000x^6 + 17338163200x^5 + 9842108416x^4 - 7584175488x^3 - 10367779296x^2 + 18864088884x + 49479262983) + 60886698033\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{352321536}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^4/Sqrt[3 - x + 2\*x^2], x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(49479262983 + 18864088884\*x - 10367779296\*x^2 - 7584175488\*x^3 + 9842108416\*x^4 + 17338163200\*x^5 + 11280384000\*x^6 + 3440640000\*x^7) + 60886698033\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/352321536

**IntegrateAlgebraic [A]** time = 0.80, size = 90, normalized size = 0.49

$$\frac{2899366573 \log(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1)}{8388608\sqrt{2}} + \frac{\sqrt{2x^2 - x + 3} (3440640000x^7 + 11280384000x^6 + 17338163200x^5 + 9842108416x^4 - 7584175488x^3 - 10367779296x^2 + 18864088884x + 49479262983)}{88080384}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^4/Sqrt[3 - x + 2\*x^2], x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(49479262983 + 18864088884\*x - 10367779296\*x^2 - 7584175488\*x^3 + 9842108416\*x^4 + 17338163200\*x^5 + 11280384000\*x^6 + 3440640000\*x^7))/88080384 + (2899366573\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(8388608\*Sqrt[2])

**fricas** [A] time = 0.43, size = 88, normalized size = 0.48

$$\frac{1}{88080384} (3440640000x^7 + 11280384000x^6 + 17338163200x^5 + 9842108416x^4 - 7584175488x^3 - 10367779296x^2 + 18864088884x + 49479262983)\sqrt{2x^2 - x + 3} + \frac{2899366573}{33554432} \sqrt{2} \log\left(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/88080384\*(3440640000\*x^7 + 11280384000\*x^6 + 17338163200\*x^5 + 9842108416\*x^4 - 7584175488\*x^3 - 10367779296\*x^2 + 18864088884\*x + 49479262983)\*sqrt(2\*x^2 - x + 3) + 2899366573/33554432\*sqrt(2)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.50, size = 83, normalized size = 0.45

$$\frac{1}{88080384} (4(8(4(16(100(120(140x + 459)x + 84659)x + 4805717)x - 59251371)x - 323993103)x + 4716022221)x + 49479262983)\sqrt{2x^2 - x + 3} + \frac{2899366573}{16777216} \sqrt{2} \log(-2\sqrt{2}(\sqrt{2x^2 - x + 3}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] 1/88080384\*(4\*(8\*(4\*(16\*(100\*(120\*(140\*x + 459)\*x + 84659)\*x + 4805717)\*x - 59251371)\*x - 323993103)\*x + 4716022221)\*x + 49479262983)\*sqrt(2\*x^2 - x + 3) + 2899366573/16777216\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.02, size = 147, normalized size = 0.79

$$\frac{625\sqrt{2x^2-x+3}x^7}{16} + \frac{57375\sqrt{2x^2-x+3}x^6}{448} + \frac{2116475\sqrt{2x^2-x+3}x^5}{10752} + \frac{686531\sqrt{2x^2-x+3}x^4}{6144} - \frac{19750457\sqrt{2x^2-x+3}x^3}{229376} - \frac{15428243\sqrt{2x^2-x+3}x^2}{131072} + \frac{1572007407\sqrt{2x^2-x+3}x}{7340032} - \frac{2899366573\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}(x-1)}{23}\right)}{16777216} + \frac{16493087661\sqrt{2x^2-x+3}}{29360128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(1/2),x)

[Out] 16493087661/29360128\*(2\*x^2-x+3)^(1/2)-2899366573/16777216\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+625/16\*x^7\*(2\*x^2-x+3)^(1/2)+57375/448\*x^6\*(2\*x^2-x+3)^(1/2)+2116475/10752\*x^5\*(2\*x^2-x+3)^(1/2)+686531/6144\*x^4\*(2\*x^2-x+3)^(1/2)-19750457/229376\*x^3\*(2\*x^2-x+3)^(1/2)-15428243/131072\*x^2\*(2\*x^2-x+3)^(1/2)+1572007407/7340032\*x\*(2\*x^2-x+3)^(1/2)

**maxima** [A] time = 1.00, size = 148, normalized size = 0.80

$$\frac{625}{16} \sqrt{2x^2-x+3}x^7 + \frac{57375}{448} \sqrt{2x^2-x+3}x^6 + \frac{2116475}{10752} \sqrt{2x^2-x+3}x^5 + \frac{686531}{6144} \sqrt{2x^2-x+3}x^4 - \frac{19750457}{229376} \sqrt{2x^2-x+3}x^3 - \frac{15428243}{131072} \sqrt{2x^2-x+3}x^2 + \frac{1572007407}{7340032} \sqrt{2x^2-x+3}x - \frac{2899366573}{16777216} \sqrt{2} \operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{16493087661}{29360128} \sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] 625/16\*sqrt(2\*x^2 - x + 3)\*x^7 + 57375/448\*sqrt(2\*x^2 - x + 3)\*x^6 + 2116475/10752\*sqrt(2\*x^2 - x + 3)\*x^5 + 686531/6144\*sqrt(2\*x^2 - x + 3)\*x^4 - 19750457/229376\*sqrt(2\*x^2 - x + 3)\*x^3 - 15428243/131072\*sqrt(2\*x^2 - x + 3)\*x^2 + 1572007407/7340032\*sqrt(2\*x^2 - x + 3)\*x - 2899366573/16777216\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) + 16493087661/29360128\*sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^4}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(1/2), x)`

[Out] `int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^4}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(1/2), x)`

[Out] `Integral((5*x**2 + 3*x + 2)**4/sqrt(2*x**2 - x + 3), x)`



$$3.77 \quad \int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}} dx$$

Optimal. Leaf size=143

$$\frac{3387\sqrt{2x^2-x+3}x^2}{1024} - \frac{372783\sqrt{2x^2-x+3}x}{8192} - \frac{203373\sqrt{2x^2-x+3}}{32768} + \frac{125}{12}\sqrt{2x^2-x+3}x^5 + \frac{1355}{48}\sqrt{2x^2-x+3}$$

**Rubi [A]** time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1661, 640, 619, 215}

$$\frac{125}{12}\sqrt{2x^2-x+3}x^5 + \frac{1355}{48}\sqrt{2x^2-x+3}x^4 + \frac{8185}{256}\sqrt{2x^2-x+3}x^3 - \frac{3387\sqrt{2x^2-x+3}x^2}{1024} - \frac{372783\sqrt{2x^2-x+3}x}{8192} - \frac{203373\sqrt{2x^2-x+3}}{32768} - \frac{9267707 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^3/Sqrt[3 - x + 2\*x^2], x]

[Out] (-203373\*Sqrt[3 - x + 2\*x^2])/32768 - (372783\*x\*Sqrt[3 - x + 2\*x^2])/8192 - (3387\*x^2\*Sqrt[3 - x + 2\*x^2])/1024 + (8185\*x^3\*Sqrt[3 - x + 2\*x^2])/256 + (1355\*x^4\*Sqrt[3 - x + 2\*x^2])/48 + (125\*x^5\*Sqrt[3 - x + 2\*x^2])/12 - (9267707\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(65536\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^3}{\sqrt{3 - x + 2x^2}} dx &= \frac{125}{12} x^5 \sqrt{3 - x + 2x^2} + \frac{1}{12} \int \frac{96 + 432x + 1368x^2 + 2484x^3 + 1545x^4 + \frac{6775x^5}{2}}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{1355}{48} x^4 \sqrt{3 - x + 2x^2} + \frac{125}{12} x^5 \sqrt{3 - x + 2x^2} + \frac{1}{120} \int \frac{960 + 4320x + 13680x^2 - 15810x^3}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} + \frac{1355}{48} x^4 \sqrt{3 - x + 2x^2} + \frac{125}{12} x^5 \sqrt{3 - x + 2x^2} + \frac{1}{960} \int \frac{7680 + 32640x - 15810x^2}{\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} + \frac{1355}{48} x^4 \sqrt{3 - x + 2x^2} + \frac{125}{12} x^5 \sqrt{3 - x + 2x^2} \\
&= -\frac{372783x \sqrt{3 - x + 2x^2}}{8192} - \frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} + \frac{1355}{48} x^4 \sqrt{3 - x + 2x^2} \\
&= -\frac{203373 \sqrt{3 - x + 2x^2}}{32768} - \frac{372783x \sqrt{3 - x + 2x^2}}{8192} - \frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} \\
&= -\frac{203373 \sqrt{3 - x + 2x^2}}{32768} - \frac{372783x \sqrt{3 - x + 2x^2}}{8192} - \frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} \\
&= -\frac{203373 \sqrt{3 - x + 2x^2}}{32768} - \frac{372783x \sqrt{3 - x + 2x^2}}{8192} - \frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 65, normalized size = 0.45

$$\frac{4\sqrt{2x^2 - x + 3} (1024000x^5 + 2775040x^4 + 3143040x^3 - 325152x^2 - 4473396x - 610119) - 27803121\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{393216}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^3/Sqrt[3 - x + 2\*x^2], x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(-610119 - 4473396\*x - 325152\*x^2 + 3143040\*x^3 + 2775040\*x^4 + 1024000\*x^5) - 27803121\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/393216

**IntegrateAlgebraic [A]** time = 0.60, size = 80, normalized size = 0.56

$$\frac{\sqrt{2x^2 - x + 3} (1024000x^5 + 2775040x^4 + 3143040x^3 - 325152x^2 - 4473396x - 610119)}{98304} - \frac{9267707 \log(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1)}{65536\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^3/Sqrt[3 - x + 2\*x^2], x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(-610119 - 4473396\*x - 325152\*x^2 + 3143040\*x^3 + 2775040\*x^4 + 1024000\*x^5))/98304 - (9267707\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(65536\*Sqrt[2])

**fricas [A]** time = 0.41, size = 78, normalized size = 0.55

$$\frac{1}{98304} (1024000x^5 + 2775040x^4 + 3143040x^3 - 325152x^2 - 4473396x - 610119)\sqrt{2x^2 - x + 3} + \frac{9267707}{262144} \sqrt{2} \log\left(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/98304\*(1024000\*x^5 + 2775040\*x^4 + 3143040\*x^3 - 325152\*x^2 - 4473396\*x - 610119)\*sqrt(2\*x^2 - x + 3) + 9267707/262144\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.52, size = 73, normalized size = 0.51

$$\frac{1}{98304} (4 (8 (20 (16 (100 x + 271) x + 4911) x - 10161) x - 1118349) x - 610119) \sqrt{2 x^2 - x + 3} - \frac{9267707}{131072} \sqrt{2} \log \left( -2 \sqrt{2} \left( \sqrt{2} x - \sqrt{2 x^2 - x + 3} \right) + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] 1/98304\*(4\*(8\*(20\*(16\*(100\*x + 271)\*x + 4911)\*x - 10161)\*x - 1118349)\*x - 610119)\*sqrt(2\*x^2 - x + 3) - 9267707/131072\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 113, normalized size = 0.79

$$\frac{125\sqrt{2x^2-x+3}x^5}{12} + \frac{1355\sqrt{2x^2-x+3}x^4}{48} + \frac{8185\sqrt{2x^2-x+3}x^3}{256} - \frac{3387\sqrt{2x^2-x+3}x^2}{1024} - \frac{372783\sqrt{2x^2-x+3}x}{8192} + \frac{9267707\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{131072} - \frac{203373\sqrt{2x^2-x+3}}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(1/2),x)

[Out] -203373/32768\*(2\*x^2-x+3)^(1/2)+9267707/131072\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+125/12\*(2\*x^2-x+3)^(1/2)\*x^5+1355/48\*(2\*x^2-x+3)^(1/2)\*x^4+8185/256\*(2\*x^2-x+3)^(1/2)\*x^3-3387/1024\*(2\*x^2-x+3)^(1/2)\*x^2-372783/8192\*(2\*x^2-x+3)^(1/2)\*x

**maxima** [A] time = 0.98, size = 114, normalized size = 0.80

$$\frac{125}{12} \sqrt{2x^2-x+3}x^5 + \frac{1355}{48} \sqrt{2x^2-x+3}x^4 + \frac{8185}{256} \sqrt{2x^2-x+3}x^3 - \frac{3387}{1024} \sqrt{2x^2-x+3}x^2 - \frac{372783}{8192} \sqrt{2x^2-x+3}x + \frac{9267707}{131072} \sqrt{2} \operatorname{arsinh}\left(\frac{1}{23} \sqrt{23}(4x-1)\right) - \frac{203373}{32768} \sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] 125/12\*sqrt(2\*x^2 - x + 3)\*x^5 + 1355/48\*sqrt(2\*x^2 - x + 3)\*x^4 + 8185/256\*sqrt(2\*x^2 - x + 3)\*x^3 - 3387/1024\*sqrt(2\*x^2 - x + 3)\*x^2 - 372783/8192\*sqrt(2\*x^2 - x + 3)\*x + 9267707/131072\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 203373/32768\*sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^3}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^3/(2\*x^2 - x + 3)^(1/2),x)

[Out] int((3\*x + 5\*x^2 + 2)^3/(2\*x^2 - x + 3)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^3}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*3/(2\*x\*\*2-x+3)\*\*(1/2),x)

[Out] Integral((5\*x\*\*2 + 3\*x + 2)\*\*3/sqrt(2\*x\*\*2 - x + 3), x)

$$3.78 \quad \int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}} dx$$

Optimal. Leaf size=101

$$\frac{655}{96}\sqrt{2x^2-x+3}x^2 + \frac{3443}{768}\sqrt{2x^2-x+3}x - \frac{11373\sqrt{2x^2-x+3}}{1024} + \frac{25}{8}\sqrt{2x^2-x+3}x^3 + \frac{30725 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

**Rubi [A]** time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1661, 640, 619, 215}

$$\frac{25}{8}\sqrt{2x^2-x+3}x^3 + \frac{655}{96}\sqrt{2x^2-x+3}x^2 + \frac{3443}{768}\sqrt{2x^2-x+3}x - \frac{11373\sqrt{2x^2-x+3}}{1024} + \frac{30725 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^2/Sqrt[3 - x + 2\*x^2], x]

[Out] (-11373\*Sqrt[3 - x + 2\*x^2])/1024 + (3443\*x\*Sqrt[3 - x + 2\*x^2])/768 + (655\*x^2\*Sqrt[3 - x + 2\*x^2])/96 + (25\*x^3\*Sqrt[3 - x + 2\*x^2])/8 + (30725\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(2048\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[Rt[b, 2]\*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}} dx &= \frac{25}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{8} \int \frac{32+96x+7x^2+\frac{655x^3}{2}}{\sqrt{3-x+2x^2}} dx \\
&= \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{48} \int \frac{192-1389x+\frac{3443x^2}{4}}{\sqrt{3-x+2x^2}} dx \\
&= \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{192} \int \frac{-\frac{7257}{4}-3}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{11373\sqrt{3-x+2x^2}}{1024} + \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+2x^2} \\
&= -\frac{11373\sqrt{3-x+2x^2}}{1024} + \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+2x^2} \\
&= -\frac{11373\sqrt{3-x+2x^2}}{1024} + \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 55, normalized size = 0.54

$$\frac{4\sqrt{2x^2-x+3} (9600x^3 + 20960x^2 + 13772x - 34119) + 92175\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{12288}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^2/Sqrt[3 - x + 2\*x^2], x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(-34119 + 13772\*x + 20960\*x^2 + 9600\*x^3) + 92175\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/12288

**IntegrateAlgebraic [A]** time = 0.49, size = 70, normalized size = 0.69

$$\frac{30725 \log\left(2\sqrt{2}\sqrt{2x^2-x+3} - 4x + 1\right)}{2048\sqrt{2}} + \frac{\sqrt{2x^2-x+3} (9600x^3 + 20960x^2 + 13772x - 34119)}{3072}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^2/Sqrt[3 - x + 2\*x^2], x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(-34119 + 13772\*x + 20960\*x^2 + 9600\*x^3))/3072 + (30725\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(2048\*Sqrt[2])

**fricas [A]** time = 0.41, size = 68, normalized size = 0.67

$$\frac{1}{3072} (9600x^3 + 20960x^2 + 13772x - 34119)\sqrt{2x^2-x+3} + \frac{30725}{8192} \sqrt{2} \log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(1/2), x, algorithm="fricas")

[Out] 1/3072\*(9600\*x^3 + 20960\*x^2 + 13772\*x - 34119)\*sqrt(2\*x^2 - x + 3) + 30725/8192\*sqrt(2)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac [A]** time = 0.49, size = 63, normalized size = 0.62

$$\frac{1}{3072} (4(40(60x + 131)x + 3443)x - 34119)\sqrt{2x^2 - x + 3} + \frac{30725}{4096} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] 1/3072\*(4\*(40\*(60\*x + 131)\*x + 3443)\*x - 34119)\*sqrt(2\*x^2 - x + 3) + 30725/4096\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple [A]** time = 0.01, size = 79, normalized size = 0.78

$$\frac{25\sqrt{2x^2 - x + 3}x^3}{8} + \frac{655\sqrt{2x^2 - x + 3}x^2}{96} + \frac{3443\sqrt{2x^2 - x + 3}x}{768} - \frac{30725\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096} - \frac{11373\sqrt{2x^2 - x + 3}}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(1/2),x)

[Out] 25/8\*(2\*x^2-x+3)^(1/2)\*x^3+655/96\*(2\*x^2-x+3)^(1/2)\*x^2+3443/768\*(2\*x^2-x+3)^(1/2)\*x-11373/1024\*(2\*x^2-x+3)^(1/2)-30725/4096\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))

**maxima [A]** time = 0.97, size = 80, normalized size = 0.79

$$\frac{25}{8} \sqrt{2x^2 - x + 3}x^3 + \frac{655}{96} \sqrt{2x^2 - x + 3}x^2 + \frac{3443}{768} \sqrt{2x^2 - x + 3}x - \frac{30725}{4096} \sqrt{2} \operatorname{arsinh}\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) - \frac{11373}{1024} \sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] 25/8\*sqrt(2\*x^2 - x + 3)\*x^3 + 655/96\*sqrt(2\*x^2 - x + 3)\*x^2 + 3443/768\*sqrt(2\*x^2 - x + 3)\*x - 30725/4096\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 11373/1024\*sqrt(2\*x^2 - x + 3)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^2}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^2/(2\*x^2 - x + 3)^(1/2),x)

[Out] int((3\*x + 5\*x^2 + 2)^2/(2\*x^2 - x + 3)^(1/2), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^2}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*2/(2\*x\*\*2-x+3)\*\*(1/2),x)

[Out] Integral((5\*x\*\*2 + 3\*x + 2)\*\*2/sqrt(2\*x\*\*2 - x + 3), x)

$$3.79 \quad \int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}} dx$$

Optimal. Leaf size=59

$$\frac{5}{4}\sqrt{2x^2-x+3}x + \frac{39}{16}\sqrt{2x^2-x+3} + \frac{17 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1661, 640, 619, 215}

$$\frac{5}{4}\sqrt{2x^2-x+3}x + \frac{39}{16}\sqrt{2x^2-x+3} + \frac{17 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)/Sqrt[3 - x + 2\*x^2], x]

[Out] (39\*Sqrt[3 - x + 2\*x^2])/16 + (5\*x\*Sqrt[3 - x + 2\*x^2])/4 + (17\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(32\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}} dx &= \frac{5}{4}x\sqrt{3-x+2x^2} + \frac{1}{4} \int \frac{-7+\frac{39x}{2}}{\sqrt{3-x+2x^2}} dx \\
&= \frac{39}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} - \frac{17}{32} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
&= \frac{39}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} - \frac{17 \operatorname{Subst} \left( \int \frac{1}{\sqrt{1+\frac{x^2}{23}} dx, x, -1+4x \right)}{32\sqrt{46}} \\
&= \frac{39}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} + \frac{17 \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{32\sqrt{2}}
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 45, normalized size = 0.76

$$\frac{1}{64} \left( 4\sqrt{2x^2-x+3}(20x+39) + 17\sqrt{2} \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)/Sqrt[3 - x + 2\*x^2], x]

[Out] (4\*(39 + 20\*x)\*Sqrt[3 - x + 2\*x^2] + 17\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/64

**IntegrateAlgebraic** [A] time = 0.28, size = 60, normalized size = 1.02

$$\frac{1}{16} \sqrt{2x^2-x+3}(20x+39) + \frac{17 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)/Sqrt[3 - x + 2\*x^2], x]

[Out] ((39 + 20\*x)\*Sqrt[3 - x + 2\*x^2])/16 + (17\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(32\*Sqrt[2])

**fricas** [A] time = 0.41, size = 58, normalized size = 0.98

$$\frac{1}{16} \sqrt{2x^2-x+3}(20x+39) + \frac{17}{128} \sqrt{2} \log(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(1/2), x, algorithm="fricas")

[Out] 1/16\*sqrt(2\*x^2 - x + 3)\*(20\*x + 39) + 17/128\*sqrt(2)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.53, size = 53, normalized size = 0.90

$$\frac{1}{16} \sqrt{2x^2-x+3}(20x+39) + \frac{17}{64} \sqrt{2} \log(-2\sqrt{2}(\sqrt{2}x - \sqrt{2x^2-x+3}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(1/2), x, algorithm="giac")



[Out]  $1/16*\sqrt{2*x^2 - x + 3}*(20*x + 39) + 17/64*\sqrt{2}*\log(-2*\sqrt{2}*(\sqrt{2}*x - \sqrt{2*x^2 - x + 3}) + 1)$

**maple** [A] time = 0.01, size = 45, normalized size = 0.76

$$\frac{5\sqrt{2x^2 - x + 3}x}{4} - \frac{17\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{64} + \frac{39\sqrt{2x^2 - x + 3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x)`

[Out]  $5/4*(2*x^2-x+3)^(1/2)*x+39/16*(2*x^2-x+3)^(1/2)-17/64*2^(1/2)*\operatorname{arcsinh}(4/23*23^(1/2)*(x-1/4))$

**maxima** [A] time = 0.96, size = 46, normalized size = 0.78

$$\frac{5}{4}\sqrt{2x^2 - x + 3}x - \frac{17}{64}\sqrt{2} \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + \frac{39}{16}\sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")`

[Out]  $5/4*\sqrt{2*x^2 - x + 3}*x - 17/64*\sqrt{2}*\operatorname{arcsinh}(1/23*\sqrt{23}*(4*x - 1)) + 39/16*\sqrt{2*x^2 - x + 3}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{5x^2 + 3x + 2}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(1/2),x)`

[Out] `int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2 + 3x + 2}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(1/2),x)`

[Out] `Integral((5*x**2 + 3*x + 2)/sqrt(2*x**2 - x + 3), x)`

$$3.80 \quad \int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx$$

**Optimal.** Leaf size=148

$$\sqrt{\frac{1}{682}(13+10\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(13+10\sqrt{2})}}((13+10\sqrt{2})x+3\sqrt{2}+7)}{\sqrt{2x^2-x+3}} \right) - \sqrt{\frac{1}{682}(10\sqrt{2}-13)} \tanh^{-1} \left( \frac{\sqrt{\frac{11}{31(10\sqrt{2}-13)}}((13-10\sqrt{2})x-3\sqrt{2}+7)}{\sqrt{2x^2-x+3}} \right)$$

**Rubi [A]** time = 0.31, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {986, 1029, 204, 206}

$$\sqrt{\frac{1}{682}(13+10\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(13+10\sqrt{2})}}((13+10\sqrt{2})x+3\sqrt{2}+7)}{\sqrt{2x^2-x+3}} \right) - \sqrt{\frac{1}{682}(10\sqrt{2}-13)} \tanh^{-1} \left( \frac{\sqrt{\frac{11}{31(10\sqrt{2}-13)}}((13-10\sqrt{2})x-3\sqrt{2}+7)}{\sqrt{2x^2-x+3}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)), x]

[Out] Sqrt[(13 + 10\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(13 + 10\*Sqrt[2]))])\*(7 + 3\*Sqrt[2] + (13 + 10\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2] - Sqrt[(-13 + 10\*Sqrt[2])/682]\*ArcTanh[(Sqrt[11/(31\*(-13 + 10\*Sqrt[2]))])\*(7 - 3\*Sqrt[2] + (13 - 10\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 986

Int[1/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] :> With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 2]}, Dist[1/(2\*q), Int[(c\*d - a\*f + q + (c\*e - b\*f)\*x)/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[1/(2\*q), Int[(c\*d - a\*f - q + (c\*e - b\*f)\*x)/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[c\*e - b\*f, 0] && NegQ[b^2 - 4\*a\*c]

#### Rule 1029

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] :> Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

#### Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx = -\frac{\int \frac{11-11\sqrt{2}-11x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{22\sqrt{2}} + \frac{\int \frac{11+11\sqrt{2}-11x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{22\sqrt{2}}$$

$$= -\left(\frac{1}{2}(11(20-13\sqrt{2}))\right) \text{Subst}\left(\int \frac{1}{-3751(13-10\sqrt{2})-11x^2} dx, x, \frac{11}{\sqrt{3-x+2x^2}}\right)$$

$$= \sqrt{\frac{1}{682}(13+10\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31(13+10\sqrt{2})}}(7+3\sqrt{2}+(13+10\sqrt{2})x)}{\sqrt{3-x+2x^2}}\right)$$

**Mathematica [C]** time = 0.30, size = 176, normalized size = 1.19

$$\frac{\sqrt{13+i\sqrt{31}}(\sqrt{31}+13i) \tanh^{-1}\left(\frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) + \sqrt{13-i\sqrt{31}}(\sqrt{31}-13i) \tanh^{-1}\left(\frac{(-22+4i\sqrt{31})x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{20\sqrt{682}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)), x]

[Out] -1/20\*(Sqrt[13 + I\*Sqrt[31]]\*(13\*I + Sqrt[31])\*ArcTanh[(63 + I\*Sqrt[31] + (-22 - (4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])] + Sqrt[13 - I\*Sqrt[31]]\*(-13\*I + Sqrt[31])\*ArcTanh[(63 - I\*Sqrt[31] + (-22 + (4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])]/Sqrt[682]

**IntegrateAlgebraic [C]** time = 0.36, size = 135, normalized size = 0.91

$$\text{RootSum}\left[-5\#1^4 + 6\sqrt{2}\#1^3 + 17\#1^2 - 26\sqrt{2}\#1 - 56\&, \frac{2\sqrt{2}\#1 \log(-\#1 + \sqrt{2x^2-x+3} - \sqrt{2}x) + \log(-\#1 + \sqrt{2x^2-x+3} - \sqrt{2}x)}{-10\#1^3 + 9\sqrt{2}\#1^2 + 17\#1 - 13\sqrt{2}}\right]\&$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)), x]

[Out] RootSum[-56 - 26\*Sqrt[2]\*#1 + 17\*#1^2 + 6\*Sqrt[2]\*#1^3 - 5\*#1^4 & , (Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1] + 2\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1)/(-13\*Sqrt[2] + 17\*#1 + 9\*Sqrt[2]\*#1^2 - 10\*#1^3) & ]

**fricas [B]** time = 1.13, size = 2002, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)/(2\*x^2-x+3)^(1/2), x, algorithm="fricas")

[Out] -1/845680\*sqrt(341)\*200^(1/4)\*sqrt(31)\*sqrt(5)\*sqrt(13\*sqrt(2) + 20)\*(13\*sqrt(2) - 20)\*log(1240\*(sqrt(341)\*200^(1/4)\*sqrt(31)\*sqrt(5)\*sqrt(2\*x^2 - x + 3)\*(sqrt(2)\*(4\*x - 1) - 3\*x - 5)\*sqrt(13\*sqrt(2) + 20) + 7595\*x^2 + 6820\*sqrt(2)\*(2\*x^2 - x + 3) - 23405\*x + 31000)/x^2) + 1/845680\*sqrt(341)\*200^(1/4)\*sqrt(31)\*sqrt(5)\*sqrt(13\*sqrt(2) + 20)\*(13\*sqrt(2) - 20)\*log(-1240\*(sqrt(341)\*200^(1/4)\*sqrt(31)\*sqrt(5)\*sqrt(2\*x^2 - x + 3)\*(sqrt(2)\*(4\*x - 1) - 3\*x - 5)\*sqrt(13\*sqrt(2) + 20) - 7595\*x^2 - 6820\*sqrt(2)\*(2\*x^2 - x + 3) + 23405\*x - 31000)/x^2) - 1/6820\*sqrt(341)\*200^(1/4)\*sqrt(5)\*sqrt(2)\*sqrt(13\*sqrt(2) + 20)\*arctan(1/2762875\*(14260\*sqrt(341)\*sqrt(5)\*sqrt(2\*x^2 - x + 3)\*(11\*200^(3/4)\*(8056\*x^7 - 28976\*x^6 + 61838\*x^5 - 93342\*x^4 + 45376\*x^3 - 1

$$\begin{aligned}
& 8288x^2 - \sqrt{2}*(4702x^7 - 19541x^6 + 40352x^5 - 68777x^4 + 35480x^3 - 19080x^2 - 34560x + 27648) - 55296x + 34560) + 5*200^{(1/4)}*(18463x^7 - 280047x^6 + 1453472x^5 - 3238500x^4 + 4140576x^3 - 2378592x^2 - \sqrt{2}*(11418x^7 - 177633x^6 + 957180x^5 - 2237548x^4 + 2920320x^3 - 2005920x^2 - 1990656x + 1534464) - 3068928x + 1990656))*\sqrt{13*\sqrt{2} + 20} + 7843000*\sqrt{31}*\sqrt{2}*(28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2}*(8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - 2*\sqrt{310}*(\sqrt{341}*\sqrt{5}*\sqrt{2x^2 - x + 3}*(11*200^{(3/4)}*(30876x^7 - 44014x^6 + 139674x^5 - 42464x^4 + 38736x^3 + 89856x^2 - \sqrt{2}*(15454x^7 - 22399x^6 + 73509x^5 - 37360x^4 + 52200x^3 + 13824x^2 - 13824x) - 89856x) + 5*200^{(1/4)}*(69479x^7 - 898236x^6 + 3454740x^5 - 4394304x^4 + 5347296x^3 + 4478976x^2 - \sqrt{2}*(38627x^7 - 500012x^6 + 1934180x^5 - 2560320x^4 + 3506400x^3 + 1202688x^2 - 1202688x) - 4478976x))*\sqrt{13*\sqrt{2} + 20} + 550*\sqrt{31})*\sqrt{2}*(123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2}*(15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) + 25*\sqrt{31}*(254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488*\sqrt{2}*(4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x))*\sqrt{-(\sqrt{341}*200^{(1/4)}*\sqrt{31}*\sqrt{5}*\sqrt{2x^2 - x + 3}*(\sqrt{2}*(4x - 1) - 3x - 5)*\sqrt{13*\sqrt{2} + 20} - 7595x^2 - 6820*\sqrt{2}*(2x^2 - x + 3) + 23405x - 31000)/x^2) + 89125*\sqrt{31}*(2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744*\sqrt{2}*(1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936) )/(2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 4249088x^3 - 34615296x^2 - 24772608x + 18579456)) - 1/6820*\sqrt{341}*200^{(1/4)}*\sqrt{5}*\sqrt{2}*\sqrt{13*\sqrt{2} + 20}*\arctan(1/2762875*(14260*\sqrt{341})*\sqrt{5}*\sqrt{2x^2 - x + 3}*(11*200^{(3/4)}*(8056x^7 - 28976x^6 + 61838x^5 - 93342x^4 + 45376x^3 - 18288x^2 - \sqrt{2}*(4702x^7 - 19541x^6 + 40352x^5 - 68777x^4 + 35480x^3 - 19080x^2 - 34560x + 27648) - 55296x + 34560) + 5*200^{(1/4)}*(18463x^7 - 280047x^6 + 1453472x^5 - 3238500x^4 + 4140576x^3 - 2378592x^2 - \sqrt{2}*(11418x^7 - 177633x^6 + 957180x^5 - 2237548x^4 + 2920320x^3 - 2005920x^2 - 1990656x + 1534464) - 3068928x + 1990656))*\sqrt{13*\sqrt{2} + 20} - 7843000*\sqrt{31}*\sqrt{2}*(28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2}*(8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - 2*\sqrt{310}*(\sqrt{341}*\sqrt{5}*\sqrt{2x^2 - x + 3}*(11*200^{(3/4)}*(30876x^7 - 44014x^6 + 139674x^5 - 42464x^4 + 38736x^3 + 89856x^2 - \sqrt{2}*(15454x^7 - 22399x^6 + 73509x^5 - 37360x^4 + 52200x^3 + 13824x^2 - 13824x) - 89856x) + 5*200^{(1/4)}*(69479x^7 - 898236x^6 + 3454740x^5 - 4394304x^4 + 5347296x^3 + 4478976x^2 - \sqrt{2}*(38627x^7 - 500012x^6 + 1934180x^5 - 2560320x^4 + 3506400x^3 + 1202688x^2 - 1202688x) - 4478976x))*\sqrt{13*\sqrt{2} + 20} - 550*\sqrt{31})*\sqrt{2}*(123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2}*(15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) - 25*\sqrt{31}*(254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488*\sqrt{2}*(4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x))*\sqrt{(\sqrt{341}*200^{(1/4)}*\sqrt{31})*\sqrt{5}*\sqrt{2x^2 - x + 3}*(\sqrt{2}*(4x - 1) - 3x - 5)*\sqrt{13*\sqrt{2} + 20} + 7595x^2 + 6820*\sqrt{2}*(2x^2 - x + 3) - 23405x + 31000)/x^2) - 89125*\sqrt{31}*(2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744*\sqrt{2}*(1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936) )/(2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 4249088x^3 - 34615296x^2 - 24772608x + 18579456))
\end{aligned}$$

+ 490880\*x^5 - 13562944\*x^4 + 44249088\*x^3 - 34615296\*x^2 - 24772608\*x + 18579456))

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)/(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Francis algorithm failure for[-1.0,infinity, infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity ]proot error [1.0,infinity,infinity,infinity,infinity]Evaluation time: 7.57 Done

**maple** [B] time = 0.00, size = 684, normalized size = 4.62

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5\*x^2+3\*x+2)/(2\*x^2-x+3)^(1/2),x)

[Out] 
$$\frac{1}{21142} \cdot (8 \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 3 \cdot 2^{1/2} \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 8 - 3 \cdot 2^{1/2})^{1/2} \cdot 2^{1/2} \cdot (369 \cdot 2^{1/2} \cdot (-8866 + 6820 \cdot 2^{1/2}))^{1/2} \cdot (-775687 + 549362 \cdot 2^{1/2})^{1/2} \cdot \arctan(1/11692487 \cdot (-775687 + 549362 \cdot 2^{1/2}))^{1/2} \cdot (-23 \cdot (8 + 3 \cdot 2^{1/2})) \cdot (-23 \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 24 \cdot 2^{1/2} - 41)^{1/2} \cdot (6485 \cdot 2^{1/2} \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 10368 \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 22379 \cdot 2^{1/2} + 32016) / (23 \cdot (x+2)^{1/2} - 1)^4 / (-x+2)^{1/2} + 1)^4 + 82 \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 23) \cdot (x+2)^{1/2} - 1) / (-x+2)^{1/2} + 1) \cdot (8 + 3 \cdot 2^{1/2})) + 520 \cdot (-8866 + 6820 \cdot 2^{1/2})^{1/2} \cdot (-775687 + 549362 \cdot 2^{1/2})^{1/2} \cdot \arctan(1/11692487 \cdot (-775687 + 549362 \cdot 2^{1/2}))^{1/2} \cdot (-23 \cdot (8 + 3 \cdot 2^{1/2})) \cdot (-23 \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 24 \cdot 2^{1/2} - 41)^{1/2} \cdot (6485 \cdot 2^{1/2} \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 10368 \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 22379 \cdot 2^{1/2} + 32016) / (23 \cdot (x+2)^{1/2} - 1)^4 / (-x+2)^{1/2} + 1)^4 + 82 \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 23) \cdot (x+2)^{1/2} - 1) / (-x+2)^{1/2} + 1) \cdot (8 + 3 \cdot 2^{1/2})) + 465124 \cdot \operatorname{arctanh}(3/2 \cdot (8 \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 3 \cdot 2^{1/2} \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 8 - 3 \cdot 2^{1/2})^{1/2} / (-8866 + 6820 \cdot 2^{1/2})^{1/2} \cdot 2^{1/2} - 866822 \cdot \operatorname{arctanh}(3/2 \cdot (8 \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 3 \cdot 2^{1/2} \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 8 - 3 \cdot 2^{1/2})^{1/2} / (-8866 + 6820 \cdot 2^{1/2})^{1/2} \cdot 2^{1/2} / ((8 \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 3 \cdot 2^{1/2} \cdot (x+2)^{1/2} - 1)^2 / (-x+2)^{1/2} + 1)^2 + 8 - 3 \cdot 2^{1/2})^{1/2} / (1 + (x+2)^{1/2} - 1) / (-x+2)^{1/2} + 1)^2)^{1/2} / (1 + (x+2)^{1/2} - 1) / (-x+2)^{1/2} + 1) / (8 + 3 \cdot 2^{1/2}) / (-8866 + 6820 \cdot 2^{1/2})^{1/2} \cdot 2^{1/2}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)/(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((5\*x^2 + 3\*x + 2)\*sqrt(2\*x^2 - x + 3)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)), x)`

[Out] `int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5*x**2+3*x+2)/(2*x**2-x+3)**(1/2), x)`

[Out] `Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)), x)`

$$3.81 \quad \int \frac{1}{\sqrt{3-x+2x^2} (2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=188

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{682(5x^2+3x+2)} + \frac{\sqrt{\frac{1}{682}(2343727+1678700\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31(2343727+1678700\sqrt{2})}}((5751+3935\sqrt{2})x+1816\sqrt{2}+2119)}{\sqrt{2x^2-x+3}}\right)}{1364}$$

**Rubi [A]** time = 0.43, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {974, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{682(5x^2+3x+2)} + \frac{\sqrt{\frac{1}{682}(2343727+1678700\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31(2343727+1678700\sqrt{2})}}((5751+3935\sqrt{2})x+1816\sqrt{2}+2119)}{\sqrt{2x^2-x+3}}\right)}{1364} - \frac{\sqrt{\frac{1}{682}(1678700\sqrt{2}-2343727)} \tanh^{-1}\left(\frac{\sqrt{\frac{11}{31(1678700\sqrt{2}-2343727)}}((5751-3935\sqrt{2})x-1816\sqrt{2}+2119)}{\sqrt{2x^2-x+3}}\right)}{1364}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] ((4 + 65\*x)\*Sqrt[3 - x + 2\*x^2])/(682\*(2 + 3\*x + 5\*x^2)) + (Sqrt[(2343727 + 1678700\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(2343727 + 1678700\*Sqrt[2]))])\*(2119 + 1816\*Sqrt[2] + (5751 + 3935\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]])/1364 - (Sqrt[(-2343727 + 1678700\*Sqrt[2])/682]\*ArcTanh[(Sqrt[11/(31\*(-2343727 + 1678700\*Sqrt[2]))])\*(2119 - 1816\*Sqrt[2] + (5751 - 3935\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]])/1364

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^(q+1))/(b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), Int[(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p+1) - c\*d\*(p+2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p+q+2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p+q+2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p+1) - c\*e\*(2\*p+q+4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p+2\*q+5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && (!(IntegerQ[p] && ILtQ[q, -1])) && !IGtQ[q, 0]

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2} (2+3x+5x^2)^2} dx = \frac{(4+65x)\sqrt{3-x+2x^2}}{682(2+3x+5x^2)} - \frac{\int \frac{-1826 + \frac{2255x}{2}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{7502}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682(2+3x+5x^2)} - \frac{\int \frac{\frac{121}{2}(537-332\sqrt{2}) - \frac{121}{2}(127-205\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{165044\sqrt{2}} + \frac{\int \frac{121}{2}(537-332\sqrt{2})}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{1364}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682(2+3x+5x^2)} - \frac{1}{496} \left( 11 \left( 3357400 - 2343727\sqrt{2} \right) \right) \text{Subst} \left( \int \frac{1}{\sqrt{3-x+2x^2}} dx \right)$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682(2+3x+5x^2)} + \frac{\sqrt{\frac{1}{682} (2343727 + 1678700\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{31(2343727 + 1678700\sqrt{2})}}{1364} \right)}{1364}$$

**Mathematica [C]** time = 1.00, size = 287, normalized size = 1.53

$$25 \left[ \frac{i\sqrt{286+22i\sqrt{31}} (224\sqrt{31}+1023i) \tanh^{-1} \left( \frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}} \sqrt{2x^2-x+3}} \right)}{(\sqrt{31}-13i)^2} + \frac{10i \left( 1364(\sqrt{31}+13i)(65x+4)\sqrt{2x^2-x+3} - 5\sqrt{286-22i\sqrt{31}}(787\sqrt{31}-1271i)(5x^2+3x+2) \tanh^{-1} \left( \frac{(-22+4i\sqrt{31})x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}} \sqrt{2x^2-x+3}} \right) \right)}{(\sqrt{31}+13i)^2(10ix+\sqrt{31}+3i)(5(\sqrt{31}-13i)x+8\sqrt{31}-4i)} \right]$$

116281

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2), x]
[Out] (25*((I*Sqrt[286 + (22*I)*Sqrt[31]]*(1023*I + 224*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x]/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2]))]/(-13*I + Sqrt[31])^2 + ((10*I)*(1364*(13*I + Sqrt[31])*(4 + 65*x)*Sqrt[3 - x + 2*x^2] - 5*Sqrt[286 - (22*I)*Sqrt[31]]*(-1271*I + 787*Sqrt[31]))*(2 + 3*x + 5*x^2)*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x]/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])))/((13*I + Sqrt[31])^2*(3*I + Sqrt[31] + (10*I)*x)*(-4*I + 8*Sqrt[31] + 5*(-13*I + Sqrt[31])*x)))/116281
```



**IntegrateAlgebraic [C]** time = 0.54, size = 230, normalized size = 1.22

$$\text{RootSum}\left[-5\#1^4 + 44\sqrt{2}\#1^3 + 68\#1^2 - 2024\sqrt{2}\#1 - 10580\&, \frac{205\sqrt{2}\#1^2 \log(\#1 - 4\sqrt{2x^2 - x + 3} + \sqrt{2}(4x - 1)) + 4492\#1 \log(\#1 - 4\sqrt{2x^2 - x + 3} + \sqrt{2}(4x - 1)) - 9430\sqrt{2} \log(\#1 - 4\sqrt{2x^2 - x + 3} + \sqrt{2}(4x - 1))}{-5\#1^3 + 33\sqrt{2}\#1^2 + 34\#1 - 506\sqrt{2}}\&\right] + \frac{\sqrt{2x^2 - x + 3}(65x + 4)}{682(5x^2 + 3x + 2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] ((4 + 65\*x)\*Sqrt[3 - x + 2\*x^2])/(682\*(2 + 3\*x + 5\*x^2)) + RootSum[-10580 - 2024\*Sqrt[2]\*#1 + 68\*#1^2 + 44\*Sqrt[2]\*#1^3 - 5\*#1^4 & , (-9430\*Sqrt[2]\*Log[Sqrt[2]\*(-1 + 4\*x) - 4\*Sqrt[3 - x + 2\*x^2] + #1] + 4492\*Log[Sqrt[2]\*(-1 + 4\*x) - 4\*Sqrt[3 - x + 2\*x^2] + #1]\*#1 + 205\*Sqrt[2]\*Log[Sqrt[2]\*(-1 + 4\*x) - 4\*Sqrt[3 - x + 2\*x^2] + #1]\*#1^2)/(-506\*Sqrt[2] + 34\*#1 + 33\*Sqrt[2]\*#1^2 - 5\*#1^3) & ]/(682\*Sqrt[2])

**fricas [B]** time = 1.25, size = 2102, normalized size = 11.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(1/2), x, algorithm="fricas")

[Out] 1/263043507934399808\*(8422204\*563606738^(1/4)\*sqrt(33574)\*sqrt(341)\*sqrt(2)\*(5\*x^2 + 3\*x + 2)\*sqrt(2343727\*sqrt(2) + 3357400)\*arctan(1/7101900221517254683789\*(47876524\*sqrt(33574)\*(22\*563606738^(3/4)\*sqrt(341)\*(2950932\*x^7 - 11691762\*x^6 + 24397746\*x^5 - 40053004\*x^4 + 20309552\*x^3 - 10145376\*x^2 - sqrt(2)\*(2248634\*x^7 - 8421787\*x^6 + 17801494\*x^5 - 27869789\*x^4 + 13808040\*x^3 - 6172200\*x^2 - 15724800\*x + 10596096) - 21192192\*x + 15724800) + 520397\*563606738^(1/4)\*sqrt(341)\*(226651\*x^7 - 3496629\*x^6 + 18614024\*x^5 - 42860780\*x^4 + 55586592\*x^3 - 36274464\*x^2 - sqrt(2)\*(168871\*x^7 - 2579646\*x^6 + 13533020\*x^5 - 30582616\*x^4 + 39345120\*x^3 - 23947200\*x^2 - 28449792\*x + 19450368) - 38900736\*x + 28449792))\*sqrt(2\*x^2 - x + 3)\*sqrt(2343727\*sqrt(2) + 3357400) + 20160232886887690715272\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x^8 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752088\*x^3 + 396144\*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) - 2\*sqrt(33574/2191)\*(sqrt(33574)\*(22\*563606738^(3/4)\*sqrt(341)\*(10257392\*x^7 - 14773368\*x^6 + 47877288\*x^5 - 20710528\*x^4 + 26321472\*x^3 + 17079552\*x^2 - sqrt(2)\*(8292238\*x^7 - 11867543\*x^6 + 37968813\*x^5 - 13449840\*x^4 + 14570280\*x^3 + 20176128\*x^2 - 20176128\*x) - 17079552\*x) + 520397\*563606738^(1/4)\*sqrt(341)\*(795513\*x^7 - 10292932\*x^6 + 39734380\*x^5 - 51864768\*x^4 + 68281632\*x^3 + 34255872\*x^2 - 8\*sqrt(2)\*(77213\*x^7 - 998548\*x^6 + 3846220\*x^5 - 4943520\*x^4 + 6215760\*x^3 + 4318272\*x^2 - 4318272\*x) - 34255872\*x))\*sqrt(2\*x^2 - x + 3)\*sqrt(2343727\*sqrt(2) + 3357400) + 421088065768678\*sqrt(31)\*sqrt(2)\*(123408\*x^8 - 914152\*x^7 + 1578888\*x^6 - 3293072\*x^5 + 396480\*x^4 + 798336\*x^3 - 3822336\*x^2 - sqrt(2)\*(15550\*x^8 - 118051\*x^7 + 244047\*x^6 - 707374\*x^5 + 1053960\*x^4 - 1667952\*x^3 + 1209600\*x^2 - 1036800\*x) + 3276288\*x) + 19140366625849\*sqrt(31)\*(254591\*x^8 - 4815126\*x^7 + 32303580\*x^6 - 90866808\*x^5 + 108781920\*x^4 - 74219328\*x^3 - 168956928\*x^2 - 15488\*sqrt(2)\*(4\*x^8 - 76\*x^7 + 517\*x^6 - 1536\*x^5 + 2385\*x^4 - 3618\*x^3 + 2268\*x^2 - 1944\*x) + 144820224\*x))\*sqrt(-(563606738^(1/4)\*sqrt(33574)\*sqrt(341)\*sqrt(31)\*sqrt(2\*x^2 - x + 3)\*(sqrt(2)\*(1123\*x + 898) - 2021\*x - 225)\*sqrt(2343727\*sqrt(2) + 3357400) - 1731948347213\*x^2 - 1555218924028\*sqrt(2)\*(2\*x^2 - x + 3) + 5337228580187\*x - 7069176927400)/x^2) + 229093555532814667219\*sqrt(31)\*(2828123\*x^8 - 9696916\*x^7 + 53385560\*x^6 - 142835344\*x^5 + 254146592\*x^4 - 249300096\*x^3 + 37981440\*x^2 - 7744\*sqrt(2)\*(1348\*x^8 - 2692\*x^7 + 9789\*x^6 - 10070\*x^5 + 15569\*x^4 - 5568\*x^3 + 1080\*x^2 + 4320\*x - 5184) + 223064064\*x - 94887936))/(2585191\*x^8 - 4661200\*x^7 + 14191920\*x^6 + 490880\*x^5 - 13562944\*x^4 + 44249088\*x^3 - 34615296\*x^2 - 24772608\*x + 18579456)) + 8422204\*563606738^(1/4)\*sqrt(33574)\*sqrt(341)\*sqrt(2)\*(5\*x^2 + 3\*x + 2)\*sqrt(2343727\*sqrt(2)

```

+ 3357400)*arctan(1/7101900221517254683789*(47876524*sqrt(33574)*(22*56360
6738^(3/4)*sqrt(341)*(2950932*x^7 - 11691762*x^6 + 24397746*x^5 - 40053004*
x^4 + 20309552*x^3 - 10145376*x^2 - sqrt(2)*(2248634*x^7 - 8421787*x^6 + 17
801494*x^5 - 27869789*x^4 + 13808040*x^3 - 6172200*x^2 - 15724800*x + 10596
096) - 21192192*x + 15724800) + 520397*563606738^(1/4)*sqrt(341)*(226651*x^
7 - 3496629*x^6 + 18614024*x^5 - 42860780*x^4 + 55586592*x^3 - 36274464*x^2
- sqrt(2)*(168871*x^7 - 2579646*x^6 + 13533020*x^5 - 30582616*x^4 + 393451
20*x^3 - 23947200*x^2 - 28449792*x + 19450368) - 38900736*x + 28449792))*sq
rt(2*x^2 - x + 3)*sqrt(2343727*sqrt(2) + 3357400) - 20160232886887690715272
*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549
144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*
x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 53913
6) + 1154304*x - 456192) - 2*sqrt(33574/2191)*(sqrt(33574)*(22*563606738^(3
/4)*sqrt(341)*(10257392*x^7 - 14773368*x^6 + 47877288*x^5 - 20710528*x^4 +
26321472*x^3 + 17079552*x^2 - sqrt(2)*(8292238*x^7 - 11867543*x^6 + 3796881
3*x^5 - 13449840*x^4 + 14570280*x^3 + 20176128*x^2 - 20176128*x) - 17079552
*x) + 520397*563606738^(1/4)*sqrt(341)*(795513*x^7 - 10292932*x^6 + 3973438
0*x^5 - 51864768*x^4 + 68281632*x^3 + 34255872*x^2 - 8*sqrt(2)*(77213*x^7 -
998548*x^6 + 3846220*x^5 - 4943520*x^4 + 6215760*x^3 + 4318272*x^2 - 43182
72*x) - 34255872*x))*sqrt(2*x^2 - x + 3)*sqrt(2343727*sqrt(2) + 3357400) -
421088065768678*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3
293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 1
18051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x
^2 - 1036800*x) + 3276288*x) - 19140366625849*sqrt(31)*(254591*x^8 - 481512
6*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956
928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3
618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((563606738^(1/4)*sqrt(335
74)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x
- 225)*sqrt(2343727*sqrt(2) + 3357400) + 1731948347213*x^2 + 1555218924028
*sqrt(2)*(2*x^2 - x + 3) - 5337228580187*x + 7069176927400)/x^2) - 22909355
5532814667219*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14283534
4*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x
^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 43
20*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 141919
20*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608
*x + 18579456)) + 563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(31)*(16787000*
x^2 - 2343727*sqrt(2)*(5*x^2 + 3*x + 2) + 10072200*x + 6714800)*sqrt(234372
7*sqrt(2) + 3357400)*log(335740000/2191*(563606738^(1/4)*sqrt(33574)*sqrt(3
41)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x - 225)*sq
rt(2343727*sqrt(2) + 3357400) + 1731948347213*x^2 + 1555218924028*sqrt(2)*(
2*x^2 - x + 3) - 5337228580187*x + 7069176927400)/x^2) - 563606738^(1/4)*sq
rt(33574)*sqrt(341)*sqrt(31)*(16787000*x^2 - 2343727*sqrt(2)*(5*x^2 + 3*x +
2) + 10072200*x + 6714800)*sqrt(2343727*sqrt(2) + 3357400)*log(-335740000/
2191*(563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(s
qrt(2)*(1123*x + 898) - 2021*x - 225)*sqrt(2343727*sqrt(2) + 3357400) - 173
1948347213*x^2 - 1555218924028*sqrt(2)*(2*x^2 - x + 3) + 5337228580187*x -
7069176927400)/x^2) + 385694293158944*sqrt(2*x^2 - x + 3)*(65*x + 4))/(5*x^
2 + 3*x + 2)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,inf
inity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
```

]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity]Evaluation time: 27.71Done

**maple [B]** time = 0.01, size = 5225, normalized size = 27.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(1/2),x)

[Out] result too large to display

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)^2 \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((5\*x^2 + 3\*x + 2)^2\*sqrt(2\*x^2 - x + 3)), x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^(1/2)\*(3\*x + 5\*x^2 + 2)^2),x)

[Out] int(1/((2\*x^2 - x + 3)^(1/2)\*(3\*x + 5\*x^2 + 2)^2), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x\*\*2+3\*x+2)\*\*2/(2\*x\*\*2-x+3)\*\*(1/2),x)

[Out] Integral(1/(sqrt(2\*x\*\*2 - x + 3)\*(5\*x\*\*2 + 3\*x + 2)\*\*2), x)

$$3.82 \quad \int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)^3} dx$$

**Optimal.** Leaf size=223

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{1364(5x^2+3x+2)^2} + \frac{(86265x+26794)\sqrt{2x^2-x+3}}{1860496(5x^2+3x+2)} + \frac{25\sqrt{\frac{1}{682}(6414867847+4536374600\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31}(6414867847+4536374600\sqrt{2})}}{\sqrt{2x^2-x+3}}\right)}{3720992}$$

**Rubi [A]** time = 0.47, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{1364(5x^2+3x+2)^2} + \frac{(86265x+26794)\sqrt{2x^2-x+3}}{1860496(5x^2+3x+2)} + \frac{25\sqrt{\frac{1}{682}(6414867847+4536374600\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31}(6414867847+4536374600\sqrt{2})}}{\sqrt{2x^2-x+3}}\right)}{3720992} - \frac{25\sqrt{\frac{1}{682}(4536374600\sqrt{2}-6414867847)} \tanh^{-1}\left(\frac{\sqrt{\frac{11}{31}(4536374600\sqrt{2}-6414867847)}}{\sqrt{2x^2-x+3}}\right)}{3720992}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] ((4 + 65\*x)\*Sqrt[3 - x + 2\*x^2])/(1364\*(2 + 3\*x + 5\*x^2)^2) + ((26794 + 862 65\*x)\*Sqrt[3 - x + 2\*x^2])/(1860496\*(2 + 3\*x + 5\*x^2)) + (25\*Sqrt[(64148678 47 + 4536374600\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(6414867847 + 4536374600\* Sqrt[2]))])\*(123161 + 85754\*Sqrt[2] + (294669 + 208915\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]])/3720992 - (25\*Sqrt[(-6414867847 + 4536374600\*Sqrt[2])/682]\*Ar cTanh[(Sqrt[11/(31\*(-6414867847 + 4536374600\*Sqrt[2]))])\*(123161 - 85754\*Sqr t[2] + (294669 - 208915\*Sqrt[2])\*x)]/Sqrt[3 - x + 2\*x^2]])/3720992

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

#### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

#### Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]
```

### Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{3-x+2x^2} (2+3x+5x^2)^3} dx &= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} - \frac{\int \frac{-5775+\frac{6479x}{2}-2860x^2}{\sqrt{3-x+2x^2}(2+3x+5x^2)^2} dx}{15004} \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496(2+3x+5x^2)} - \frac{\int \frac{-28220225+\frac{28220225x}{2}}{\sqrt{3-x+2x^2}(2+3x+5x^2)^2} dx}{11256} \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496(2+3x+5x^2)} - \frac{\int \frac{\frac{33275}{4}(26103x-11256)}{\sqrt{3-x+2x^2}(2+3x+5x^2)^2} dx}{11256} \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496(2+3x+5x^2)} - \frac{(6875(9072x-11256))\sqrt{3-x+2x^2}}{11256} \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496(2+3x+5x^2)} + \frac{25\sqrt{\frac{1}{682}}(64x-11256)}{11256}
\end{aligned}$$

**Mathematica** [C] time = 6.23, size = 1277, normalized size = 5.73

result too large to display

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] (2500\*Sqrt[3 - x + 2\*x^2])/(341\*Sqrt[31]\*(13\*I + Sqrt[31])\*(3 - I\*Sqrt[31] + 10\*x)^2) + (7500\*Sqrt[3 - x + 2\*x^2])/(10571\*(13 - I\*Sqrt[31])\*(3 - I\*Sqrt[31] + 10\*x)) - (2500\*Sqrt[3 - x + 2\*x^2])/(341\*Sqrt[31]\*(13\*I - Sqrt[31])\*(3 + I\*Sqrt[31] + 10\*x)^2) + (7500\*Sqrt[3 - x + 2\*x^2])/(10571\*(13 + I\*Sqrt[31])\*(3 + I\*Sqrt[31] + 10\*x)) - (375\*Sqrt[(2\*(13 - I\*Sqrt[31]))/11]\*(11 - (2\*I)\*Sqrt[31])\*ArcTanh[(63 - I\*Sqrt[31] - 2\*(11 - (2\*I)\*Sqrt[31])\*x)/(2\*Sqrt[22\*(13 - I\*Sqrt[31]])\*Sqrt[3 - x + 2\*x^2])])/(10571\*(13\*I + Sqrt[31])^2) - (750\*Sqrt[(2\*(13 - I\*Sqrt[31]))/341]\*ArcTanh[(63 - I\*Sqrt[31] - 2\*(11 - (2\*I)\*Sqrt[31])\*x)/(2\*Sqrt[22\*(13 - I\*Sqrt[31]])\*Sqrt[3 - x + 2\*x^2])])/(961\*(13\*I + Sqrt[31])) + (750\*Sqrt[(2\*(13 + I\*Sqrt[31]))/341]\*ArcTanh[(63 + I\*Sqrt[31] - 2\*(11 + (2\*I)\*Sqrt[31])\*x)/(2\*Sqrt[22\*(13 + I\*Sqrt[31]])\*Sqrt[3 - x + 2\*x^2])])/(961\*(13\*I - Sqrt[31])) - (375\*Sqrt[(2\*(13 + I\*Sqrt[31]))/11]\*(11 + (2\*I)\*Sqrt[31])\*ArcTanh[(63 + I\*Sqrt[31] - 2\*(11 + (2\*I)\*Sqrt[31])\*x)/(2\*Sqrt[22\*(13 + I\*Sqrt[31]])\*Sqrt[3 - x + 2\*x^2])])/(10571\*(13\*I - Sqrt[31])^2) + (((500\*I)/31)\*(((20\*(-3 + I\*Sqrt[31])) + 10\*(27 - (4\*I)\*Sqrt[31]))\*Sqrt[3 - x + 2\*x^2])/((300 - 10\*(-3 + I\*Sqrt[31]) + 2\*(-3 + I\*Sqrt[31])^2)\*(-3 + I\*Sqrt[31] - 10\*x)) + (2\*Sqrt[22\*(13 - I\*Sqrt[31])]\*(20\*(-3 + I\*Sqrt[31]) + 10\*(27 - (4\*I)\*Sqrt[31]) - 2\*(600 + 2\*(-3 + I\*Sqrt[31])\*(27 - (4\*I)\*Sqrt[31])))\*ArcTanh[(-63 + I\*Sqrt[31] - (-10 + 4\*(-3 + I\*Sqrt[31]))\*x)/(2\*Sqrt[22\*(13 - I\*Sqrt[31]])\*Sqrt[3 - x + 2\*x^2])])/((300 - 10\*(-3 + I\*Sqrt[31]) + 2\*(-3 + I\*Sqrt[31])^2)\*(1200 - 40\*(-3 + I\*Sqrt[31]) + 8\*(-3 + I\*Sqrt[31])^2)))/(Sqrt[31]\*(300 - 10\*(-3 + I\*Sqrt[31]) + 2\*(-3 + I\*Sqrt[31])^2)) + (((500\*I)/31)\*(((20\*(3 + I\*Sqrt[31])) - 10\*(-27 - (4\*I)\*Sqrt[31]))\*Sqrt[3 - x + 2\*x^2])/((300 + 10\*(3 + I\*Sqrt[31]) + 2\*(3 + I\*Sqrt[31])^2)\*(3 + I\*Sqrt[31] + 10\*x)) + (2\*Sqrt[22\*(13 + I\*Sqrt[31])]\*(-20\*(3 + I\*Sqrt[31]) - 10\*(-27 - (4\*I)\*Sqrt[31]) - 2\*(600 + 2\*(3 + I\*Sqrt[31])\*(-27 - (4\*I)\*Sqrt[31])))\*ArcTanh[(63 + I\*Sqrt[31] - (10 + 4\*(3 + I\*Sqrt[31]))\*x)/(2\*Sqrt[22\*(13 + I\*Sqrt[31])]\*Sqrt[3 - x + 2\*x^2])])/(10571\*(13 + I\*Sqrt[31])^2)

$$\frac{(13 + I\sqrt{31})\sqrt{3 - x + 2x^2}}{((300 + 10(3 + I\sqrt{31}) + 2(3 + I\sqrt{31})^2)(1200 + 40(3 + I\sqrt{31}) + 8(3 + I\sqrt{31})^2))} / (\sqrt{31}(300 + 10(3 + I\sqrt{31}) + 2(3 + I\sqrt{31})^2))$$

**IntegrateAlgebraic [C]** time = 0.84, size = 396, normalized size = 1.78

$$\frac{\sqrt{31}\sqrt{-5x^4 + 6\sqrt{31}x^3 + 17x^2 - 26\sqrt{31}x - 56}, \frac{-2092729x^2\sqrt{4x^2 - 2\sqrt{31}x - 56} - 2092729\sqrt{4x^2 - 2\sqrt{31}x - 56}}{-2092729x^2 - 2092729}, \frac{16\sqrt{31}\sqrt{-5x^4 + 6\sqrt{31}x^3 + 17x^2 - 26\sqrt{31}x - 56}}{4909725}}{\sqrt{31} - 3} \frac{(431325x^2 + 392765x + 341572 + 59044)}{1860496(x^2 + 3x + 2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(59044 + 341572\*x + 392765\*x^2 + 431325\*x^3))/(1860496\*(2 + 3\*x + 5\*x^2)^2) + (3\*RootSum[-56 - 26\*Sqrt[2]\*#1 + 17\*#1^2 + 6\*Sqrt[2]\*#1^3 - 5\*#1^4 & , (-42330420383\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1] + 11629301740\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1 - 2992879225\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1^2)/(-13\*Sqrt[2] + 17\*#1 + 9\*Sqrt[2]\*#1^2 - 10\*#1^3) & ])/49210119200 - (16\*RootSum[-56 - 26\*Sqrt[2]\*#1 + 17\*#1^2 + 6\*Sqrt[2]\*#1^3 - 5\*#1^4 & , (-720397\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1] + 129160\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1 - 65525\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1^2)/(-13\*Sqrt[2] + 17\*#1 + 9\*Sqrt[2]\*#1^2 - 10\*#1^3) & ])/4509725

**fricas [B]** time = 1.28, size = 2183, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(1/2), x, algorithm="fricas")

[Out] -1/212344000027477426346822144\*(46113488900\*4115738902305032^(1/4)\*sqrt(22681873)\*sqrt(341)\*sqrt(2)\*(25\*x^4 + 30\*x^3 + 29\*x^2 + 12\*x + 4)\*sqrt(6414867847\*sqrt(2) + 9072749200)\*arctan(1/3836668309294009530058322373948769\*(64688701796\*sqrt(22681873)\*(11\*4115738902305032^(3/4)\*sqrt(341)\*(160344708\*x^7 - 615873378\*x^6 + 1294230774\*x^5 - 2070733376\*x^4 + 1037098288\*x^3 - 489164544\*x^2 - sqrt(2)\*(112700446\*x^7 - 434839553\*x^6 + 912850886\*x^5 - 1466127691\*x^4 + 735661560\*x^3 - 350098200\*x^2 - 799200000\*x + 567316224) - 1134632448\*x + 799200000) + 703138063\*4115738902305032^(1/4)\*sqrt(341)\*(12162569\*x^7 - 186616851\*x^6 + 985490056\*x^5 - 2246141620\*x^4 + 2900382048\*x^3 - 1823848416\*x^2 - sqrt(2)\*(8564099\*x^7 - 131508024\*x^6 + 695288980\*x^5 - 1587105104\*x^4 + 2050714080\*x^3 - 1296806400\*x^2 - 1457077248\*x + 1033108992) - 2066217984\*x + 1457077248))\*sqrt(2\*x^2 - x + 3)\*sqrt(6414867847\*sqrt(2) + 9072749200) + 10891187458641059311133302222822312\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x^8 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752088\*x^3 + 396144\*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) - 2\*sqrt(45363746/479849)\*(sqrt(22681873)\*(11\*4115738902305032^(3/4)\*sqrt(341)\*(576322648\*x^7 - 827050092\*x^6 + 2660713572\*x^5 - 1032439232\*x^4 + 1211604768\*x^3 + 1213394688\*x^2 - sqrt(2)\*(403157522\*x^7 - 578844217\*x^6 + 1864129347\*x^5 - 735062160\*x^4 + 873708120\*x^3 + 823986432\*x^2 - 823986432\*x) - 1213394688\*x) + 703138063\*4115738902305032^(1/4)\*sqrt(341)\*(43684647\*x^7 - 565067708\*x^6 + 2178643220\*x^5 - 2819241792\*x^4 + 3618371808\*x^3 + 2197767168\*x^2 - 2\*sqrt(2)\*(15328963\*x^7 - 198290348\*x^6 + 764653220\*x^5 - 990717120\*x^4 + 1276256160\*x^3 + 755350272\*x^2 - 755350272\*x) - 2197767168\*x))\*sqrt(2\*x^2 - x + 3)\*sqrt(6414867847\*sqrt(2) + 9072749200) + 168363055004367262339322\*sqrt(31)\*sqrt(2)\*(123408\*x^8 - 914152\*x^7 + 1578888\*x^6 - 3293072\*x^5 + 396480\*x^4 + 798336\*x^3 - 3822336\*x^2 - sqrt(2)\*(15550\*x^8 - 118051\*x^7 + 244047\*x^6 - 707374\*x^5 + 1053960\*x^4 - 1667952\*x^3 + 1209600\*x^2 - 1036800\*x) + 3276288\*x) + 7652866136562148288151\*sqrt(31)\*(254591\*x^8 - 4815126\*x^7 + 32303580\*x^6 - 90866808\*x^5 + 108781920\*x^4 - 74219328\*x^3 - 168956928\*x^2 - 15488\*sqrt(2)\*(4\*x^8 - 76\*x^7 + 517\*x^6 - 1536\*x^5 + 2385\*x^4 - 3618\*x

$$\begin{aligned}
&^3 + 2268x^2 - 1944x) + 144820224x))\sqrt{-(4115738902305032^{1/4})\sqrt{(22681873)\sqrt{(341)\sqrt{(31)\sqrt{(2x^2 - x + 3)(\sqrt{2}(67187x + 26012) - 93199x - 41175)\sqrt{(6414867847\sqrt{2} + 9072749200) - 512510746420187753x^2 - 460213731479352268\sqrt{2}(2x^2 - x + 3) + 1579369851213231647x - 2091880597633419400)/x^2) + 123763493848193855808332979804799\sqrt{31}(2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744\sqrt{2}(1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936)))/(2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456)) + 46113488900*4115738902305032^{1/4})\sqrt{(22681873)\sqrt{(341)\sqrt{(2)(25x^4 + 30x^3 + 29x^2 + 12x + 4)\sqrt{(6414867847\sqrt{2} + 9072749200)\arctan(1/3836668309294009530058322373948769(64688701796\sqrt{(22681873)(11*4115738902305032^{3/4})\sqrt{(341)(160344708x^7 - 615873378x^6 + 1294230774x^5 - 2070733376x^4 + 1037098288x^3 - 489164544x^2 - \sqrt{2}(112700446x^7 - 434839553x^6 + 912850886x^5 - 1466127691x^4 + 735661560x^3 - 350098200x^2 - 799200000x + 567316224) - 1134632448x + 799200000) + 703138063*4115738902305032^{1/4})\sqrt{(341)(12162569x^7 - 186616851x^6 + 985490056x^5 - 2246141620x^4 + 2900382048x^3 - 1823848416x^2 - \sqrt{2}(8564099x^7 - 131508024x^6 + 695288980x^5 - 1587105104x^4 + 2050714080x^3 - 1296806400x^2 - 1457077248x + 1033108992) - 2066217984x + 1457077248))\sqrt{(2x^2 - x + 3)\sqrt{(6414867847\sqrt{2} + 9072749200) - 1089118745864105931113330222822312\sqrt{(31)\sqrt{(2)(28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2}(8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - 2\sqrt{(45363746/479849)(\sqrt{(22681873)(11*4115738902305032^{3/4})\sqrt{(341)(576322648x^7 - 827050092x^6 + 2660713572x^5 - 1032439232x^4 + 1211604768x^3 + 1213394688x^2 - \sqrt{2}(403157522x^7 - 578844217x^6 + 1864129347x^5 - 735062160x^4 + 873708120x^3 + 823986432x^2 - 823986432x) - 1213394688x) + 703138063*4115738902305032^{1/4})\sqrt{(341)(43684647x^7 - 565067708x^6 + 2178643220x^5 - 2819241792x^4 + 3618371808x^3 + 2197767168x^2 - 2\sqrt{(2)(15328963x^7 - 198290348x^6 + 764653220x^5 - 990717120x^4 + 1276256160x^3 + 755350272x^2 - 755350272x) - 2197767168x))\sqrt{(2x^2 - x + 3)\sqrt{(6414867847\sqrt{2} + 9072749200) - 168363055004367262339322\sqrt{(31)\sqrt{(2)(123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2}(15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) - 7652866136562148288151\sqrt{(31)(254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488\sqrt{(2)(4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x))\sqrt{(4115738902305032^{1/4})\sqrt{(22681873)\sqrt{(341)\sqrt{(31)\sqrt{(2x^2 - x + 3)(\sqrt{2}(67187x + 26012) - 93199x - 41175)\sqrt{(6414867847\sqrt{2} + 9072749200) + 512510746420187753x^2 + 460213731479352268\sqrt{2}(2x^2 - x + 3) - 1579369851213231647x + 2091880597633419400)/x^2) - 123763493848193855808332979804799\sqrt{31}(2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744\sqrt{2}(1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936)))/(2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456)) - 25*4115738902305032^{1/4})\sqrt{(22681873)\sqrt{(341)\sqrt{(31)(226818730000x^4 + 272182476000x^3 + 263109726800x^2 - 6414867847\sqrt{2}(25x^4 + 30x^3 + 29x^2 + 12x + 4) + 108872990400x + 36290996800)\sqrt{(6414867847\sqrt{2} + 9072749200)\log(1134093650000000/479849(4115738902305032^{1/4})\sqrt{(22681873)\sqrt{(341)\sqrt{(31)\sqrt{(2x^2 - x + 3)(\sqrt{2}(67187x + 26012) - 93199x - 41175)\sqrt{(6414867847\sqrt{2} + 9072749200) + 512510746420187753x^2 + 460213731479352268\sqrt{2}(2x^2 - x + 3) - 1579369851213231647x + 2091880597633419400)/x^2) + 25*4115738902305032^{1/4})\sqrt{(22681873)\sqrt{(341)\sqrt{(31)(226818730000x^4 + 2721}
\end{aligned}$$



```
82476000*x^3 + 263109726800*x^2 - 6414867847*sqrt(2)*(25*x^4 + 30*x^3 + 29*
x^2 + 12*x + 4) + 108872990400*x + 36290996800)*sqrt(6414867847*sqrt(2) + 9
072749200)*log(-1134093650000000/479849*(4115738902305032^(1/4)*sqrt(226818
73)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(67187*x + 26012) - 931
99*x - 41175)*sqrt(6414867847*sqrt(2) + 9072749200) - 512510746420187753*x^
2 - 460213731479352268*sqrt(2)*(2*x^2 - x + 3) + 1579369851213231647*x - 20
91880597633419400)/x^2) - 114133005406879362464*(431325*x^3 + 392765*x^2 +
341572*x + 59044)*sqrt(2*x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4
)
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity]
]Evaluation time: 42.09Done
```

**maple** [B] time = 0.01, size = 13040, normalized size = 58.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)^3 \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*sqrt(2*x^2 - x + 3)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2), x)
```

```
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3), x)
```

$$3.83 \quad \int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=166

$$\frac{111315\sqrt{2x^2-x+3}x^2}{2048} - \frac{8992487\sqrt{2x^2-x+3}x}{16384} - \frac{31009685\sqrt{2x^2-x+3}}{65536} - \frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}} + \frac{625}{24}\sqrt{2x^2-x+3}$$

**Rubi [A]** time = 0.20, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{625}{24}\sqrt{2x^2-x+3}x^5 + \frac{10075}{96}\sqrt{2x^2-x+3}x^4 + \frac{79425}{512}\sqrt{2x^2-x+3}x^3 - \frac{111315\sqrt{2x^2-x+3}x^2}{2048} - \frac{8992487\sqrt{2x^2-x+3}x}{16384} - \frac{31009685\sqrt{2x^2-x+3}}{65536} - \frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}} - \frac{310445587 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{131072\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^(3/2), x]

[Out] (-14641\*(101 + 79\*x))/(1472\*Sqrt[3 - x + 2\*x^2]) - (31009685\*Sqrt[3 - x + 2\*x^2])/65536 - (8992487\*x\*Sqrt[3 - x + 2\*x^2])/16384 - (111315\*x^2\*Sqrt[3 - x + 2\*x^2])/2048 + (79425\*x^3\*Sqrt[3 - x + 2\*x^2])/512 + (10075\*x^4\*Sqrt[3 - x + 2\*x^2])/96 + (625\*x^5\*Sqrt[3 - x + 2\*x^2])/24 - (310445587\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(131072\*Sqrt[2])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a +

$b*x + c*x^2)^p \text{ExpandToSum}[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^{(q - 2)} - b*e*(q + p)*x^{(q - 1)} - c*e*(q + 2*p + 1)*x^q, x], x] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{LeQ}[p, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{(2 + 3x + 5x^2)^4}{(3 - x + 2x^2)^{3/2}} dx &= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} + \frac{2}{23} \int \frac{\frac{2821893}{256} - \frac{661181x}{128} - \frac{488267x^2}{64} + \frac{143635x^3}{32} + \frac{213325x^4}{16} + \frac{83375x^5}{8}}{\sqrt{3 - x + 2x^2}} \\ &= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} + \frac{625}{24}x^5\sqrt{3 - x + 2x^2} + \frac{1}{138} \int \frac{\frac{8465679}{64} - \frac{1983543x}{32} - \frac{1464801x^2}{16} + \frac{430000x^3}{8}}{\sqrt{3 - x + 2x^2}} \\ &= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} + \frac{10075}{96}x^4\sqrt{3 - x + 2x^2} + \frac{625}{24}x^5\sqrt{3 - x + 2x^2} + \int \frac{\frac{42328395}{32} - \frac{9917715x}{16}}{\sqrt{3 - x + 2x^2}} \\ &= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} + \frac{79425}{512}x^3\sqrt{3 - x + 2x^2} + \frac{10075}{96}x^4\sqrt{3 - x + 2x^2} + \frac{625}{24}x^5\sqrt{3 - x + 2x^2} \\ &= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048} + \frac{79425}{512}x^3\sqrt{3 - x + 2x^2} + \frac{10075}{96}x^4\sqrt{3 - x + 2x^2} \\ &= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{8992487x\sqrt{3 - x + 2x^2}}{16384} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048} + \frac{79425}{512}x^3\sqrt{3 - x + 2x^2} \\ &= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{31009685\sqrt{3 - x + 2x^2}}{65536} - \frac{8992487x\sqrt{3 - x + 2x^2}}{16384} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048} \\ &= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{31009685\sqrt{3 - x + 2x^2}}{65536} - \frac{8992487x\sqrt{3 - x + 2x^2}}{16384} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048} \\ &= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{31009685\sqrt{3 - x + 2x^2}}{65536} - \frac{8992487x\sqrt{3 - x + 2x^2}}{16384} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048} \end{aligned}$$

**Mathematica [A]** time = 0.37, size = 95, normalized size = 0.57

$$\sqrt{2x^2 - x + 3} \left( \frac{625x^5}{24} + \frac{10075x^4}{96} + \frac{79425x^3}{512} - \frac{111315x^2}{2048} - \frac{14641(79x + 101)}{1472(2x^2 - x + 3)} - \frac{8992487x}{16384} - \frac{31009685}{65536} \right) + \frac{310445587 \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{131072\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^(3/2), x]

[Out] Sqrt[3 - x + 2\*x^2]\*(-31009685/65536 - (8992487\*x)/16384 - (111315\*x^2)/2048 + (79425\*x^3)/512 + (10075\*x^4)/96 + (625\*x^5)/24 - (14641\*(101 + 79\*x))/(1472\*(3 - x + 2\*x^2))) + (310445587\*ArcSinh[(-1 + 4\*x)/Sqrt[23]])/(131072\*Sqrt[2])

**IntegrateAlgebraic [A]** time = 1.32, size = 90, normalized size = 0.54

$$\frac{235520000x^7 + 831385600x^6 + 1281670400x^5 + 230669760x^4 - 2613624504x^3 - 2534760678x^2 - 8859305979x - 10961697147}{4521984\sqrt{2x^2 - x + 3}} - \frac{310445587 \log(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1)}{131072\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^(3/2),x]

[Out] (-10961697147 - 8859305979\*x - 2534760678\*x^2 - 2613624504\*x^3 + 230669760\*x^4 + 1281670400\*x^5 + 831385600\*x^6 + 235520000\*x^7)/(4521984\*Sqrt[3 - x + 2\*x^2]) - (310445587\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(131072\*Sqrt[2])

**fricas** [A] time = 0.41, size = 112, normalized size = 0.67

$$\frac{21420745503\sqrt{2}(2x^2-x+3)\log(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25)+8(235520000x^7+831385600x^6+1281670400x^5+230669760x^4-2613624504x^3-2534760678x^2-8859305979x-10961697147)\sqrt{2x^2-x+3}}{36175872(2x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(3/2),x, algorithm="fricas")

[Out] 1/36175872\*(21420745503\*sqrt(2)\*(2\*x^2 - x + 3)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 8\*(235520000\*x^7 + 831385600\*x^6 + 1281670400\*x^5 + 230669760\*x^4 - 2613624504\*x^3 - 2534760678\*x^2 - 8859305979\*x - 10961697147)\*sqrt(2\*x^2 - x + 3))/(2\*x^2 - x + 3)

**giac** [A] time = 0.25, size = 82, normalized size = 0.49

$$-\frac{310445587}{262144}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2x-x+3}\right)+1\right)+\frac{(46(4(40(20(16(100x+353)x+8707)x+31341)x-14204481)x-55103493)x-8859305979)x-10961697147}{4521984\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(3/2),x, algorithm="giac")

[Out] -310445587/262144\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) + 1/4521984\*((46\*(4\*(40\*(20\*(16\*(100\*x + 353)\*x + 8707)\*x + 31341)\*x - 14204481)\*x - 55103493)\*x - 8859305979)\*x - 10961697147)/sqrt(2\*x^2 - x + 3)

**maple** [A] time = 0.03, size = 166, normalized size = 1.00

$$\frac{625x^7}{12\sqrt{2x^2-x+3}} + \frac{8825x^6}{48\sqrt{2x^2-x+3}} + \frac{217675x^5}{768\sqrt{2x^2-x+3}} + \frac{52235x^4}{1024\sqrt{2x^2-x+3}} - \frac{4734827x^3}{8192\sqrt{2x^2-x+3}} - \frac{18367831x^2}{32768\sqrt{2x^2-x+3}} - \frac{310445587x}{131072\sqrt{2x^2-x+3}} + \frac{310445587\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}(x-1)}{23}\right)}{262144} - \frac{1217267299}{524288\sqrt{2x^2-x+3}} + \frac{1234044515x - 1234044515}{12058624\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(3/2),x)

[Out] -1217267299/524288/(2\*x^2-x+3)^(1/2)+1234044515/12058624\*(4\*x-1)/(2\*x^2-x+3)^(1/2)+310445587/262144\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+217675/768\*x^5/(2\*x^2-x+3)^(1/2)+52235/1024\*x^4/(2\*x^2-x+3)^(1/2)-4734827/8192\*x^3/(2\*x^2-x+3)^(1/2)+625/12\*x^7/(2\*x^2-x+3)^(1/2)+8825/48\*x^6/(2\*x^2-x+3)^(1/2)-18367831/32768\*x^2/(2\*x^2-x+3)^(1/2)-310445587/131072\*x/(2\*x^2-x+3)^(1/2)

**maxima** [A] time = 0.98, size = 148, normalized size = 0.89

$$\frac{625x^7}{12\sqrt{2x^2-x+3}} + \frac{8825x^6}{48\sqrt{2x^2-x+3}} + \frac{217675x^5}{768\sqrt{2x^2-x+3}} + \frac{52235x^4}{1024\sqrt{2x^2-x+3}} - \frac{4734827x^3}{8192\sqrt{2x^2-x+3}} - \frac{18367831x^2}{32768\sqrt{2x^2-x+3}} + \frac{310445587}{262144}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{2953101993x}{1507328\sqrt{2x^2-x+3}} - \frac{3653899049}{1507328\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(3/2),x, algorithm="maxima")

[Out] 625/12\*x^7/sqrt(2\*x^2 - x + 3) + 8825/48\*x^6/sqrt(2\*x^2 - x + 3) + 217675/768\*x^5/sqrt(2\*x^2 - x + 3) + 52235/1024\*x^4/sqrt(2\*x^2 - x + 3) - 4734827/8192\*x^3/sqrt(2\*x^2 - x + 3) - 18367831/32768\*x^2/sqrt(2\*x^2 - x + 3) + 310445587/262144\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 2953101993/1507328\*x/sqrt(2\*x^2 - x + 3) - 3653899049/1507328/sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^4}{(2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^4/(2\*x^2 - x + 3)^(3/2), x)

[Out] int((3\*x + 5\*x^2 + 2)^4/(2\*x^2 - x + 3)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^4}{(2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*4/(2\*x\*\*2-x+3)\*\*(3/2), x)

[Out] Integral((5\*x\*\*2 + 3\*x + 2)\*\*4/(2\*x\*\*2 - x + 3)\*\*(3/2), x)

$$3.84 \quad \int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=124

$$\frac{1825}{64} \sqrt{2x^2 - x + 3} x^2 + \frac{15565}{512} \sqrt{2x^2 - x + 3} x - \frac{181561 \sqrt{2x^2 - x + 3}}{2048} - \frac{1331(17 - 45x)}{368 \sqrt{2x^2 - x + 3}} + \frac{125}{16} \sqrt{2x^2 - x + 3} x^3 +$$

**Rubi [A]** time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{125}{16} \sqrt{2x^2 - x + 3} x^3 + \frac{1825}{64} \sqrt{2x^2 - x + 3} x^2 + \frac{15565}{512} \sqrt{2x^2 - x + 3} x - \frac{181561 \sqrt{2x^2 - x + 3}}{2048} - \frac{1331(17 - 45x)}{368 \sqrt{2x^2 - x + 3}} + \frac{1168881 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^(3/2), x]

[Out] (-1331\*(17 - 45\*x))/(368\*sqrt[3 - x + 2\*x^2]) - (181561\*sqrt[3 - x + 2\*x^2])/2048 + (15565\*x\*sqrt[3 - x + 2\*x^2])/512 + (1825\*x^2\*sqrt[3 - x + 2\*x^2])/64 + (125\*x^3\*sqrt[3 - x + 2\*x^2])/16 + (1168881\*ArcSinh[(1 - 4\*x)/sqrt[23]])/(4096\*sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*

$e*(q + p)*x^{(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{PolyQ}[\text{Pq}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{LeQ}[p, -1]$

Rubi steps

$$\begin{aligned} \int \frac{(2 + 3x + 5x^2)^3}{(3 - x + 2x^2)^{3/2}} dx &= -\frac{1331(17 - 45x)}{368\sqrt{3 - x + 2x^2}} + \frac{2}{23} \int \frac{-\frac{110285}{64} - \frac{19067x}{32} + \frac{22195x^2}{16} + \frac{13225x^3}{8} + \frac{2875x^4}{4}}{\sqrt{3 - x + 2x^2}} dx \\ &= -\frac{1331(17 - 45x)}{368\sqrt{3 - x + 2x^2}} + \frac{125}{16}x^3\sqrt{3 - x + 2x^2} + \frac{1}{92} \int \frac{-\frac{110285}{8} - \frac{19067x}{4} + \frac{18515x^2}{4} + \frac{125925x^3}{8}}{\sqrt{3 - x + 2x^2}} \\ &= -\frac{1331(17 - 45x)}{368\sqrt{3 - x + 2x^2}} + \frac{1825}{64}x^2\sqrt{3 - x + 2x^2} + \frac{125}{16}x^3\sqrt{3 - x + 2x^2} + \frac{1}{552} \int \frac{-\frac{330855}{4} - \frac{110285x}{4} + \frac{18515x^2}{4} + \frac{125925x^3}{8}}{\sqrt{3 - x + 2x^2}} \\ &= -\frac{1331(17 - 45x)}{368\sqrt{3 - x + 2x^2}} + \frac{15565}{512}x\sqrt{3 - x + 2x^2} + \frac{1825}{64}x^2\sqrt{3 - x + 2x^2} + \frac{125}{16}x^3\sqrt{3 - x + 2x^2} \\ &= -\frac{1331(17 - 45x)}{368\sqrt{3 - x + 2x^2}} - \frac{181561\sqrt{3 - x + 2x^2}}{2048} + \frac{15565}{512}x\sqrt{3 - x + 2x^2} + \frac{1825}{64}x^2\sqrt{3 - x + 2x^2} \\ &= -\frac{1331(17 - 45x)}{368\sqrt{3 - x + 2x^2}} - \frac{181561\sqrt{3 - x + 2x^2}}{2048} + \frac{15565}{512}x\sqrt{3 - x + 2x^2} + \frac{1825}{64}x^2\sqrt{3 - x + 2x^2} \\ &= -\frac{1331(17 - 45x)}{368\sqrt{3 - x + 2x^2}} - \frac{181561\sqrt{3 - x + 2x^2}}{2048} + \frac{15565}{512}x\sqrt{3 - x + 2x^2} + \frac{1825}{64}x^2\sqrt{3 - x + 2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.23, size = 65, normalized size = 0.52

$$\frac{4(736000x^5 + 2318400x^4 + 2624760x^3 - 5754186x^2 + 16138403x - 15423965)}{\sqrt{2x^2 - x + 3}} - 26884263\sqrt{2} \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)$$

188416

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^(3/2), x]

[Out] ((4\*(-15423965 + 16138403\*x - 5754186\*x^2 + 2624760\*x^3 + 2318400\*x^4 + 736000\*x^5))/Sqrt[3 - x + 2\*x^2] - 26884263\*Sqrt[2]\*ArcSinh[(-1 + 4\*x)/Sqrt[23]])/188416

**IntegrateAlgebraic [A]** time = 0.84, size = 80, normalized size = 0.65

$$\frac{1168881 \log(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1)}{4096\sqrt{2}} + \frac{736000x^5 + 2318400x^4 + 2624760x^3 - 5754186x^2 + 16138403x - 15423965}{47104\sqrt{2x^2 - x + 3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^(3/2), x]

[Out] (-15423965 + 16138403\*x - 5754186\*x^2 + 2624760\*x^3 + 2318400\*x^4 + 736000\*x^5)/(47104\*Sqrt[3 - x + 2\*x^2]) + (1168881\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(4096\*Sqrt[2])

**fricas [A]** time = 0.42, size = 102, normalized size = 0.82

$$\frac{26884263\sqrt{2}(2x^2 - x + 3) \log(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 8(736000x^5 + 2318400x^4 + 2624760x^3 - 5754186x^2 + 16138403x - 15423965)\sqrt{2x^2 - x + 3}}{376832(2x^2 - x + 3)}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(3/2),x, algorithm="fricas")

[Out] 1/376832\*(26884263\*sqrt(2)\*(2\*x^2 - x + 3)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 8\*(736000\*x^5 + 2318400\*x^4 + 2624760\*x^3 - 5754186\*x^2 + 16138403\*x - 15423965)\*sqrt(2\*x^2 - x + 3))/(2\*x^2 - x + 3)

**giac** [A] time = 0.28, size = 72, normalized size = 0.58

$$\frac{1168881}{8192} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2x-x+3}\right)+1\right) + \frac{(46(20(40(20x+63)x+2853)x-125091)x+16138403)x-15423965}{47104\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(3/2),x, algorithm="giac")

[Out] 1168881/8192\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) + 1/47104\*((46\*(20\*(40\*(20\*x + 63)\*x + 2853)\*x - 125091)\*x + 16138403)\*x - 15423965)/sqrt(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 132, normalized size = 1.06

$$\frac{125x^5}{8\sqrt{2x^2-x+3}} + \frac{1575x^4}{32\sqrt{2x^2-x+3}} + \frac{14265x^3}{256\sqrt{2x^2-x+3}} - \frac{125091x^2}{1024\sqrt{2x^2-x+3}} + \frac{1168881x}{4096\sqrt{2x^2-x+3}} - \frac{1168881\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192} - \frac{5130399}{16384\sqrt{2x^2-x+3}} + \frac{5392543x - 5392543}{\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(3/2),x)

[Out] -5130399/16384/(2\*x^2-x+3)^(1/2)+5392543/376832\*(4\*x-1)/(2\*x^2-x+3)^(1/2)-1168881/8192\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+125/8/(2\*x^2-x+3)^(1/2)\*x^5+1575/32/(2\*x^2-x+3)^(1/2)\*x^4+14265/256/(2\*x^2-x+3)^(1/2)\*x^3-125091/1024/(2\*x^2-x+3)^(1/2)\*x^2+1168881/4096/(2\*x^2-x+3)^(1/2)\*x

**maxima** [A] time = 0.97, size = 114, normalized size = 0.92

$$\frac{125x^5}{8\sqrt{2x^2-x+3}} + \frac{1575x^4}{32\sqrt{2x^2-x+3}} + \frac{14265x^3}{256\sqrt{2x^2-x+3}} - \frac{125091x^2}{1024\sqrt{2x^2-x+3}} - \frac{1168881}{8192} \sqrt{2} \operatorname{arsinh}\left(\frac{1}{23} \sqrt{23}(4x-1)\right) + \frac{16138403x}{47104\sqrt{2x^2-x+3}} - \frac{15423965}{47104\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(3/2),x, algorithm="maxima")

[Out] 125/8\*x^5/sqrt(2\*x^2 - x + 3) + 1575/32\*x^4/sqrt(2\*x^2 - x + 3) + 14265/256\*x^3/sqrt(2\*x^2 - x + 3) - 125091/1024\*x^2/sqrt(2\*x^2 - x + 3) - 1168881/8192\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) + 16138403/47104\*x/sqrt(2\*x^2 - x + 3) - 15423965/47104/sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^3}{(2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^3/(2\*x^2 - x + 3)^(3/2),x)

[Out] int((3\*x + 5\*x^2 + 2)^3/(2\*x^2 - x + 3)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^3}{(2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(3/2), x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(3/2), x)
```

$$3.85 \quad \int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=82

$$\frac{121(19-7x)}{92\sqrt{2x^2-x+3}} + \frac{25}{8}x\sqrt{2x^2-x+3} + \frac{415}{32}\sqrt{2x^2-x+3} - \frac{223 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

**Rubi [A]** time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{121(19-7x)}{92\sqrt{2x^2-x+3}} + \frac{25}{8}x\sqrt{2x^2-x+3} + \frac{415}{32}\sqrt{2x^2-x+3} - \frac{223 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^(3/2), x]

[Out] (121\*(19 - 7\*x))/(92\*Sqrt[3 - x + 2\*x^2]) + (415\*Sqrt[3 - x + 2\*x^2])/32 + (25\*x\*Sqrt[3 - x + 2\*x^2])/8 - (223\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(64\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1660

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*

$e^{*(q+p)*x^{(q-1)} - c*e^{*(q+2*p+1)*x^q}, x], x], x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{LeQ}[p, -1]$

Rubi steps

$$\begin{aligned} \int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}} dx &= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{2}{23} \int \frac{\frac{1173}{16} + \frac{1955x}{8} + \frac{575x^2}{4}}{\sqrt{3-x+2x^2}} dx \\ &= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{25}{8}x\sqrt{3-x+2x^2} + \frac{1}{46} \int \frac{-138 + \frac{9545x}{8}}{\sqrt{3-x+2x^2}} dx \\ &= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{415}{32}\sqrt{3-x+2x^2} + \frac{25}{8}x\sqrt{3-x+2x^2} + \frac{223}{64} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\ &= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{415}{32}\sqrt{3-x+2x^2} + \frac{25}{8}x\sqrt{3-x+2x^2} + \frac{223 \text{Subst} \left( \int \frac{1}{\sqrt{1+\frac{x^2}{23}}} dx, x, \right)}{64\sqrt{46}} \\ &= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{415}{32}\sqrt{3-x+2x^2} + \frac{25}{8}x\sqrt{3-x+2x^2} - \frac{223 \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{64\sqrt{2}} \end{aligned}$$

**Mathematica** [A] time = 0.14, size = 55, normalized size = 0.67

$$\frac{4600x^3 + 16790x^2 - 9421x + 47027}{736\sqrt{2x^2 - x + 3}} + \frac{223 \sinh^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^(3/2), x]

[Out] (47027 - 9421\*x + 16790\*x^2 + 4600\*x^3)/(736\*sqrt[3 - x + 2\*x^2]) + (223\*ArcSinh[(-1 + 4\*x)/sqrt[23]])/(64\*sqrt[2])

**IntegrateAlgebraic** [A] time = 0.73, size = 70, normalized size = 0.85

$$\frac{4600x^3 + 16790x^2 - 9421x + 47027}{736\sqrt{2x^2 - x + 3}} - \frac{223 \log \left( 2\sqrt{2} \sqrt{2x^2 - x + 3} - 4x + 1 \right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^(3/2), x]

[Out] (47027 - 9421\*x + 16790\*x^2 + 4600\*x^3)/(736\*sqrt[3 - x + 2\*x^2]) - (223\*Log[1 - 4\*x + 2\*sqrt[2]\*sqrt[3 - x + 2\*x^2]])/(64\*sqrt[2])

**fricas** [A] time = 0.41, size = 92, normalized size = 1.12

$$\frac{5129\sqrt{2}(2x^2 - x + 3)\log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 8(4600x^3 + 16790x^2 - 9421x + 47027)\sqrt{2x^2 - x + 3}}{5888(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(3/2), x, algorithm="fricas")

[Out] 1/5888\*(5129\*sqrt(2)\*(2\*x^2 - x + 3)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 8\*(4600\*x^3 + 16790\*x^2 - 9421\*x + 47027)\*sqrt(2\*x^2 - x + 3))/(2\*x^2 - x + 3)

**giac** [A] time = 0.24, size = 62, normalized size = 0.76

$$-\frac{223}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{(230(20x+73)x-9421)x+47027}{736\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(3/2),x, algorithm="giac")

[Out] -223/128\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) + 1/736\*((230\*(20\*x + 73)\*x - 9421)\*x + 47027)/sqrt(2\*x^2 - x + 3)

**maple** [A] time = 0.01, size = 98, normalized size = 1.20

$$\frac{25x^3}{4\sqrt{2x^2-x+3}} + \frac{365x^2}{16\sqrt{2x^2-x+3}} - \frac{223x}{64\sqrt{2x^2-x+3}} + \frac{223\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128} + \frac{15761}{256\sqrt{2x^2-x+3}} - \frac{13713(4x-1)}{5888\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(3/2),x)

[Out] 25/4/(2\*x^2-x+3)^(1/2)\*x^3+365/16/(2\*x^2-x+3)^(1/2)\*x^2-223/64/(2\*x^2-x+3)^(1/2)\*x+15761/256/(2\*x^2-x+3)^(1/2)-13713/5888\*(4\*x-1)/(2\*x^2-x+3)^(1/2)+223/128\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))

**maxima** [A] time = 0.96, size = 80, normalized size = 0.98

$$\frac{25x^3}{4\sqrt{2x^2-x+3}} + \frac{365x^2}{16\sqrt{2x^2-x+3}} + \frac{223}{128}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{9421x}{736\sqrt{2x^2-x+3}} + \frac{47027}{736\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(3/2),x, algorithm="maxima")

[Out] 25/4\*x^3/sqrt(2\*x^2 - x + 3) + 365/16\*x^2/sqrt(2\*x^2 - x + 3) + 223/128\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 9421/736\*x/sqrt(2\*x^2 - x + 3) + 47027/736/sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^2}{(2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^2/(2\*x^2 - x + 3)^(3/2),x)

[Out] int((3\*x + 5\*x^2 + 2)^2/(2\*x^2 - x + 3)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^2}{(2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*2/(2\*x\*\*2-x+3)\*\*(3/2),x)

[Out] Integral((5\*x\*\*2 + 3\*x + 2)\*\*2/(2\*x\*\*2 - x + 3)\*\*(3/2), x)

$$3.86 \quad \int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=45

$$\frac{11(3x+5)}{23\sqrt{2x^2-x+3}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1660, 12, 619, 215}

$$\frac{11(3x+5)}{23\sqrt{2x^2-x+3}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^(3/2), x]

[Out] (-11\*(5 + 3\*x))/(23\*sqrt[3 - x + 2\*x^2]) - (5\*ArcSinh[(1 - 4\*x)/sqrt[23]])/(2\*sqrt[2])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 215

Int[1/sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[Rt[b, 2]\*x]/sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}} dx &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}} + \frac{2}{23} \int \frac{115}{4\sqrt{3-x+2x^2}} dx \\
&= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}} + \frac{5}{2} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}} + \frac{5 \operatorname{Subst} \left( \int \frac{1}{\sqrt{1+\frac{x^2}{23}}} dx, x, -1+4x \right)}{2\sqrt{46}} \\
&= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}} - \frac{5 \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{2\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 45, normalized size = 1.00

$$\frac{5 \sinh^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{2\sqrt{2}} - \frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^(3/2), x]

[Out] (-11\*(5 + 3\*x))/(23\*sqrt[3 - x + 2\*x^2]) + (5\*ArcSinh[(-1 + 4\*x)/sqrt[23]])/(2\*sqrt[2])

**IntegrateAlgebraic [A]** time = 0.42, size = 60, normalized size = 1.33

$$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}} - \frac{5 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^(3/2), x]

[Out] (-11\*(5 + 3\*x))/(23\*sqrt[3 - x + 2\*x^2]) - (5\*Log[1 - 4\*x + 2\*sqrt[2]\*sqrt[3 - x + 2\*x^2]])/(2\*sqrt[2])

**fricas [B]** time = 0.41, size = 82, normalized size = 1.82

$$\frac{115\sqrt{2}(2x^2-x+3)\log(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25)-88\sqrt{2x^2-x+3}(3x+5)}{184(2x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(3/2), x, algorithm="fricas")

[Out] 1/184\*(115\*sqrt(2)\*(2\*x^2 - x + 3)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) - 88\*sqrt(2\*x^2 - x + 3)\*(3\*x + 5))/(2\*x^2 - x + 3)

**giac [A]** time = 0.23, size = 53, normalized size = 1.18

$$-\frac{5}{4}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(3/2),x, algorithm="giac")

[Out]  $-5/4*\sqrt{2}*\log(-2*\sqrt{2}*(\sqrt{2}*x - \sqrt{2*x^2 - x + 3}) + 1) - 11/23*(3*x + 5)/\sqrt{2*x^2 - x + 3}$

**maple** [A] time = 0.01, size = 64, normalized size = 1.42

$$-\frac{5x}{2\sqrt{2x^2-x+3}} + \frac{5\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4} - \frac{17}{8\sqrt{2x^2-x+3}} + \frac{\frac{49x}{46} - \frac{49}{184}}{\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(3/2),x)

[Out]  $-5/2/(2*x^2-x+3)^(1/2)*x-17/8/(2*x^2-x+3)^(1/2)+49/184*(4*x-1)/(2*x^2-x+3)^(1/2)+5/4*2^(1/2)*\operatorname{arcsinh}(4/23*23^(1/2)*(x-1/4))$

**maxima** [A] time = 0.95, size = 46, normalized size = 1.02

$$\frac{5}{4}\sqrt{2} \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{33x}{23\sqrt{2x^2-x+3}} - \frac{55}{23\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(3/2),x, algorithm="maxima")

[Out]  $5/4*\sqrt{2}*\operatorname{arcsinh}(1/23*\sqrt{23}*(4*x - 1)) - 33/23*x/\sqrt{2*x^2 - x + 3} - 55/23/\sqrt{2*x^2 - x + 3}$

**mupad** [B] time = 0.23, size = 87, normalized size = 1.93

$$\frac{5\sqrt{2} \ln\left(\sqrt{2x^2-x+3} + \frac{\sqrt{2}\left(2x-\frac{1}{2}\right)}{2}\right)}{4} + \frac{3(2x-12)}{23\sqrt{2x^2-x+3}} - \frac{10\left(\frac{11x}{2} + \frac{3}{2}\right)}{23\sqrt{2x^2-x+3}} + \frac{16x-4}{23\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)/(2\*x^2 - x + 3)^(3/2),x)

[Out]  $(5*2^(1/2)*\log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/4 + (3*(2*x - 12))/(23*(2*x^2 - x + 3)^(1/2)) - (10*((11*x)/2 + 3/2))/(23*(2*x^2 - x + 3)^(1/2)) + (16*x - 4)/(23*(2*x^2 - x + 3)^(1/2))$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2 + 3x + 2}{(2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)/(2\*x\*\*2-x+3)\*\*(3/2),x)

[Out] Integral((5\*x\*\*2 + 3\*x + 2)/(2\*x\*\*2 - x + 3)\*\*(3/2), x)



$$3.87 \quad \int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)} dx$$

Optimal. Leaf size=176

$$\frac{13-6x}{253\sqrt{2x^2-x+3}} + \frac{1}{22} \sqrt{\frac{1}{682}(247+500\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(247+500\sqrt{2})}}((69+65\sqrt{2})x+4\sqrt{2}+61)}{\sqrt{2x^2-x+3}} \right) - \frac{1}{22} \sqrt{\frac{1}{682}(247+500\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(500\sqrt{2}-247)}}((69-65\sqrt{2})x-4\sqrt{2}+61)}{\sqrt{2x^2-x+3}} \right)$$

Rubi [A] time = 0.41, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {974, 1035, 1029, 206, 204}

$$\frac{13-6x}{253\sqrt{2x^2-x+3}} + \frac{1}{22} \sqrt{\frac{1}{682}(247+500\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(247+500\sqrt{2})}}((69+65\sqrt{2})x+4\sqrt{2}+61)}{\sqrt{2x^2-x+3}} \right) - \frac{1}{22} \sqrt{\frac{1}{682}(500\sqrt{2}-247)} \tanh^{-1} \left( \frac{\sqrt{\frac{11}{31(500\sqrt{2}-247)}}((69-65\sqrt{2})x-4\sqrt{2}+61)}{\sqrt{2x^2-x+3}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/((3-x+2\*x^2)^(3/2)\*(2+3\*x+5\*x^2)),x]

[Out] (13-6\*x)/(253\*Sqrt[3-x+2\*x^2])+(Sqrt[(247+500\*Sqrt[2])/682]\*ArcTan[(Sqrt[11/(31\*(247+500\*Sqrt[2]))])\*(61+4\*Sqrt[2]+(69+65\*Sqrt[2])\*x)/Sqrt[3-x+2\*x^2]])/22-(Sqrt[(-247+500\*Sqrt[2])/682]\*ArcTanh[(Sqrt[11/(31\*(-247+500\*Sqrt[2]))])\*(61-4\*Sqrt[2]+(69-65\*Sqrt[2])\*x)/Sqrt[3-x+2\*x^2]])/22

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^(q+1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), Int[(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^q\*Simp[p[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p+1) - c\*d\*(p+2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p+q+2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p+q+2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p+1) - c\*e\*(2\*p+q+4))]\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p+2\*q+5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int
[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

**Rule 1035**

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^{3/2} (2+3x+5x^2)} dx = \frac{13-6x}{253\sqrt{3-x+2x^2}} - \frac{\int \frac{-1012-\frac{1265x}{2}}{\sqrt{3-x+2x^2} (2+3x+5x^2)} dx}{2783}$$

$$= \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{\int \frac{\frac{2783}{2}(3+8\sqrt{2})-\frac{2783}{2}(13-5\sqrt{2})x}{\sqrt{3-x+2x^2} (2+3x+5x^2)} dx}{61226\sqrt{2}} - \frac{\int \frac{\frac{2783}{2}(3-8\sqrt{2})-\frac{2783}{2}(13-5\sqrt{2})x}{\sqrt{3-x+2x^2} (2+3x+5x^2)} dx}{61226\sqrt{2}}$$

$$= \frac{13-6x}{253\sqrt{3-x+2x^2}} - \frac{1}{8} \left( 253 \left( 1000 - 247\sqrt{2} \right) \right) \text{Subst} \left( \int \frac{-\frac{240097759}{4} (247\sqrt{2} - 1000)}{\sqrt{31(247+500\sqrt{2})} (247\sqrt{2} - 1000 + x)} dx, \frac{247\sqrt{2} - 1000 + x}{\sqrt{31(247+500\sqrt{2})}} \right)$$

$$= \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{1}{22} \sqrt{\frac{1}{682} (247+500\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{31(247+500\sqrt{2})} (247\sqrt{2} - 1000 + x)}{247\sqrt{2} - 1000 + x} \right)$$

**Mathematica [C]** time = 1.29, size = 202, normalized size = 1.15

$$\frac{-\frac{27280(6x-13)}{\sqrt{2x^2-x+3}} - 23\sqrt{682(13+i\sqrt{31})} (13\sqrt{31} + 69i) \tanh^{-1}\left(\frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}} \sqrt{2x^2-x+3}}\right) - 23\sqrt{682(13-i\sqrt{31})} (13\sqrt{31} - 69i) \tanh^{-1}\left(\frac{(-22+4i\sqrt{31})x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}} \sqrt{2x^2-x+3}}\right)}{6901840}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)), x]
```

```
[Out] ((-27280*(-13 + 6*x))/Sqrt[3 - x + 2*x^2] - 23*Sqrt[682*(13 + I*Sqrt[31])]*
(69*I + 13*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x]/(
2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])) - 23*Sqrt[682*(13 - I*S
qrt[31])]*(-69*I + 13*Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqr
t[31])*x]/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2]))]/6901840
```

**IntegrateAlgebraic [C]** time = 0.49, size = 199, normalized size = 1.13

$$\frac{1}{22} \text{RootSum} \left[ -5\#1^4 + 6\sqrt{2}\#1^3 + 17\#1^2 - 26\sqrt{2}\#1 - 56\&, \frac{-5\#1^2 \log(-\#1 + \sqrt{2x^2-x+3} - \sqrt{2}x) + 16\sqrt{2}\#1 \log(-\#1 + \sqrt{2x^2-x+3} - \sqrt{2}x) + 23 \log(-\#1 + \sqrt{2x^2-x+3} - \sqrt{2}x)}{-10\#1^3 + 9\sqrt{2}\#1^2 + 17\#1 - 13\sqrt{2}} \right] + \frac{13-6x}{253\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(253*Sqrt[3 - x + 2*x^2]) + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (23*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 16*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 5*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/22
```

**fricas [B]** time = 1.17, size = 2083, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] -1/50921775520*(339388*sqrt(341)*50^(1/4)*sqrt(10)*sqrt(2)*(2*x^2 - x + 3)*sqrt(247*sqrt(2) + 1000)*arctan(1/328782125*(14260*sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*50^(3/4)*(57708*x^7 - 181278*x^6 + 400374*x^5 - 525676*x^4 + 235088*x^3 - 46944*x^2 - sqrt(2)*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 + 234360*x^3 - 156600*x^2 - 172800*x + 186624) - 373248*x + 172800) + 5*50^(1/4)*(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 10862496*x^2 - sqrt(2)*(56119*x^7 - 908994*x^6 + 5175980*x^5 - 12895624*x^4 + 17261280*x^3 - 14184000*x^2 - 10533888*x + 9994752) - 19989504*x + 10533888))*sqrt(247*sqrt(2) + 1000) + 933317000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(310/119)*(sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*50^(3/4)*(246848*x^7 - 348192*x^6 + 1080672*x^5 - 178432*x^4 - 18432*x^3 + 1029888*x^2 - sqrt(2)*(46522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*x^3 - 269568*x^2 + 269568*x) - 1029888*x) + 5*50^(1/4)*(516957*x^7 - 6676948*x^6 + 25569820*x^5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 4*sqrt(2)*(38689*x^7 - 502244*x^6 + 1967660*x^5 - 2828160*x^4 + 4711680*x^3 - 1689984*x^2 + 1689984*x) - 46199808*x))*sqrt(247*sqrt(2) + 1000) + 65450*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 2975*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 38) + x - 75)*sqrt(247*sqrt(2) + 1000) - 903805*x^2 - 811580*sqrt(2)*(2*x^2 - x + 3) + 2785195*x - 3689000)/x^2) + 10605875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 339388*sqrt(341)*50^(1/4)*sqrt(10)*sqrt(2)*(2*x^2 - x + 3)*sqrt(247*sqrt(2) + 1000)*arctan(1/328782125*(14260*sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*50^(3/4)*(57708*x^7 - 181278*x^6 + 400374*x^5 - 525676*x^4 + 235088*x^3 - 46944*x^2 - sqrt(2)*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 + 234360*x^3 - 156600*x^2 - 172800*x + 186624) - 373248*x + 172800) + 5*50^(1/4)*(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 10862496*x^2 - sqrt(2)*(56119*x^7 - 908994*x^6 + 5175980*x^5 - 12895624*x^4 + 17261280*x^3 - 14184000*x^2 - 10533888*x + 9994752) - 19989504*x + 10533888))*sqrt(247*sqrt(2) + 1000) - 933317000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
```

```

088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(31
0/119)*(sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*50^(3/4)*(246848*x^7 - 3
48192*x^6 + 1080672*x^5 - 178432*x^4 - 18432*x^3 + 1029888*x^2 - sqrt(2)*(4
6522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*x^3 - 269568*x^2 +
269568*x) - 1029888*x) + 5*50^(1/4)*(516957*x^7 - 6676948*x^6 + 25569820*x^
5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 4*sqrt(2)*(38689*x^7 - 502
244*x^6 + 1967660*x^5 - 2828160*x^4 + 4711680*x^3 - 1689984*x^2 + 1689984*x
) - 46199808*x))*sqrt(247*sqrt(2) + 1000) - 65450*sqrt(31)*sqrt(2)*(123408*
x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38
22336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105
3960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 2975*sqrt(
31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4
- 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 -
1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((
sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 3
8) + x - 75)*sqrt(247*sqrt(2) + 1000) + 903805*x^2 + 811580*sqrt(2)*(2*x^2
- x + 3) - 2785195*x + 3689000)/x^2) - 10605875*sqrt(31)*(2828123*x^8 - 969
6916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3
7981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15
569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(
2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4424
9088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 23*sqrt(341)*50^(1/4)*s
qrt(31)*sqrt(10)*(2000*x^2 - 247*sqrt(2)*(2*x^2 - x + 3) - 1000*x + 3000)*s
qrt(247*sqrt(2) + 1000)*log(3100000/119*(sqrt(341)*50^(1/4)*sqrt(31)*sqrt(1
0)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 38) + x - 75)*sqrt(247*sqrt(2) + 10
00) + 903805*x^2 + 811580*sqrt(2)*(2*x^2 - x + 3) - 2785195*x + 3689000)/x^
2) + 23*sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*(2000*x^2 - 247*sqrt(2)*(2*x^2
- x + 3) - 1000*x + 3000)*sqrt(247*sqrt(2) + 1000)*log(-3100000/119*(sqrt(
341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 38) +
x - 75)*sqrt(247*sqrt(2) + 1000) - 903805*x^2 - 811580*sqrt(2)*(2*x^2 - x +
3) + 2785195*x - 3689000)/x^2) + 201271840*sqrt(2*x^2 - x + 3)*(6*x - 13))
/(2*x^2 - x + 3)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2),x, algorithm="giac")

```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infini
ty]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity]
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 9.49Done

```

**maple** [B] time = 0.03, size = 718, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2),x)

```

[Out] 1/465124*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+
2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(2197*2^(1/2)*(-8866+6820*2^(1/2))

```

$(1/2)*(-775687+549362*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})))+3070*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})))+1712502*\arctanh(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})*2^{(1/2)}-6617446*\arctanh(31/2*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})/((8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1))^2)^{(1/2)}/(1+(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1))/(8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}+1/22/(2*x^2-x+3)^{(1/2)}-3/506*(4*x-1)/(2*x^2-x+3)^{(1/2)}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)(2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2),x, algorithm="maxima")

[Out] integrate(1/((5\*x^2 + 3\*x + 2)\*(2\*x^2 - x + 3)^(3/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(2x^2 - x + 3)^{3/2} (5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)),x)

[Out] int(1/((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*(3/2)/(5\*x\*\*2+3\*x+2),x)

[Out] Integral(1/((2\*x\*\*2 - x + 3)\*\*(3/2)\*(5\*x\*\*2 + 3\*x + 2)), x)

$$3.88 \quad \int \frac{1}{(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} dx$$

**Optimal.** Leaf size=211

$$\frac{6315 - 2306x}{345092\sqrt{2x^2 - x + 3}} + \frac{65x + 4}{682\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)} + \frac{\sqrt{\frac{1}{682} (129694447 + 103775000\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{31(129694447 + 103775000\sqrt{2})}}{\sqrt{2x^2 - x + 3}} \right)}{30008}$$

**Rubi [A]** time = 0.47, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{6315 - 2306x}{345092\sqrt{2x^2 - x + 3}} + \frac{65x + 4}{682\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)} + \frac{\sqrt{\frac{1}{682} (129694447 + 103775000\sqrt{2})} \tan^{-1} \left( \frac{\sqrt{31(129694447 + 103775000\sqrt{2})} ((45519 + 29065\sqrt{2})x + 16454\sqrt{2} + 12611)}{\sqrt{2x^2 - x + 3}} \right)}{30008} - \frac{\sqrt{\frac{1}{682} (103775000\sqrt{2} - 129694447)} \tanh^{-1} \left( \frac{\sqrt{31(103775000\sqrt{2} - 129694447)} ((45519 - 29065\sqrt{2})x - 16454\sqrt{2} + 12611)}{\sqrt{2x^2 - x + 3}} \right)}{30008}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] -(6315 - 2306\*x)/(345092\*sqrt[3 - x + 2\*x^2]) + (4 + 65\*x)/(682\*sqrt[3 - x + 2\*x^2]\*(2 + 3\*x + 5\*x^2)) + (sqrt[(129694447 + 103775000\*sqrt[2])/682]\*ArcTan[(sqrt[11/(31\*(129694447 + 103775000\*sqrt[2]))]\*(12611 + 16454\*sqrt[2] + (45519 + 29065\*sqrt[2])\*x))/sqrt[3 - x + 2\*x^2]])/30008 - (sqrt[(-12969447 + 103775000\*sqrt[2])/682]\*ArcTanh[(sqrt[11/(31\*(-129694447 + 103775000\*sqrt[2]))]\*(12611 - 16454\*sqrt[2] + (45519 - 29065\*sqrt[2])\*x))/sqrt[3 - x + 2\*x^2]])/30008

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

#### Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]
```

### Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

### Rubi steps

$$\int \frac{1}{(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} dx = \frac{4+65x}{682\sqrt{3-x+2x^2} (2+3x+5x^2)} - \frac{\int \frac{-1782+\frac{3333x}{2}-2860x^2}{(3-x+2x^2)^{3/2} (2+3x+5x^2)} dx}{7502}$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2} (2+3x+5x^2)} - \frac{\int \frac{-30028\frac{1}{2}}{\sqrt{3-x+2x^2}}}{20}$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2} (2+3x+5x^2)} + \frac{\int \frac{30613}{4}(46)}{20}$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2} (2+3x+5x^2)} - \frac{(253(207)}{20}$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2} (2+3x+5x^2)} + \frac{\sqrt{\frac{1}{682}}(12)}{20}$$

**Mathematica [C]** time = 1.53, size = 740, normalized size = 3.51

$$\frac{100 \left( \frac{682(52 + \sqrt{31}) + (22 - (4I)\sqrt{31})x}{(3I + \sqrt{31} + (10I)x)\sqrt{3-x+2x^2}} + \frac{682(52 - \sqrt{31}) + (22 + (4I)\sqrt{31})x}{(-13I + \sqrt{31})(-3I + \sqrt{31} - (10I)x)\sqrt{3-x+2x^2}} + \frac{22(31I + 52\sqrt{31}) + 2*(-62I + 11\sqrt{31})x}{(13I + \sqrt{31})\sqrt{3-x+2x^2}} + \frac{22(-31I + 52\sqrt{31}) + 2*(62I + 11\sqrt{31})x}{(-13I + \sqrt{31})\sqrt{3-x+2x^2}} + \frac{(575I)\sqrt{682(13 + I\sqrt{31})} \operatorname{ArcTanh}[(63 + I\sqrt{31}) + (-22 - (4I)\sqrt{31})x]}{(2\sqrt{286 + (22I)\sqrt{31}})\sqrt{3-x+2x^2}} \right)}{(-13I + \sqrt{31})^2 + (155*(44*(16353 + (581I)\sqrt{31}))\sqrt{3-x+2x^2} + 345\sqrt{286 + (22I)\sqrt{31}})*(-29 + (17I)\sqrt{31} + 10*(11 + (2I)\sqrt{31})x) \operatorname{ArcTanh}[(63 + I\sqrt{31}) + (-22 - (4I)\sqrt{31})x]} + \frac{(155*(44*(16353 - (581I)\sqrt{31}))\sqrt{3-x+2x^2} + 345\sqrt{286 - (22I)\sqrt{31}})*(29 + (17I)\sqrt{31} + (-110 + (20I)\sqrt{31})x) \operatorname{ArcTanh}[(-63 + I\sqrt{31}) + (22 - (4I)\sqrt{31})x]}{(2\sqrt{286 - (22I)\sqrt{31}})\sqrt{3-x+2x^2}} \right)}{(22*(-13I + \sqrt{31})^3*(-3I + \sqrt{31} - (10I)x)) + (155*(44*(16353 - (581I)\sqrt{31}))\sqrt{3-x+2x^2} + 345\sqrt{286 - (22I)\sqrt{31}})*(29 + (17I)\sqrt{31} + (-110 + (20I)\sqrt{31})x) \operatorname{ArcTanh}[(-63 + I\sqrt{31}) + (22 - (4I)\sqrt{31})x]} + \frac{((575I)\sqrt{682(13 - I\sqrt{31})} \operatorname{ArcTanh}[(63 - I\sqrt{31}) + (-22 + (4I)\sqrt{31})x]}{(2\sqrt{286 - (22I)\sqrt{31}})\sqrt{3-x+2x^2}} \right)}{(13I + \sqrt{31})^2)} / 2674463$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2), x]
[Out] (100*((682*(52 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x))/((13*I + Sqrt[31]))*(3*I + Sqrt[31] + (10*I)*x)*Sqrt[3 - x + 2*x^2]) + (682*(52 - I*Sqrt[31] + (22 + (4*I)*Sqrt[31])*x))/((-13*I + Sqrt[31])*(-3*I + Sqrt[31] - (10*I)*x)*Sqrt[3 - x + 2*x^2]) + (22*(31*I + 52*Sqrt[31] + 2*(-62*I + 11*Sqrt[31])*x))/((13*I + Sqrt[31])*Sqrt[3 - x + 2*x^2]) + (22*(-31*I + 52*Sqrt[31] + 2*(62*I + 11*Sqrt[31])*x))/((-13*I + Sqrt[31])*Sqrt[3 - x + 2*x^2]) + ((575*I)*Sqrt[682*(13 + I*Sqrt[31])]*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x]/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(-13*I + Sqrt[31])^2 + (155*(44*(16353 + (581*I)*Sqrt[31])*Sqrt[3 - x + 2*x^2] + 345*Sqrt[286 + (22*I)*Sqrt[31]])*(-29 + (17*I)*Sqrt[31] + 10*(11 + (2*I)*Sqrt[31])*x)*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x]/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/((22*(-13*I + Sqrt[31])^3*(-3*I + Sqrt[31] - (10*I)*x)) + (155*(44*(16353 - (581*I)*Sqrt[31])*Sqrt[3 - x + 2*x^2] + 345*Sqrt[286 - (22*I)*Sqrt[31]])*(29 + (17*I)*Sqrt[31] + (-110 + (20*I)*Sqrt[31])*x)*ArcTanh[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x]/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/((22*(13*I + Sqrt[31])^3*(3*I + Sqrt[31] + (10*I)*x)) - ((575*I)*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x]/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/((13*I + Sqrt[31])^2))/2674463
```

**IntegrateAlgebraic [C]** time = 0.95, size = 406, normalized size = 1.92

$$\frac{1}{484} \operatorname{RootSum}[-581^4 + 6\sqrt{2}81^3 + 1781^2 - 26\sqrt{2}81 - 564, \frac{-1581^2 \log(-81 + \sqrt{2x^2 - x + 3} - \sqrt{2x}) + 8\sqrt{2}81 \log(-81 + \sqrt{2x^2 - x + 3} - \sqrt{2x}) + 225 \log(-81 + \sqrt{2x^2 - x + 3} - \sqrt{2x})}{-1081^2 + 9\sqrt{2}81^2 + 1781 - 13\sqrt{2}}] + \frac{\operatorname{RootSum}[-581^4 + 6\sqrt{2}81^3 + 1781^2 - 26\sqrt{2}81 - 564, \frac{108\sqrt{2}81^2 \log(-81 + \sqrt{2x^2 - x + 3} - \sqrt{2x}) - 108\sqrt{2}81 \log(-81 + \sqrt{2x^2 - x + 3} - \sqrt{2x})}{30008\sqrt{2}}]}{30008\sqrt{2}} + \frac{11530x^2 - 24657x + 18557z - 10606}{345092\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2), x]
```



```
[Out] (-10606 + 18557*x - 24657*x^2 + 11530*x^3)/(345092*Sqrt[3 - x + 2*x^2]*(2 +
  3*x + 5*x^2)) - RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5
  *#1^4 & , (225*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 8*Sqrt[2]*Log
  [-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 15*Log[-(Sqrt[2]*x) + Sqrt[3
  - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3)
  & ]/484 + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 &
  , (8623*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 9624*Log[-(
  Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 1565*Sqrt[2]*Log[-(Sqrt[2]*x) +
  Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10
  *#1^3) & ]/(30008*Sqrt[2])
```

**fricas [B]** time = 1.26, size = 2173, normalized size = 10.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/29889247038841109870720*(35183643812*3446160200^(1/4)*sqrt(20755)*sqrt(34
  1)*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(129694447*sqrt(2) + 20755
  0000)*arctan(1/2437871055247532640924125*(59193260*sqrt(20755)*(11*34461602
  00^(3/4)*sqrt(341)*(20748108*x^7 - 87744678*x^6 + 180517074*x^5 - 311740976
  *x^4 + 161753488*x^3 - 89046144*x^2 - sqrt(2)*(18515146*x^7 - 65709803*x^6
  + 140687186*x^5 - 209710441*x^4 + 101256360*x^3 - 39198600*x^2 - 126316800*
  x + 76909824) - 153819648*x + 126316800) + 643405*3446160200^(1/4)*sqrt(341
  )*(1637219*x^7 - 25548801*x^6 + 138274456*x^5 - 324967420*x^4 + 425065248*x
  ^3 - 297030816*x^2 - sqrt(2)*(1361849*x^7 - 20608224*x^6 + 106575580*x^5 -
  236322704*x^4 + 301502880*x^3 - 169632000*x^2 - 225358848*x + 143534592) -
  287069184*x + 225358848))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 2075
  50000) + 6920408156831705561333000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7
  + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2
  )*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x
  ^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(41510/3
  97951)*(sqrt(20755)*(11*3446160200^(3/4)*sqrt(341)*(66710248*x^7 - 96938292
  *x^6 + 319739772*x^5 - 172116032*x^4 + 247423968*x^3 + 38700288*x^2 - sqrt(
  2)*(71827622*x^7 - 102266467*x^6 + 323714097*x^5 - 93357360*x^4 + 79054920*
  x^3 + 219532032*x^2 - 219532032*x) - 38700288*x) + 643405*3446160200^(1/4)*
  sqrt(341)*(5462397*x^7 - 70721108*x^6 + 273784220*x^5 - 364358592*x^4 + 506
  287008*x^3 + 144903168*x^2 - 2*sqrt(2)*(2586013*x^7 - 33428948*x^6 + 128512
  220*x^5 - 162918720*x^4 + 196126560*x^3 + 173705472*x^2 - 173705472*x) - 14
  4903168*x))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 207550000) + 11691
  2097033204550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 329
  3072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118
  051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2
  - 1036800*x) + 3276288*x) + 5314186228782025*sqrt(31)*(254591*x^8 - 481512
  6*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956
  928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3
  618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(3446160200^(1/4)*sqrt(2
  0755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x + 12812) - 18
  949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) - 388930324332445*x^2 - 3
  49243556543420*sqrt(2)*(2*x^2 - x + 3) + 1198540387228555*x - 1587470711561
  000)/x^2) + 78641001782178472287875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 5
  3385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2
  - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 55
  68*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8
  - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 3
  4615296*x^2 - 24772608*x + 18579456) + 35183643812*3446160200^(1/4)*sqrt(2
  0755)*sqrt(341)*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(129694447*sq
  rt(2) + 207550000)*arctan(1/2437871055247532640924125*(59193260*sqrt(20755)
  *(11*3446160200^(3/4)*sqrt(341)*(20748108*x^7 - 87744678*x^6 + 180517074*x^
```

```

5 - 311740976*x^4 + 161753488*x^3 - 89046144*x^2 - sqrt(2)*(18515146*x^7 -
65709803*x^6 + 140687186*x^5 - 209710441*x^4 + 101256360*x^3 - 39198600*x^2
- 126316800*x + 76909824) - 153819648*x + 126316800) + 643405*3446160200^(
1/4)*sqrt(341)*(1637219*x^7 - 25548801*x^6 + 138274456*x^5 - 324967420*x^4
+ 425065248*x^3 - 297030816*x^2 - sqrt(2)*(1361849*x^7 - 20608224*x^6 + 106
575580*x^5 - 236322704*x^4 + 301502880*x^3 - 169632000*x^2 - 225358848*x +
143534592) - 287069184*x + 225358848))*sqrt(2*x^2 - x + 3)*sqrt(129694447*s
qrt(2) + 207550000) - 6920408156831705561333000*sqrt(31)*sqrt(2)*(28180*x^8
- 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496
*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x
^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2
*sqrt(41510/397951)*(sqrt(20755)*(11*3446160200^(3/4)*sqrt(341)*(66710248*x
^7 - 96938292*x^6 + 319739772*x^5 - 172116032*x^4 + 247423968*x^3 + 3870028
8*x^2 - sqrt(2)*(71827622*x^7 - 102266467*x^6 + 323714097*x^5 - 93357360*x^
4 + 79054920*x^3 + 219532032*x^2 - 219532032*x) - 38700288*x) + 643405*3446
160200^(1/4)*sqrt(341)*(5462397*x^7 - 70721108*x^6 + 273784220*x^5 - 364358
592*x^4 + 506287008*x^3 + 144903168*x^2 - 2*sqrt(2)*(2586013*x^7 - 33428948
*x^6 + 128512220*x^5 - 162918720*x^4 + 196126560*x^3 + 173705472*x^2 - 1737
05472*x) - 144903168*x))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 20755
0000) - 116912097033204550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578
888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15
550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3
+ 1209600*x^2 - 1036800*x) + 3276288*x) - 5314186228782025*sqrt(31)*(254591
*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328
*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 +
2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((3446160200^(
1/4)*sqrt(20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x +
12812) - 18949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) + 38893032433
2445*x^2 + 349243556543420*sqrt(2)*(2*x^2 - x + 3) - 1198540387228555*x + 1
587470711561000)/x^2) - 78641001782178472287875*sqrt(31)*(2828123*x^8 - 969
6916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3
7981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15
569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/((
2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4424
9088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 23*3446160200^(1/4)*sqr
t(20755)*sqrt(341)*sqrt(31)*(2075500000*x^4 + 207550000*x^3 + 3320800000*x^
2 - 129694447*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6) + 1452850000*x + 12
45300000)*sqrt(129694447*sqrt(2) + 207550000)*log(1037750000000/397951*(344
6160200^(1/4)*sqrt(20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(
6137*x + 12812) - 18949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) + 388
930324332445*x^2 + 349243556543420*sqrt(2)*(2*x^2 - x + 3) - 11985403872285
55*x + 1587470711561000)/x^2) - 23*3446160200^(1/4)*sqrt(20755)*sqrt(341)*s
qrt(31)*(2075500000*x^4 + 207550000*x^3 + 3320800000*x^2 - 129694447*sqrt(2
)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6) + 1452850000*x + 1245300000)*sqrt(12969
4447*sqrt(2) + 207550000)*log(-1037750000000/397951*(3446160200^(1/4)*sqrt(
20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x + 12812) - 1
8949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) - 388930324332445*x^2 -
349243556543420*sqrt(2)*(2*x^2 - x + 3) + 1198540387228555*x - 158747071156
1000)/x^2) + 86612402022768160*(11530*x^3 - 24657*x^2 + 18557*x - 10606)*sq
rt(2*x^2 - x + 3))/(10*x^4 + x^3 + 16*x^2 + 7*x + 6)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x);;OUTPUT:Francis algorithm failure for[-1.0,infinity,

infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi  
 nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity  
 ]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai  
 lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity  
 ,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf  
 inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infini  
 ty]Evaluation time: 36.15Done

**maple** [B] time = 0.10, size = 5942, normalized size = 28.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^2,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)^2 (2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^2,x, algorithm="maxima")

[Out] integrate(1/((5\*x^2 + 3\*x + 2)^2\*(2\*x^2 - x + 3)^(3/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)^2),x)

[Out] int(1/((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*(3/2)/(5\*x\*\*2+3\*x+2)\*\*2,x)

[Out] Integral(1/((2\*x\*\*2 - x + 3)\*\*(3/2)\*(5\*x\*\*2 + 3\*x + 2)\*\*2), x)

3.89  $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3} dx$

Optimal. Leaf size=246

$$\frac{4353943 - 6508666x}{941410976\sqrt{2x^2 - x + 3}} + \frac{5(17315x + 7318)}{1860496\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)} + \frac{65x + 4}{1364\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^2} + \frac{3\sqrt{\frac{1}{682}}}{1364\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^2}$$

**Rubi [A]** time = 0.53, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{4353943 - 6508666x}{941410976\sqrt{2x^2 - x + 3}} + \frac{5(17315x + 7318)}{1860496\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)} + \frac{65x + 4}{1364\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^2} + \frac{3\sqrt{\frac{1}{682}} \operatorname{arctan}\left(\frac{\sqrt{11(13874275807943 + 9819738650000\sqrt{2})}}{\sqrt{2x^2 - x + 3}}\right)}{81861824} + \frac{3\sqrt{\frac{1}{682}} \operatorname{arctanh}\left(\frac{\sqrt{11(13874275807943 + 9819738650000\sqrt{2})}}{\sqrt{2x^2 - x + 3}}\right)}{81861824}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3), x]
[Out] -(4353943 - 6508666*x)/(941410976*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(1364*Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) + (5*(7318 + 17315*x))/(1860496*Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)) + (3*Sqrt[(13874275807943 + 9819738650000*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13874275807943 + 9819738650000*Sqrt[2])])]*(5538393 + 4123702*Sqrt[2] + (13785797 + 9662095*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2])/81861824 - (3*Sqrt[(-13874275807943 + 9819738650000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-13874275807943 + 9819738650000*Sqrt[2])])]*(5538393 - 4123702*Sqrt[2] + (13785797 - 9662095*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2])/81861824
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && (!(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0])
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((A_.) + (B_.)*(x_) + (C_.)*(x
_)^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_.), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^{3/2} (2+3x+5x^2)^3} dx = \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} - \frac{\int \frac{-5731+\frac{7557x}{2}-5720x^2}{(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} dx}{15004}$$

$$= \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2} (2+3x+5x^2)^2}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2} (2+3x+5x^2)^2}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2} (2+3x+5x^2)^2}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2} (2+3x+5x^2)^2}$$

**Mathematica [C]** time = 2.21, size = 231, normalized size = 0.94

$$\frac{69\sqrt{286-22i\sqrt{31}}(13785797\sqrt{31}+14026539i)\tan^{-1}\left(\frac{-2(2\sqrt{31}+11i)+\sqrt{51}+63i}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)-69i\sqrt{286+22i\sqrt{31}}(13785797\sqrt{31}-14026539i)\tanh^{-1}\left(\frac{-22-4i\sqrt{31}+\sqrt{51}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)+\frac{27280(162716650x^5+86411405x^4+277167774x^3+175833195x^2+161806828x+22374044)}{\sqrt{2x^2-x+3}(5x^2+3x+2)^2}}{25681691425280}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2\*x^2)^(3/2)\*(2+3\*x+5\*x^2)^3),x]

[Out] ((27280\*(22374044+161806828\*x+175833195\*x^2+277167774\*x^3+86411405\*x^4+162716650\*x^5))/(Sqrt[3-x+2\*x^2]\*(2+3\*x+5\*x^2)^2)+69\*Sqrt[286-(22\*I)\*Sqrt[31]]\*(14026539\*I+13785797\*Sqrt[31])\*ArcTan[(63\*I+Sqrt[31]-2\*(11\*I+2\*Sqrt[31])\*x)/(2\*Sqrt[286-(22\*I)\*Sqrt[31]]\*Sqrt[3-x+2\*x^2])] - (69\*I)\*Sqrt[286+(22\*I)\*Sqrt[31]]\*(-14026539\*I+13785797\*Sqrt[31])\*ArcTanh[(63+I\*Sqrt[31]+(-22-(4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286+(22\*I)\*Sqrt[31]]\*Sqrt[3-x+2\*x^2])])/25681691425280

**IntegrateAlgebraic [C]** time = 1.51, size = 611, normalized size = 2.48

$$\frac{22374044+161806828x+175833195x^2+277167774x^3+86411405x^4+162716650x^5}{(941410976\sqrt{3-x+2x^2}(2+3x+5x^2)^2)-\text{RootSum}[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4 \& , (-491\sqrt{2}*\text{Log}[-(\sqrt{2}*x)+\sqrt{3-x+2x^2}]-\#1)+208*\text{Log}[-(\sqrt{2}*x)+\sqrt{3-x+2x^2}]-\#1]\#1+5\sqrt{2}*\text{Log}[-(\sqrt{2}*x)+\sqrt{3-x+2x^2}]-\#1]\#1^2)/(-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3) \& ]}{(10648\sqrt{2}+17\sqrt{2}\#1+9\sqrt{2}\#1^2-10\#1^3) \& ]}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((3-x+2\*x^2)^(3/2)\*(2+3\*x+5\*x^2)^3),x]

[Out] (22374044+161806828\*x+175833195\*x^2+277167774\*x^3+86411405\*x^4+162716650\*x^5)/(941410976\*Sqrt[3-x+2\*x^2]\*(2+3\*x+5\*x^2)^2)-RootSum[-56-26\*Sqrt[2]\*#1+17\*#1^2+6\*Sqrt[2]\*#1^3-5\*#1^4 & , (-491\*Sqrt[2]\*Log[-(Sqrt[2]\*x)+Sqrt[3-x+2\*x^2]-#1]+208\*Log[-(Sqrt[2]\*x)+Sqrt[3-x+2\*x^2]-#1]\*#1+5\*Sqrt[2]\*Log[-(Sqrt[2]\*x)+Sqrt[3-x+2\*x^2]-#1]\*#1^2)/(-13\*Sqrt[2]+17\*#1+9\*Sqrt[2]\*#1^2-10\*#1^3) & ]/(10648\*Sqrt[2]+17\*Sqrt[2]\*#1+9\*Sqrt[2]\*#1^2-10\*#1^3) & ]

[2]) + RootSum[-56 - 26\*Sqrt[2]\*#1 + 17\*#1^2 + 6\*Sqrt[2]\*#1^3 - 5\*#1^4 & ,  
 (7194481\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1] - 3798456\*Log  
 [-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1 + 575915\*Sqrt[2]\*Log[-(Sqrt[2]  
 \*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1^2)/(-13\*Sqrt[2] + 17\*#1 + 9\*Sqrt[2]\*#1^2  
 - 10\*#1^3) & ]/(15184048\*Sqrt[2]) - (7\*RootSum[-56 - 26\*Sqrt[2]\*#1 + 17\*#1  
 ^2 + 6\*Sqrt[2]\*#1^3 - 5\*#1^4 & , (143178771\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt  
 [3 - x + 2\*x^2] - #1] - 105962920\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] -  
 #1]\*#1 + 6180225\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1^2)  
 /(-13\*Sqrt[2] + 17\*#1 + 9\*Sqrt[2]\*#1^2 - 10\*#1^3) & ])/(1882821952\*Sqrt[2])

**fricas** [B] time = 1.39, size = 2263, normalized size = 9.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^3,x, algorithm="fricas")

[Out] 1/33652296632397026886019646994897920\*(920746859815884\*1928545343086076450^  
 (1/4)\*sqrt(1963947730)\*sqrt(341)\*sqrt(2)\*(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^  
 3 + 83\*x^2 + 32\*x + 12)\*sqrt(13874275807943\*sqrt(2) + 19639477300000)\*arcta  
 n(1/2252270155289097943751876925347228391692375\*(2800589462980\*sqrt(1963947  
 730)\*(22\*1928545343086076450^(3/4)\*sqrt(341)\*(7361410004\*x^7 - 28555361914\*  
 x^6 + 59872788262\*x^5 - 96593638888\*x^4 + 48573560944\*x^3 - 23355012672\*x^2  
 - sqrt(2)\*(5311119598\*x^7 - 20292577289\*x^6 + 42695479118\*x^5 - 6800681868  
 3\*x^4 + 33985514680\*x^3 - 15860251800\*x^2 - 37489478400\*x + 26167456512) -  
 52334913024\*x + 37489478400) + 30441189815\*1928545343086076450^(1/4)\*sqrt(3  
 41)\*(560592897\*x^7 - 8616399363\*x^6 + 45618625128\*x^5 - 104316505460\*x^4 +  
 134890825824\*x^3 - 85859939808\*x^2 - sqrt(2)\*(402019087\*x^7 - 6162703212\*x^  
 6 + 32499503540\*x^5 - 73942829952\*x^4 + 95407993440\*x^3 - 59600016000\*x^2 -  
 68177562624\*x + 47773380096) - 95546760192\*x + 68177562624)\*sqrt(2\*x^2 -  
 x + 3)\*sqrt(13874275807943\*sqrt(2) + 19639477300000) + 63935410859819554532  
 31134497759874144159000\*sqrt(31)\*sqrt(2)\*(28180\*x^8 - 254666\*x^7 + 704270\*x  
 ^6 - 1385256\*x^5 + 1549144\*x^4 - 642048\*x^3 - 98496\*x^2 - sqrt(2)\*(8746\*x^8  
 - 102335\*x^7 + 396104\*x^6 - 783113\*x^5 + 1320710\*x^4 - 752088\*x^3 + 396144  
 \*x^2 + 546048\*x - 539136) + 1154304\*x - 456192) - 2\*sqrt(1963947730/3471424  
 919)\*(sqrt(1963947730)\*(22\*1928545343086076450^(3/4)\*sqrt(341)\*(26184810824  
 \*x^7 - 37618468196\*x^6 + 121297463436\*x^5 - 48741866816\*x^4 + 58784153184\*x  
 ^3 + 51583129344\*x^2 - sqrt(2)\*(19194187986\*x^7 - 27528525721\*x^6 + 8845761  
 3411\*x^5 - 33685377680\*x^4 + 38926767960\*x^3 + 41764674816\*x^2 - 4176467481  
 6\*x) - 51583129344\*x) + 30441189815\*1928545343086076450^(1/4)\*sqrt(341)\*(19  
 98926311\*x^7 - 25858659004\*x^6 + 99738083860\*x^5 - 129415692096\*x^4 + 16744  
 6420704\*x^3 + 96037622784\*x^2 - 22\*sqrt(2)\*(65886479\*x^7 - 852213084\*x^6 +  
 3285070260\*x^5 - 4244909760\*x^4 + 5424792480\*x^3 + 3393259776\*x^2 - 3393259  
 776\*x) - 96037622784\*x)\*sqrt(2\*x^2 - x + 3)\*sqrt(13874275807943\*sqrt(2) +  
 19639477300000) + 2282926923240949861309948624550\*sqrt(31)\*sqrt(2)\*(123408\*  
 x^8 - 914152\*x^7 + 1578888\*x^6 - 3293072\*x^5 + 396480\*x^4 + 798336\*x^3 - 38  
 22336\*x^2 - sqrt(2)\*(15550\*x^8 - 118051\*x^7 + 244047\*x^6 - 707374\*x^5 + 105  
 3960\*x^4 - 1667952\*x^3 + 1209600\*x^2 - 1036800\*x) + 3276288\*x) + 1037694056  
 01861357332270392025\*sqrt(31)\*(254591\*x^8 - 4815126\*x^7 + 32303580\*x^6 - 90  
 866808\*x^5 + 108781920\*x^4 - 74219328\*x^3 - 168956928\*x^2 - 15488\*sqrt(2)\*(  
 4\*x^8 - 76\*x^7 + 517\*x^6 - 1536\*x^5 + 2385\*x^4 - 3618\*x^3 + 2268\*x^2 - 1944  
 \*x) + 144820224\*x)\*sqrt(-(1928545343086076450^(1/4)\*sqrt(1963947730)\*sqrt(  
 341)\*sqrt(31)\*sqrt(2\*x^2 - x + 3)\*(sqrt(2)\*(2995431\*x + 1523456) - 4518887\*  
 x - 1471975)\*sqrt(13874275807943\*sqrt(2) + 19639477300000) - 16051926912456  
 8199977215\*x^2 - 144139751866959199979540\*sqrt(2)\*(2\*x^2 - x + 3) + 4946614  
 21179791799929785\*x - 655180690304359999907000)/x^2) + 72653875977067675604  
 899255656362206183625\*sqrt(31)\*(2828123\*x^8 - 9696916\*x^7 + 53385560\*x^6 -  
 142835344\*x^5 + 254146592\*x^4 - 249300096\*x^3 + 37981440\*x^2 - 7744\*sqrt(2)  
 \*(1348\*x^8 - 2692\*x^7 + 9789\*x^6 - 10070\*x^5 + 15569\*x^4 - 5568\*x^3 + 1080\*  
 x^2 + 4320\*x - 5184) + 223064064\*x - 94887936)/(2585191\*x^8 - 4661200\*x^7

$$\begin{aligned}
& + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - \\
& 24772608*x + 18579456) + 920746859815884*1928545343086076450^{(1/4)}*\sqrt{19} \\
& 63947730)*\sqrt{341)*\sqrt{2}*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + \\
& 32*x + 12)*\sqrt{13874275807943*\sqrt{2} + 19639477300000)*\arctan(1/225227015 \\
& 5289097943751876925347228391692375*(2800589462980*\sqrt{1963947730})*(22*1928 \\
& 545343086076450^{(3/4)}*\sqrt{341}*(7361410004*x^7 - 28555361914*x^6 + 5987278 \\
& 8262*x^5 - 96593638888*x^4 + 48573560944*x^3 - 23355012672*x^2 - \sqrt{2}*(5 \\
& 311119598*x^7 - 20292577289*x^6 + 42695479118*x^5 - 68006818683*x^4 + 33985 \\
& 514680*x^3 - 15860251800*x^2 - 37489478400*x + 26167456512) - 52334913024*x \\
& + 37489478400) + 30441189815*1928545343086076450^{(1/4)}*\sqrt{341}*(56059289 \\
& 7*x^7 - 8616399363*x^6 + 45618625128*x^5 - 104316505460*x^4 + 134890825824* \\
& x^3 - 85859939808*x^2 - \sqrt{2}*(402019087*x^7 - 6162703212*x^6 + 324995035 \\
& 40*x^5 - 73942829952*x^4 + 95407993440*x^3 - 59600016000*x^2 - 68177562624* \\
& x + 47773380096) - 95546760192*x + 68177562624))*\sqrt{2*x^2 - x + 3)*\sqrt{1} \\
& 3874275807943*\sqrt{2} + 19639477300000) - 639354108598195545323113449775987 \\
& 4144159000*\sqrt{31)*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256* \\
& x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 \\
& + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048 \\
& *x - 539136) + 1154304*x - 456192) - 2*\sqrt{1963947730/3471424919}*(\sqrt{19} \\
& 63947730)*(22*1928545343086076450^{(3/4)}*\sqrt{341}*(26184810824*x^7 - 376184 \\
& 68196*x^6 + 121297463436*x^5 - 48741866816*x^4 + 58784153184*x^3 + 51583129 \\
& 344*x^2 - \sqrt{2}*(19194187986*x^7 - 27528525721*x^6 + 88457613411*x^5 - 33 \\
& 685377680*x^4 + 38926767960*x^3 + 41764674816*x^2 - 41764674816*x) - 515831 \\
& 29344*x) + 30441189815*1928545343086076450^{(1/4)}*\sqrt{341}*(1998926311*x^7 \\
& - 25858659004*x^6 + 99738083860*x^5 - 129415692096*x^4 + 167446420704*x^3 + \\
& 96037622784*x^2 - 22*\sqrt{2}*(65886479*x^7 - 852213084*x^6 + 3285070260*x^ \\
& 5 - 4244909760*x^4 + 5424792480*x^3 + 3393259776*x^2 - 3393259776*x) - 9603 \\
& 7622784*x))*\sqrt{2*x^2 - x + 3)*\sqrt{13874275807943*\sqrt{2} + 1963947730000 \\
& 0) - 2282926923240949861309948624550*\sqrt{31)*\sqrt{2}*(123408*x^8 - 914152* \\
& x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - s \\
& \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 16 \\
& 67952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 10376940560186135733227 \\
& 0392025*\sqrt{31}*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + \\
& 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^ \\
& 7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 1448202 \\
& 24*x))*\sqrt{((1928545343086076450^{(1/4)}*\sqrt{1963947730})*\sqrt{341})*\sqrt{31)* \\
& \sqrt{2*x^2 - x + 3}*(\sqrt{2}*(2995431*x + 1523456) - 4518887*x - 1471975)*s \\
& \sqrt{13874275807943*\sqrt{2} + 19639477300000) + 160519269124568199977215*x^2 \\
& + 144139751866959199979540*\sqrt{2}*(2*x^2 - x + 3) - 494661421179791799929 \\
& 785*x + 655180690304359999907000)/x^2) - 7265387597706767560489925565636220 \\
& 6183625*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 \\
& + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348*x^8 - 2 \\
& 692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - \\
& 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 \\
& + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 1 \\
& 8579456)) + 69*1928545343086076450^{(1/4)}*\sqrt{1963947730}*\sqrt{341}*\sqrt{31} \\
& )*(981973865000000*x^6 + 687381705500000*x^5 + 2022866161900000*x^4 + 16693 \\
& 55570500000*x^3 + 1630076615900000*x^2 - 13874275807943*\sqrt{2}*(50*x^6 + 3 \\
& 5*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12) + 628463273600000*x + 235673 \\
& 727600000)*\sqrt{13874275807943*\sqrt{2} + 19639477300000)*\log(17675529570000 \\
& 00000/3471424919*(1928545343086076450^{(1/4)}*\sqrt{1963947730}*\sqrt{341})*\sqrt{ \\
& 31)*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(2995431*x + 1523456) - 4518887*x - 14719 \\
& 75)*\sqrt{13874275807943*\sqrt{2} + 19639477300000) + 16051926912456819997721 \\
& 5*x^2 + 144139751866959199979540*\sqrt{2}*(2*x^2 - x + 3) - 4946614211797917 \\
& 99929785*x + 655180690304359999907000)/x^2) - 69*1928545343086076450^{(1/4)}* \\
& \sqrt{1963947730}*\sqrt{341}*\sqrt{31}*(981973865000000*x^6 + 687381705500000* \\
& x^5 + 2022866161900000*x^4 + 1669355570500000*x^3 + 1630076615900000*x^2 - \\
& 13874275807943*\sqrt{2}*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x \\
& + 12) + 628463273600000*x + 235673727600000)*\sqrt{13874275807943*\sqrt{2} +
\end{aligned}$$



19639477300000)\*log(-1767552957000000000/3471424919\*(1928545343086076450^(1/4)\*sqrt(1963947730)\*sqrt(341)\*sqrt(31)\*sqrt(2\*x^2 - x + 3)\*(sqrt(2)\*(2995431\*x + 1523456) - 4518887\*x - 1471975)\*sqrt(13874275807943\*sqrt(2) + 19639477300000) - 160519269124568199977215\*x^2 - 144139751866959199979540\*sqrt(2)\*(2\*x^2 - x + 3) + 494661421179791799929785\*x - 655180690304359999907000)/x^2) + 35746658463005881594925920\*(162716650\*x^5 + 86411405\*x^4 + 277167774\*x^3 + 175833195\*x^2 + 161806828\*x + 22374044)\*sqrt(2\*x^2 - x + 3))/(50\*x^6 + 35\*x^5 + 103\*x^4 + 85\*x^3 + 83\*x^2 + 32\*x + 12)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity, infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity ]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity ,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity ]Evaluation time: 69.9Done

**maple** [B] time = 0.21, size = 18981, normalized size = 77.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^3,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)^3 (2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(3/2)/(5\*x^2+3\*x+2)^3,x, algorithm="maxima")

[Out] integrate(1/((5\*x^2 + 3\*x + 2)^3\*(2\*x^2 - x + 3)^(3/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)^3),x)

[Out] int(1/((2\*x^2 - x + 3)^(3/2)\*(3\*x + 5\*x^2 + 2)^3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**3), x)
```

$$3.90 \quad \int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=147

$$\frac{38375}{384} \sqrt{2x^2 - x + 3} x^2 + \frac{526075 \sqrt{2x^2 - x + 3} x}{3072} - \frac{1308645 \sqrt{2x^2 - x + 3}}{4096} + \frac{1331(116368x + 7409)}{101568 \sqrt{2x^2 - x + 3}} - \frac{14641(79x + 101)}{4416 (2x^2 - x + 3)^{3/2}}$$

**Rubi [A]** time = 0.17, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{625}{32} \sqrt{2x^2 - x + 3} x^3 + \frac{38375}{384} \sqrt{2x^2 - x + 3} x^2 + \frac{526075 \sqrt{2x^2 - x + 3} x}{3072} - \frac{1308645 \sqrt{2x^2 - x + 3}}{4096} + \frac{1331(116368x + 7409)}{101568 \sqrt{2x^2 - x + 3}} - \frac{14641(79x + 101)}{4416 (2x^2 - x + 3)^{3/2}} + \frac{16955197 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^(5/2), x]

[Out] (-14641\*(101 + 79\*x))/(4416\*(3 - x + 2\*x^2)^(3/2)) + (1331\*(7409 + 116368\*x))/(101568\*sqrt[3 - x + 2\*x^2]) - (1308645\*sqrt[3 - x + 2\*x^2])/4096 + (526075\*x\*sqrt[3 - x + 2\*x^2])/3072 + (38375\*x^2\*sqrt[3 - x + 2\*x^2])/384 + (625\*x^3\*sqrt[3 - x + 2\*x^2])/32 + (16955197\*ArcSinh[(1 - 4\*x)/sqrt[23]])/(8192\*sqrt[2])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a +

```
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(2 + 3x + 5x^2)^4}{(3 - x + 2x^2)^{5/2}} dx = -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{2}{69} \int \frac{\frac{3839123}{256} - \frac{1983543x}{128} - \frac{1464801x^2}{64} + \frac{430905x^3}{32} + \frac{639975x^4}{16} + \frac{250}{16}}{(3 - x + 2x^2)^{3/2}} dx$$

$$= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} + \frac{4}{1587} \int \frac{-\frac{141812733}{256} - \frac{1880595x}{16} + \frac{15512925x^2}{64} + \frac{3372375x^3}{16}}{\sqrt{3 - x + 2x^2}} dx$$

$$= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} + \frac{625}{32} x^3 \sqrt{3 - x + 2x^2} + \frac{1}{32} \int \frac{141812733 - 1880595x + 15512925x^2 + 3372375x^3}{\sqrt{3 - x + 2x^2}} dx$$

$$= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} + \frac{38375}{384} x^2 \sqrt{3 - x + 2x^2} + \frac{625}{32} x^3 \sqrt{3 - x + 2x^2} + \frac{1}{384} \int \frac{141812733 - 1880595x + 15512925x^2 + 3372375x^3}{\sqrt{3 - x + 2x^2}} dx$$

$$= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} + \frac{526075x\sqrt{3 - x + 2x^2}}{3072} + \frac{38375}{384} x^2 \sqrt{3 - x + 2x^2} + \frac{1}{384} \int \frac{141812733 - 1880595x + 15512925x^2 + 3372375x^3}{\sqrt{3 - x + 2x^2}} dx$$

$$= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} - \frac{1308645\sqrt{3 - x + 2x^2}}{4096} + \frac{526075x\sqrt{3 - x + 2x^2}}{3072} + \frac{1}{384} \int \frac{141812733 - 1880595x + 15512925x^2 + 3372375x^3}{\sqrt{3 - x + 2x^2}} dx$$

$$= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} - \frac{1308645\sqrt{3 - x + 2x^2}}{4096} + \frac{526075x\sqrt{3 - x + 2x^2}}{3072} + \frac{1}{384} \int \frac{141812733 - 1880595x + 15512925x^2 + 3372375x^3}{\sqrt{3 - x + 2x^2}} dx$$

$$= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} - \frac{1308645\sqrt{3 - x + 2x^2}}{4096} + \frac{526075x\sqrt{3 - x + 2x^2}}{3072} + \frac{1}{384} \int \frac{141812733 - 1880595x + 15512925x^2 + 3372375x^3}{\sqrt{3 - x + 2x^2}} dx$$

**Mathematica [A]** time = 0.51, size = 75, normalized size = 0.51

$$\frac{507840000x^7 + 2090608000x^6 + 3504730800x^5 - 5076781260x^4 + 39848900984x^3 - 36481630395x^2 + 49883864262x - 18974698519}{6500352(2x^2 - x + 3)^{3/2}} - \frac{16955197 \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{8192\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2), x]
```

```
[Out] (-18974698519 + 49883864262*x - 36481630395*x^2 + 39848900984*x^3 - 5076781
260*x^4 + 3504730800*x^5 + 2090608000*x^6 + 507840000*x^7)/(6500352*(3 - x
+ 2*x^2)^(3/2)) - (16955197*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(8192*Sqrt[2])
```

**IntegrateAlgebraic [A]** time = 1.45, size = 90, normalized size = 0.61

$$\frac{16955197 \log\left(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1\right)}{8192\sqrt{2}} + \frac{507840000x^7 + 2090608000x^6 + 3504730800x^5 - 5076781260x^4 + 39848900984x^3 - 36481630395x^2 + 49883864262x - 18974698519}{6500352(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^4/(3 - x + 2\*x^2)^(5/2),x]

[Out] (-18974698519 + 49883864262\*x - 36481630395\*x^2 + 39848900984\*x^3 - 5076781260\*x^4 + 3504730800\*x^5 + 2090608000\*x^6 + 507840000\*x^7)/(6500352\*(3 - x + 2\*x^2)^(3/2)) + (16955197\*Log[1 - 4\*x + 2\*sqrt[2]\*sqrt[3 - x + 2\*x^2]])/(8192\*sqrt[2])

**fricas** [A] time = 0.42, size = 132, normalized size = 0.90

$$\frac{26907897639\sqrt{2}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\log(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 8(507840000x^7 + 2090608000x^6 + 3504730800x^5 - 5076781260x^4 + 39848900984x^3 - 36481630395x^2 - 18974698519)\sqrt{2x^2 - x + 3}}{52002816(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(5/2),x, algorithm="fricas")

[Out] 1/52002816\*(26907897639\*sqrt(2)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 8\*(507840000\*x^7 + 2090608000\*x^6 + 3504730800\*x^5 - 5076781260\*x^4 + 39848900984\*x^3 - 36481630395\*x^2 + 49883864262\*x - 18974698519)\*sqrt(2\*x^2 - x + 3))/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac** [A] time = 0.25, size = 81, normalized size = 0.55

$$\frac{16955197}{16384}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2x^2-x+3}\right)+1\right)+\frac{((4(2645(20(40(60x+247)x+16563)x-479847)x+9962225246)x-36481630395)x+49883864262)x-18974698519)}{6500352(2x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(5/2),x, algorithm="giac")

[Out] 16955197/16384\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) + 1/6500352\*((4\*(2645\*(20\*(40\*(60\*x + 247)\*x + 16563)\*x - 479847)\*x + 9962225246)\*x - 36481630395)\*x + 49883864262)\*x - 18974698519)/(2\*x^2 - x + 3)^(3/2)

**maple** [A] time = 0.03, size = 214, normalized size = 1.46

$$\frac{625x^7}{8(2x^2-x+3)^{\frac{3}{2}}} + \frac{30875x^6}{96(2x^2-x+3)^{\frac{3}{2}}} + \frac{138025x^5}{256(2x^2-x+3)^{\frac{3}{2}}} - \frac{799745x^4}{1024(2x^2-x+3)^{\frac{3}{2}}} + \frac{16955197x^3}{12288(2x^2-x+3)^{\frac{3}{2}}} - \frac{67488035x^2}{16384(2x^2-x+3)^{\frac{3}{2}}} + \frac{55167267x}{131072(2x^2-x+3)^{\frac{3}{2}}} + \frac{16955197x}{8192\sqrt{2x^2-x+3}} - \frac{16955197\sqrt{2}\arcsin\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{16384} + \frac{16955197}{32768\sqrt{2x^2-x+3}} - \frac{2149616639}{524288(2x^2-x+3)^{\frac{3}{2}}} + \frac{99296033x}{350176\sqrt{2x^2-x+3}} - \frac{99296033}{13000704} + \frac{5141612725x}{94368(2x^2-x+3)^{\frac{3}{2}}} - \frac{5141612725}{36175872(2x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(5/2),x)

[Out] 16955197/32768/(2\*x^2-x+3)^(1/2)-2149616639/524288/(2\*x^2-x+3)^(3/2)+992926033/13000704\*(4\*x-1)/(2\*x^2-x+3)^(1/2)-16955197/16384\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+5141612725/36175872\*(4\*x-1)/(2\*x^2-x+3)^(3/2)+625/8\*x^7/(2\*x^2-x+3)^(3/2)+30875/96\*x^6/(2\*x^2-x+3)^(3/2)+138025/256\*x^5/(2\*x^2-x+3)^(3/2)-799745/1024\*x^4/(2\*x^2-x+3)^(3/2)+16955197/12288\*x^3/(2\*x^2-x+3)^(3/2)-67488035/16384\*x^2/(2\*x^2-x+3)^(3/2)+55167267/131072\*x/(2\*x^2-x+3)^(3/2)+16955197/8192/(2\*x^2-x+3)^(1/2)\*x

**maxima** [B] time = 1.00, size = 253, normalized size = 1.72

$$\frac{625x^7}{8(2x^2-x+3)^{\frac{3}{2}}} + \frac{30875x^6}{96(2x^2-x+3)^{\frac{3}{2}}} + \frac{138025x^5}{256(2x^2-x+3)^{\frac{3}{2}}} - \frac{799745x^4}{1024(2x^2-x+3)^{\frac{3}{2}}} + \frac{16955197x^3}{13000704}\left(\frac{284x}{\sqrt{2x^2-x+3}} - \frac{3174x^2}{(2x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2x^2-x+3}} - \frac{805x}{(2x^2-x+3)^{\frac{3}{2}}} - \frac{3243}{(2x^2-x+3)^{\frac{3}{2}}}\right) + \frac{16955197\sqrt{2}\operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)}{16384} + \frac{1203818897\sqrt{2x^2-x+3}}{6500352} + \frac{3536205983x}{3250176\sqrt{2x^2-x+3}} - \frac{2638851x^2}{512(2x^2-x+3)^{\frac{3}{2}}} + \frac{257773037}{1083392\sqrt{2x^2-x+3}} + \frac{29484937x}{23852(2x^2-x+3)^{\frac{3}{2}}} - \frac{374445479}{70656(2x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^4/(2\*x^2-x+3)^(5/2),x, algorithm="maxima")

[Out] 625/8\*x^7/(2\*x^2 - x + 3)^(3/2) + 30875/96\*x^6/(2\*x^2 - x + 3)^(3/2) + 138025/256\*x^5/(2\*x^2 - x + 3)^(3/2) - 799745/1024\*x^4/(2\*x^2 - x + 3)^(3/2) - 16955197/13000704\*x\*(284\*x/sqrt(2\*x^2 - x + 3) - 3174\*x^2/(2\*x^2 - x + 3)^(3/2) - 71/sqrt(2\*x^2 - x + 3) + 805\*x/(2\*x^2 - x + 3)^(3/2) - 3243/(2\*x^2 - x + 3)^(3/2)) - 16955197/16384\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) +

$1203818987/6500352*\sqrt{2*x^2 - x + 3} + 3536205583/3250176*x/\sqrt{2*x^2 - x + 3} - 2638851/512*x^2/(2*x^2 - x + 3)^{(3/2)} + 257773037/1083392/\sqrt{2*x^2 - x + 3} + 29484067/23552*x/(2*x^2 - x + 3)^{(3/2)} - 374445479/70656/(2*x^2 - x + 3)^{(3/2)}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^4}{(2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^4/(2\*x^2 - x + 3)^(5/2), x)

[Out] int((3\*x + 5\*x^2 + 2)^4/(2\*x^2 - x + 3)^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^4}{(2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*4/(2\*x\*\*2-x+3)\*\*(5/2), x)

[Out] Integral((5\*x\*\*2 + 3\*x + 2)\*\*4/(2\*x\*\*2 - x + 3)\*\*(5/2), x)

$$3.91 \quad \int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=105

$$\frac{121(10679 - 6744x)}{8464\sqrt{2x^2 - x + 3}} + \frac{125}{16}x\sqrt{2x^2 - x + 3} + \frac{3175}{64}\sqrt{2x^2 - x + 3} - \frac{1331(17 - 45x)}{1104(2x^2 - x + 3)^{3/2}} - \frac{7495 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

**Rubi [A]** time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, number of rules / integrand size = 0.185, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{121(10679 - 6744x)}{8464\sqrt{2x^2 - x + 3}} + \frac{125}{16}x\sqrt{2x^2 - x + 3} + \frac{3175}{64}\sqrt{2x^2 - x + 3} - \frac{1331(17 - 45x)}{1104(2x^2 - x + 3)^{3/2}} - \frac{7495 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^(5/2), x]

[Out] (-1331\*(17 - 45\*x))/(1104\*(3 - x + 2\*x^2)^(3/2)) + (121\*(10679 - 6744\*x))/(8464\*Sqrt[3 - x + 2\*x^2]) + (3175\*Sqrt[3 - x + 2\*x^2])/64 + (125\*x\*Sqrt[3 - x + 2\*x^2])/16 - (7495\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(128\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a +

$b*x + c*x^2)^p \text{ExpandToSum}[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^{(q - 2)} - b*e*(q + p)*x^{(q - 1)} - c*e*(q + 2*p + 1)*x^q, x], x] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{LeQ}[p, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{(2 + 3x + 5x^2)^3}{(3 - x + 2x^2)^{5/2}} dx &= -\frac{1331(17 - 45x)}{1104(3 - x + 2x^2)^{3/2}} + \frac{2}{69} \int \frac{-\frac{91275}{64} - \frac{57201x}{32} + \frac{66585x^2}{16} + \frac{39675x^3}{8} + \frac{8625x^4}{4}}{(3 - x + 2x^2)^{3/2}} dx \\ &= -\frac{1331(17 - 45x)}{1104(3 - x + 2x^2)^{3/2}} + \frac{121(10679 - 6744x)}{8464\sqrt{3 - x + 2x^2}} + \frac{4 \int \frac{\frac{1452105}{64} + \frac{277725x}{8} + \frac{198375x^2}{16}}{\sqrt{3 - x + 2x^2}} dx}{1587} \\ &= -\frac{1331(17 - 45x)}{1104(3 - x + 2x^2)^{3/2}} + \frac{121(10679 - 6744x)}{8464\sqrt{3 - x + 2x^2}} + \frac{125}{16}x\sqrt{3 - x + 2x^2} + \frac{\int \frac{\frac{214245}{4} + \frac{5038725x}{32}}{\sqrt{3 - x + 2x^2}} dx}{1587} \\ &= -\frac{1331(17 - 45x)}{1104(3 - x + 2x^2)^{3/2}} + \frac{121(10679 - 6744x)}{8464\sqrt{3 - x + 2x^2}} + \frac{3175}{64}\sqrt{3 - x + 2x^2} + \frac{125}{16}x\sqrt{3 - x + 2x^2} \\ &= -\frac{1331(17 - 45x)}{1104(3 - x + 2x^2)^{3/2}} + \frac{121(10679 - 6744x)}{8464\sqrt{3 - x + 2x^2}} + \frac{3175}{64}\sqrt{3 - x + 2x^2} + \frac{125}{16}x\sqrt{3 - x + 2x^2} \\ &= -\frac{1331(17 - 45x)}{1104(3 - x + 2x^2)^{3/2}} + \frac{121(10679 - 6744x)}{8464\sqrt{3 - x + 2x^2}} + \frac{3175}{64}\sqrt{3 - x + 2x^2} + \frac{125}{16}x\sqrt{3 - x + 2x^2} \\ &= -\frac{1331(17 - 45x)}{1104(3 - x + 2x^2)^{3/2}} + \frac{121(10679 - 6744x)}{8464\sqrt{3 - x + 2x^2}} + \frac{3175}{64}\sqrt{3 - x + 2x^2} + \frac{125}{16}x\sqrt{3 - x + 2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.34, size = 65, normalized size = 0.62

$$\frac{3174000x^5 + 16980900x^4 - 29423976x^3 + 101546529x^2 - 62463282x + 89784565}{101568(2x^2 - x + 3)^{3/2}} + \frac{7495 \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^(5/2), x]

[Out] (89784565 - 62463282\*x + 101546529\*x^2 - 29423976\*x^3 + 16980900\*x^4 + 3174000\*x^5)/(101568\*(3 - x + 2\*x^2)^(3/2)) + (7495\*ArcSinh[(-1 + 4\*x)/Sqrt[23]])/(128\*Sqrt[2])

**IntegrateAlgebraic [A]** time = 1.03, size = 80, normalized size = 0.76

$$\frac{3174000x^5 + 16980900x^4 - 29423976x^3 + 101546529x^2 - 62463282x + 89784565}{101568(2x^2 - x + 3)^{3/2}} - \frac{7495 \log\left(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^3/(3 - x + 2\*x^2)^(5/2), x]

[Out] (89784565 - 62463282\*x + 101546529\*x^2 - 29423976\*x^3 + 16980900\*x^4 + 3174000\*x^5)/(101568\*(3 - x + 2\*x^2)^(3/2)) - (7495\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(128\*Sqrt[2])



**fricas [A]** time = 0.43, size = 122, normalized size = 1.16

$$\frac{11894565\sqrt{2}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 8(3174000x^5 + 16980900x^4 - 29423976x^3 - 62463282x + 89784565)\sqrt{2x^2 - x + 3}}{812544(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(5/2),x, algorithm="fricas")

[Out] 1/812544\*(11894565\*sqrt(2)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 8\*(3174000\*x^5 + 16980900\*x^4 - 29423976\*x^3 + 101546529\*x^2 - 62463282\*x + 89784565)\*sqrt(2\*x^2 - x + 3))/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac [A]** time = 0.27, size = 72, normalized size = 0.69

$$-\frac{7495}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{3((4(13225(20x + 107)x - 2451998)x + 33848843)x - 20821094)x + 89784565}{101568(2x^2 - x + 3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(5/2),x, algorithm="giac")

[Out] -7495/256\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) + 1/101568\*(3\*((4\*(13225\*(20\*x + 107)\*x - 2451998)\*x + 33848843)\*x - 20821094)\*x + 89784565)/(2\*x^2 - x + 3)^(3/2)

**maple [B]** time = 0.01, size = 180, normalized size = 1.71

$$\frac{125x^5}{4(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{2675x^4}{16(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{7495x^3}{192(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{222809x^2}{256(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{281177x}{2048(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{7495x}{128\sqrt{2x^2 - x + 3}} + \frac{7495\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x - \frac{1}{4}\right)}{23}\right)}{256} - \frac{7495}{512\sqrt{2x^2 - x + 3}} + \frac{20961031}{24576(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{3391139(4x - 1)}{203136\sqrt{2x^2 - x + 3}} - \frac{14081711(4x - 1)}{565248(2x^2 - x + 3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(5/2),x)

[Out] -7495/512/(2\*x^2-x+3)^(1/2)+20961031/24576/(2\*x^2-x+3)^(3/2)-3391139/203136\*(4\*x-1)/(2\*x^2-x+3)^(1/2)+7495/256\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))-14081711/565248\*(4\*x-1)/(2\*x^2-x+3)^(3/2)+125/4/(2\*x^2-x+3)^(3/2)\*x^5+2675/16/(2\*x^2-x+3)^(3/2)\*x^4-7495/192/(2\*x^2-x+3)^(3/2)\*x^3+222809/256/(2\*x^2-x+3)^(3/2)\*x^2-281177/2048/(2\*x^2-x+3)^(3/2)\*x-7495/128/(2\*x^2-x+3)^(1/2)\*x

**maxima [B]** time = 1.02, size = 219, normalized size = 2.09

$$\frac{125x^5}{4(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{2675x^4}{16(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{7495}{203136}\left(\frac{284x}{\sqrt{2x^2 - x + 3}} - \frac{3174x^2}{(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2x^2 - x + 3}} + \frac{805x}{(2x^2 - x + 3)^{\frac{3}{2}}} - \frac{3243}{(2x^2 - x + 3)^{\frac{3}{2}}}\right) + \frac{7495\sqrt{2}\operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}(4x - 1)\right)}{256} - \frac{532145\sqrt{2x^2 - x + 3}}{101568} - \frac{4515389x}{50784\sqrt{2x^2 - x + 3}} + \frac{7197x^2}{8(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{396211}{50784\sqrt{2x^2 - x + 3}} - \frac{269783x}{1104(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{1002137}{1104(2x^2 - x + 3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^3/(2\*x^2-x+3)^(5/2),x, algorithm="maxima")

[Out] 125/4\*x^5/(2\*x^2 - x + 3)^(3/2) + 2675/16\*x^4/(2\*x^2 - x + 3)^(3/2) + 7495/203136\*x\*(284\*x/sqrt(2\*x^2 - x + 3) - 3174\*x^2/(2\*x^2 - x + 3)^(3/2) - 71/sqrt(2\*x^2 - x + 3) + 805\*x/(2\*x^2 - x + 3)^(3/2) - 3243/(2\*x^2 - x + 3)^(3/2)) + 7495/256\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 532145/101568\*sqrt(2\*x^2 - x + 3) - 4515389/50784\*x/sqrt(2\*x^2 - x + 3) + 7197/8\*x^2/(2\*x^2 - x + 3)^(3/2) + 396211/50784/sqrt(2\*x^2 - x + 3) - 269783/1104\*x/(2\*x^2 - x + 3)^(3/2) + 1002137/1104/(2\*x^2 - x + 3)^(3/2)

**mapad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^3}{(2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(5/2), x)`

[Out] `int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^3}{(2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(5/2), x)`

[Out] `Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(5/2), x)`

$$3.92 \quad \int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=68

$$\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}} - \frac{11(2336x+7351)}{6348\sqrt{2x^2-x+3}} - \frac{25 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

**Rubi [A]** time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1660, 12, 619, 215}

$$\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}} - \frac{11(2336x+7351)}{6348\sqrt{2x^2-x+3}} - \frac{25 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^(5/2), x]

[Out] (121\*(19 - 7\*x))/(276\*(3 - x + 2\*x^2)^(3/2)) - (11\*(7351 + 2336\*x))/(6348\*Sqrt[3 - x + 2\*x^2]) - (25\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(4\*Sqrt[2])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}} dx &= \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} + \frac{2}{69} \int \frac{\frac{131}{16} + \frac{5865x}{8} + \frac{1725x^2}{4}}{(3-x+2x^2)^{3/2}} dx \\
&= \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} + \frac{4 \int \frac{39675}{16\sqrt{3-x+2x^2}} dx}{1587} \\
&= \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} + \frac{25}{4} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
&= \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} + \frac{25 \operatorname{Subst} \left( \int \frac{1}{\sqrt{1+\frac{x^2}{23}}} dx, x, -1+4x \right)}{4\sqrt{46}} \\
&= \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} - \frac{25 \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{4\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 55, normalized size = 0.81

$$\frac{25 \sinh^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{4\sqrt{2}} - \frac{11(2336x^3 + 6183x^2 + 714x + 8623)}{3174(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^(5/2), x]

[Out] (-11\*(8623 + 714\*x + 6183\*x^2 + 2336\*x^3))/(3174\*(3 - x + 2\*x^2)^(3/2)) + (25\*ArcSinh[(-1 + 4\*x)/Sqrt[23]])/(4\*Sqrt[2])

**IntegrateAlgebraic [A]** time = 0.74, size = 70, normalized size = 1.03

$$\frac{25 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{4\sqrt{2}} - \frac{11(2336x^3 + 6183x^2 + 714x + 8623)}{3174(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)^2/(3 - x + 2\*x^2)^(5/2), x]

[Out] (-11\*(8623 + 714\*x + 6183\*x^2 + 2336\*x^3))/(3174\*(3 - x + 2\*x^2)^(3/2)) - (25\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(4\*Sqrt[2])

**fricas [B]** time = 0.41, size = 112, normalized size = 1.65

$$\frac{39675\sqrt{2}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\log(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25) - 88(2336x^3 + 6183x^2 + 714x + 8623)\sqrt{2x^2-x+3}}{25392(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(5/2), x, algorithm="fricas")

[Out] 1/25392\*(39675\*sqrt(2)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) - 88\*(2336\*x^3 + 6183\*x^2 + 714\*x + 8623)\*sqrt(2\*x^2 - x + 3))/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac** [A] time = 0.27, size = 61, normalized size = 0.90

$$-\frac{25}{8}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{11\left(\left(\left(2336x+6183\right)x+714\right)x+8623\right)}{3174\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(5/2),x, algorithm="giac")

[Out] -25/8\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) - 11/3174\*(((2336\*x + 6183)\*x + 714)\*x + 8623)/(2\*x^2 - x + 3)^(3/2)

**maple** [B] time = 0.01, size = 146, normalized size = 2.15

$$\frac{25x^3}{6(2x^2-x+3)^{\frac{3}{2}}}-\frac{145x^2}{8(2x^2-x+3)^{\frac{3}{2}}}-\frac{319x}{64(2x^2-x+3)^{\frac{3}{2}}}-\frac{25x}{4\sqrt{2x^2-x+3}}+\frac{25\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8}-\frac{15775}{768(2x^2-x+3)^{\frac{3}{2}}}+\frac{8493x}{1472}-\frac{8493}{5888}+\frac{2267x}{529}-\frac{2267}{2116}-\frac{25}{16\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(5/2),x)

[Out] -25/6/(2\*x^2-x+3)^(3/2)\*x^3-145/8/(2\*x^2-x+3)^(3/2)\*x^2-319/64/(2\*x^2-x+3)^(3/2)\*x-15775/768/(2\*x^2-x+3)^(3/2)+8493/5888\*(4\*x-1)/(2\*x^2-x+3)^(3/2)+2267/2116\*(4\*x-1)/(2\*x^2-x+3)^(1/2)-25/4/(2\*x^2-x+3)^(1/2)\*x-25/16/(2\*x^2-x+3)^(1/2)+25/8\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))

**maxima** [B] time = 0.98, size = 185, normalized size = 2.72

$$\frac{25}{6348}\left(\frac{284x}{\sqrt{2x^2-x+3}}-\frac{3174x^2}{(2x^2-x+3)^{\frac{3}{2}}}-\frac{71}{\sqrt{2x^2-x+3}}+\frac{805x}{(2x^2-x+3)^{\frac{3}{2}}}-\frac{3243}{(2x^2-x+3)^{\frac{3}{2}}}\right)+\frac{25}{8}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{1775}{3174}\sqrt{2x^2-x+3}+\frac{1017x}{529\sqrt{2x^2-x+3}}-\frac{15x^2}{(2x^2-x+3)^{\frac{3}{2}}}-\frac{6413}{3174\sqrt{2x^2-x+3}}-\frac{x}{138(2x^2-x+3)^{\frac{3}{2}}}-\frac{2593}{138(2x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)^2/(2\*x^2-x+3)^(5/2),x, algorithm="maxima")

[Out] 25/6348\*x\*(284\*x/sqrt(2\*x^2 - x + 3) - 3174\*x^2/(2\*x^2 - x + 3)^(3/2) - 71/sqrt(2\*x^2 - x + 3) + 805\*x/(2\*x^2 - x + 3)^(3/2) - 3243/(2\*x^2 - x + 3)^(3/2)) + 25/8\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 1775/3174\*sqrt(2\*x^2 - x + 3) + 1017/529\*x/sqrt(2\*x^2 - x + 3) - 15\*x^2/(2\*x^2 - x + 3)^(3/2) - 6413/3174/sqrt(2\*x^2 - x + 3) - 1/138\*x/(2\*x^2 - x + 3)^(3/2) - 2593/138/(2\*x^2 - x + 3)^(3/2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2 + 3x + 2)^2}{(2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)^2/(2\*x^2 - x + 3)^(5/2),x)

[Out] int((3\*x + 5\*x^2 + 2)^2/(2\*x^2 - x + 3)^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^2}{(2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)\*\*2/(2\*x\*\*2-x+3)\*\*(5/2),x)

[Out] Integral((5\*x\*\*2 + 3\*x + 2)\*\*2/(2\*x\*\*2 - x + 3)\*\*(5/2), x)

$$3.93 \quad \int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=47

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}} - \frac{11(3x+5)}{69(2x^2-x+3)^{3/2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {1660, 12, 613}

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}} - \frac{11(3x+5)}{69(2x^2-x+3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^(5/2), x]

[Out] (-11\*(5 + 3\*x))/(69\*(3 - x + 2\*x^2)^(3/2)) - (71\*(1 - 4\*x))/(529\*Sqrt[3 - x + 2\*x^2])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 613

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-3/2), x\_Symbol] := Simp[(-2\*(b + 2\*c\*x))/((b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}} dx &= -\frac{11(5+3x)}{69(3-x+2x^2)^{3/2}} + \frac{2}{69} \int \frac{213}{4(3-x+2x^2)^{3/2}} dx \\ &= -\frac{11(5+3x)}{69(3-x+2x^2)^{3/2}} + \frac{71}{46} \int \frac{1}{(3-x+2x^2)^{3/2}} dx \\ &= -\frac{11(5+3x)}{69(3-x+2x^2)^{3/2}} - \frac{71(1-4x)}{529\sqrt{3-x+2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 33, normalized size = 0.70

$$\frac{2(852x^3 - 639x^2 + 1005x - 952)}{1587(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^(5/2), x]

[Out] (2\*(-952 + 1005\*x - 639\*x^2 + 852\*x^3))/(1587\*(3 - x + 2\*x^2)^(3/2))

**IntegrateAlgebraic [A]** time = 0.52, size = 33, normalized size = 0.70

$$\frac{2(852x^3 - 639x^2 + 1005x - 952)}{1587(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3\*x + 5\*x^2)/(3 - x + 2\*x^2)^(5/2), x]

[Out] (2\*(-952 + 1005\*x - 639\*x^2 + 852\*x^3))/(1587\*(3 - x + 2\*x^2)^(3/2))

**fricas [A]** time = 0.40, size = 51, normalized size = 1.09

$$\frac{2(852x^3 - 639x^2 + 1005x - 952)\sqrt{2x^2 - x + 3}}{1587(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(5/2), x, algorithm="fricas")

[Out] 2/1587\*(852\*x^3 - 639\*x^2 + 1005\*x - 952)\*sqrt(2\*x^2 - x + 3)/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac [A]** time = 0.23, size = 29, normalized size = 0.62

$$\frac{2(3(71(4x - 3)x + 335)x - 952)}{1587(2x^2 - x + 3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(5/2), x, algorithm="giac")

[Out] 2/1587\*(3\*(71\*(4\*x - 3)\*x + 335)\*x - 952)/(2\*x^2 - x + 3)^(3/2)

**maple [A]** time = 0.00, size = 30, normalized size = 0.64

$$\frac{\frac{568}{529}x^3 - \frac{426}{529}x^2 + \frac{670}{529}x - \frac{1904}{1587}}{(2x^2 - x + 3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(5/2), x)

[Out] 2/1587/(2\*x^2-x+3)^(3/2)\*(852\*x^3-639\*x^2+1005\*x-952)

**maxima [A]** time = 0.43, size = 59, normalized size = 1.26

$$\frac{284x}{529\sqrt{2x^2 - x + 3}} - \frac{71}{529\sqrt{2x^2 - x + 3}} - \frac{11x}{23(2x^2 - x + 3)^{3/2}} - \frac{55}{69(2x^2 - x + 3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*x+2)/(2\*x^2-x+3)^(5/2),x, algorithm="maxima")

[Out] 284/529\*x/sqrt(2\*x^2 - x + 3) - 71/529/sqrt(2\*x^2 - x + 3) - 11/23\*x/(2\*x^2 - x + 3)^(3/2) - 55/69/(2\*x^2 - x + 3)^(3/2)

mupad [B] time = 0.09, size = 29, normalized size = 0.62

$$\frac{2(852x^3 - 639x^2 + 1005x - 952)}{1587(2x^2 - x + 3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 5\*x^2 + 2)/(2\*x^2 - x + 3)^(5/2),x)

[Out] (2\*(1005\*x - 639\*x^2 + 852\*x^3 - 952))/(1587\*(2\*x^2 - x + 3)^(3/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2 + 3x + 2}{(2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*x+2)/(2\*x\*\*2-x+3)\*\*(5/2),x)

[Out] Integral((5\*x\*\*2 + 3\*x + 2)/(2\*x\*\*2 - x + 3)\*\*(5/2), x)



$$3.94 \quad \int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)} dx$$

Optimal. Leaf size=199

$$\frac{3603 - 658x}{128018\sqrt{2x^2 - x + 3}} + \frac{13 - 6x}{759(2x^2 - x + 3)^{3/2}} + \frac{1}{484} \sqrt{\frac{1}{682} (25000\sqrt{2} - 15457)} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(25000\sqrt{2} - 15457)}} \left( (247 + 345\sqrt{2})x - 98\sqrt{2} + 443 \right)}{\sqrt{2x^2 - x + 3}} \right) - \frac{1}{484} \sqrt{\frac{1}{682} (15457 + 25000\sqrt{2})} \tanh^{-1} \left( \frac{\sqrt{\frac{11}{31(15457 + 25000\sqrt{2})}} \left( (247 - 345\sqrt{2})x + 98\sqrt{2} + 443 \right)}{\sqrt{2x^2 - x + 3}} \right)$$

Rubi [A] time = 0.46, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{3603 - 658x}{128018\sqrt{2x^2 - x + 3}} + \frac{13 - 6x}{759(2x^2 - x + 3)^{3/2}} + \frac{1}{484} \sqrt{\frac{1}{682} (25000\sqrt{2} - 15457)} \tan^{-1} \left( \frac{\sqrt{\frac{11}{31(25000\sqrt{2} - 15457)}} \left( (247 + 345\sqrt{2})x - 98\sqrt{2} + 443 \right)}{\sqrt{2x^2 - x + 3}} \right) - \frac{1}{484} \sqrt{\frac{1}{682} (15457 + 25000\sqrt{2})} \tanh^{-1} \left( \frac{\sqrt{\frac{11}{31(15457 + 25000\sqrt{2})}} \left( (247 - 345\sqrt{2})x + 98\sqrt{2} + 443 \right)}{\sqrt{2x^2 - x + 3}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)), x]

[Out] (13 - 6\*x)/(759\*(3 - x + 2\*x^2)^(3/2)) + (3603 - 658\*x)/(128018\*sqrt[3 - x + 2\*x^2]) + (sqrt[(-15457 + 25000\*sqrt[2])/682]\*ArcTan[(sqrt[11/(31\*(-15457 + 25000\*sqrt[2]))])\*(443 - 98\*sqrt[2] + (247 + 345\*sqrt[2])\*x)]/sqrt[3 - x + 2\*x^2])/484 - (sqrt[(15457 + 25000\*sqrt[2])/682]\*ArcTanh[(sqrt[11/(31\*(15457 + 25000\*sqrt[2]))])\*(443 + 98\*sqrt[2] + (247 - 345\*sqrt[2])\*x)]/sqrt[3 - x + 2\*x^2])/484

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^(q+1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1)), Int[(a + b\*x + c\*x^2)^(p+1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p+1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p+1) - c\*d\*(p+2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p+q+2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p+q+2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p+1) - c\*e\*(2\*p+q+4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p+2\*q+5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && (!(IntegerQ[p] && ILtQ[q, -1])) && !IGtQ[q, 0]

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]
```

### Rule 1035

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 1060

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^{5/2} (2+3x+5x^2)} dx &= \frac{13-6x}{759(3-x+2x^2)^{3/2}} - \frac{\int \frac{-2772-\frac{3003x}{2}+660x^2}{(3-x+2x^2)^{3/2}(2+3x+5x^2)} dx}{8349} \\
&= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} - \frac{\int \frac{\frac{5184729}{2}-\frac{12481755x}{4}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{23235267} \\
&= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{\int \frac{-\frac{2112297}{4}(11-54\sqrt{2})-\frac{211}{4}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{51117587} \\
&= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} - \frac{1}{32} \left( 17457 \left( 50000 - 1 \right) \right) \\
&= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{1}{484} \sqrt{\frac{1}{682} \left( -15457 + \dots \right)}
\end{aligned}$$

**Mathematica [C]** time = 0.87, size = 218, normalized size = 1.10

$$\frac{\sqrt{\frac{1}{682}(13+i\sqrt{31})} (119\sqrt{31} + 247i) \tanh^{-1}\left(\frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{9680} + \frac{\sqrt{\frac{1}{682}(13-i\sqrt{31})} (119\sqrt{31} - 247i) \tanh^{-1}\left(\frac{(22-4i\sqrt{31})x+i\sqrt{31}-63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{9680} + \frac{-3948x^3 + 23592x^2 - 19767x + 39005}{384054(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)),x]

[Out] (39005 - 19767\*x + 23592\*x^2 - 3948\*x^3)/(384054\*(3 - x + 2\*x^2)^(3/2)) - (Sqrt[(13 + I\*Sqrt[31])/682]\*(247\*I + 119\*Sqrt[31])\*ArcTanh[(63 + I\*Sqrt[31] + (-22 - (4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])])/9680 + (Sqrt[(13 - I\*Sqrt[31])/682]\*(-247\*I + 119\*Sqrt[31])\*ArcTanh[(-63 + I\*Sqrt[31] + (22 - (4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])])/9680

**IntegrateAlgebraic [C]** time = 0.83, size = 209, normalized size = 1.05

$$\frac{1}{484} \text{RootSum}\left[-5\#1^4 + 6\sqrt{2}\#1^3 + 17\#1^2 - 26\sqrt{2}\#1 - 56\&, \frac{-65\#1^2 \log(-\#1 + \sqrt{2x^2 - x + 3} - \sqrt{2}x) + 108\sqrt{2}\#1 \log(-\#1 + \sqrt{2x^2 - x + 3} - \sqrt{2}x) + 249 \log(-\#1 + \sqrt{2x^2 - x + 3} - \sqrt{2}x)}{-10\#1^3 + 9\sqrt{2}\#1^2 + 17\#1 - 13\sqrt{2}}\&\right] + \frac{-3948x^3 + 23592x^2 - 19767x + 39005}{384054(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)),x]

[Out] (39005 - 19767\*x + 23592\*x^2 - 3948\*x^3)/(384054\*(3 - x + 2\*x^2)^(3/2)) + RootSum[-56 - 26\*Sqrt[2]\*#1 + 17\*#1^2 + 6\*Sqrt[2]\*#1^3 - 5\*#1^4 &, (249\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1] + 108\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1 - 65\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1^2)/(-13\*Sqrt[2] + 17\*#1 + 9\*Sqrt[2]\*#1^2 - 10\*#1^3) & ]/484

**fricas [B]** time = 1.24, size = 2133, normalized size = 10.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2),x, algorithm="fricas")

[Out]  $\frac{1}{370971467791584000} \cdot (1123856268 \cdot \sqrt{341}) \cdot 200^{1/4} \cdot \sqrt{2} \cdot (4x^4 - 4x^3 + 13x^2 - 6x + 9) \cdot \sqrt{-772850000 \cdot \sqrt{2} + 2500000000} \cdot \arctan(-1/7889389562500 \cdot (71300 \cdot \sqrt{341}) \cdot \sqrt{2x^2 - x + 3}) \cdot (11 \cdot 200^{3/4}) \cdot (347404x^7 - 907814x^6 + 2112962x^5 - 2166688x^4 + 787344x^3 + 304128x^2 - \sqrt{2} \cdot (35898x^7 - 441939x^6 + 782418x^5 - 2117233x^4 + 1272680x^3 - 1081800x^2 - 518400x + 1043712) - 2087424x + 518400) + 5 \cdot 200^{1/4} \cdot (712757x^7 - 10233303x^6 + 48529768x^5 - 94500260x^4 + 113086944x^3 - 22282848x^2 - \sqrt{2} \cdot (158647x^7 - 2935272x^6 + 19428740x^5 - 55765712x^4 + 78380640x^3 - 84096000x^2 - 37407744x + 53208576) - 106417152x + 37407744) \cdot \sqrt{-772850000 \cdot \sqrt{2} + 2500000000} + 22395686500000 \cdot \sqrt{31} \cdot \sqrt{2} \cdot (28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2} \cdot (8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - \sqrt{310/5711} \cdot (\sqrt{341}) \cdot \sqrt{2x^2 - x + 3} \cdot (11 \cdot 200^{3/4}) \cdot (1665224x^7 - 2325796x^6 + 7065036x^5 - 196416x^4 - 2176416x^3 + 8895744x^2 + \sqrt{2} \cdot (167914x^7 - 195429x^6 + 331239x^5 + 1685680x^4 - 3693960x^3 + 4195584x^2 - 4195584x) - 8895744x) + 5 \cdot 200^{1/4} \cdot (3246491x^7 - 41888524x^6 + 159670660x^5 - 190080576x^4 + 180496224x^3 + 376648704x^2 - 2 \cdot \sqrt{2} \cdot (40239x^7 - 558044x^6 + 2804660x^5 - 9524160x^4 + 34843680x^3 - 74006784x^2 + 74006784x) - 376648704x) \cdot \sqrt{-772850000 \cdot \sqrt{2} + 2500000000} + 314105000 \cdot \sqrt{31} \cdot \sqrt{2} \cdot (123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2} \cdot (15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) + 14277500 \cdot \sqrt{31} \cdot (254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488 \cdot \sqrt{2} \cdot (4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x) \cdot \sqrt{-(\sqrt{341}) \cdot 200^{1/4} \cdot \sqrt{31} \cdot \sqrt{2x^2 - x + 3}} \cdot (\sqrt{2} \cdot (281x - 444) + 163x - 725) \cdot \sqrt{-772850000 \cdot \sqrt{2} + 2500000000} - 4337504500x^2 - 3894902000 \cdot \sqrt{2} \cdot (2x^2 - x + 3) + 13366595500x - 17704100000) / x^2 + 254496437500 \cdot \sqrt{31} \cdot (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744 \cdot \sqrt{2} \cdot (1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456) + 1123856268 \cdot \sqrt{341} \cdot 200^{1/4} \cdot \sqrt{2} \cdot (4x^4 - 4x^3 + 13x^2 - 6x + 9) \cdot \sqrt{-772850000 \cdot \sqrt{2} + 2500000000} \cdot \arctan(-1/7889389562500 \cdot (71300 \cdot \sqrt{341}) \cdot \sqrt{2x^2 - x + 3}) \cdot (11 \cdot 200^{3/4}) \cdot (347404x^7 - 907814x^6 + 2112962x^5 - 2166688x^4 + 787344x^3 + 304128x^2 - \sqrt{2} \cdot (35898x^7 - 441939x^6 + 782418x^5 - 2117233x^4 + 1272680x^3 - 1081800x^2 - 518400x + 1043712) - 2087424x + 518400) + 5 \cdot 200^{1/4} \cdot (712757x^7 - 10233303x^6 + 48529768x^5 - 94500260x^4 + 113086944x^3 - 22282848x^2 - \sqrt{2} \cdot (158647x^7 - 2935272x^6 + 19428740x^5 - 55765712x^4 + 78380640x^3 - 84096000x^2 - 37407744x + 53208576) - 106417152x + 37407744) \cdot \sqrt{-772850000 \cdot \sqrt{2} + 2500000000} - 2239568650000 \cdot \sqrt{31} \cdot \sqrt{2} \cdot (28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2} \cdot (8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - \sqrt{310/5711} \cdot (\sqrt{341}) \cdot \sqrt{2x^2 - x + 3} \cdot (11 \cdot 200^{3/4}) \cdot (1665224x^7 - 2325796x^6 + 7065036x^5 - 196416x^4 - 2176416x^3 + 8895744x^2 + \sqrt{2} \cdot (167914x^7 - 195429x^6 + 331239x^5 + 1685680x^4 - 3693960x^3 + 4195584x^2 - 4195584x) - 8895744x) + 5 \cdot 200^{1/4} \cdot (3246491x^7 - 41888524x^6 + 159670660x^5 - 190080576x^4 + 180496224x^3 + 376648704x^2 - 2 \cdot \sqrt{2} \cdot (40239x^7 - 558044x^6 + 2804660x^5 - 9524160x^4 + 34843680x^3 - 74006784x^2 + 74006784x) - 376648704x) \cdot \sqrt{-772850000 \cdot \sqrt{2} + 2500000000} - 314105000 \cdot \sqrt{31} \cdot \sqrt{2} \cdot (123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2} \cdot (15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) - 14277500 \cdot \sqrt{31} \cdot (254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4$

```

4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6
- 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(
(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(281*x - 444) +
163*x - 725)*sqrt(-772850000*sqrt(2) + 2500000000) + 4337504500*x^2 + 38949
02000*sqrt(2)*(2*x^2 - x + 3) - 13366595500*x + 17704100000)/x^2) - 2544964
37500*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 +
254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 269
2*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5
184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 +
490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 185
79456) + 1587*sqrt(341)*200^(1/4)*sqrt(31)*(200000*x^4 - 200000*x^3 + 6500
00*x^2 + 15457*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) - 300000*x + 4500
00)*sqrt(-772850000*sqrt(2) + 2500000000)*log(77500000/5711*(sqrt(341)*200^
(1/4)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(281*x - 444) + 163*x - 725)*sq
rt(-772850000*sqrt(2) + 2500000000) + 4337504500*x^2 + 3894902000*sqrt(2)*(
2*x^2 - x + 3) - 13366595500*x + 17704100000)/x^2) - 1587*sqrt(341)*200^(1/
4)*sqrt(31)*(200000*x^4 - 200000*x^3 + 650000*x^2 + 15457*sqrt(2)*(4*x^4 -
4*x^3 + 13*x^2 - 6*x + 9) - 300000*x + 450000)*sqrt(-772850000*sqrt(2) + 25
00000000)*log(-77500000/5711*(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(2*x^2 - x +
3)*(sqrt(2)*(281*x - 444) + 163*x - 725)*sqrt(-772850000*sqrt(2) + 2500000
000) - 4337504500*x^2 - 3894902000*sqrt(2)*(2*x^2 - x + 3) + 13366595500*x
- 17704100000)/x^2) - 965935696000*(3948*x^3 - 23592*x^2 + 19767*x - 39005)
*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,  
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi  
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity  
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai  
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity  
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf  
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity  
]Evaluation time: 16.02Done

**maple** [B] time = 0.04, size = 751, normalized size = 3.77

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2),x)

[Out] 
$$1/10232728*(8*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+3*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(10111*2^{(1/2)}*(-8866+6820*2^{(1/2)}))^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+22379*2^{(1/2)}+32016)/(23*(x+2^{(1/2)}-1)^4/(-x+2^{(1/2)}+1)^4+82*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)*(x+2^{(1/2)}-1)/(-x+2^{(1/2)}+1)*(8+3*2^{(1/2)})+13910*(-8866+6820*2^{(1/2)})^{(1/2)}*(-775687+549362*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*2^{(1/2)}*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+10368*(x+2^{(1/2)}-1)^2/(-x+2^{(1/2)}+1)^2+23)$$

$$\frac{2379 \cdot 2^{1/2} + 32016}{(23 \cdot (x+2^{1/2}) - 1)^4 / (-x+2^{1/2}+1)^4 + 82 \cdot (x+2^{1/2}) - 1)^2 / (-x+2^{1/2}+1)^2 + 23} \cdot (x+2^{1/2}) - 1) / (-x+2^{1/2}+1) \cdot (8+3 \cdot 2^{1/2}) - 993674 \cdot \operatorname{arctanh}\left(\frac{31/2 \cdot (8 \cdot (x+2^{1/2}) - 1)^2 / (-x+2^{1/2}+1)^2 + 3 \cdot 2^{1/2} \cdot (x+2^{1/2}) - 1)^2 / (-x+2^{1/2}+1)^2 + 8 - 3 \cdot 2^{1/2}}{(-8866 + 6820 \cdot 2^{1/2})^{1/2}}\right) \cdot 2^{1/2} - 42685698 \cdot \operatorname{arctanh}\left(\frac{31/2 \cdot (8 \cdot (x+2^{1/2}) - 1)^2 / (-x+2^{1/2}+1)^2 + 3 \cdot 2^{1/2} \cdot (x+2^{1/2}) - 1)^2 / (-x+2^{1/2}+1)^2 + 8 - 3 \cdot 2^{1/2}}{(-8866 + 6820 \cdot 2^{1/2})^{1/2}}\right) / ((8 \cdot (x+2^{1/2}) - 1)^2 / (-x+2^{1/2}+1)^2 + 3 \cdot 2^{1/2} \cdot (x+2^{1/2}) - 1)^2 / (-x+2^{1/2}+1)^2 + 8 - 3 \cdot 2^{1/2}) / (1 + (x+2^{1/2}) - 1) / (-x+2^{1/2}+1))^{1/2} / (1 + (x+2^{1/2}) - 1) / (-x+2^{1/2}+1)) / (8+3 \cdot 2^{1/2}) / (-8866 + 6820 \cdot 2^{1/2})^{1/2} + 13/484 / (2 \cdot x^2 - x + 3)^{1/2} - 329/256036 \cdot (4 \cdot x - 1) / (2 \cdot x^2 - x + 3)^{1/2} + 1/66 / (2 \cdot x^2 - x + 3)^{3/2} - 1/506 \cdot (4 \cdot x - 1) / (2 \cdot x^2 - x + 3)^{3/2}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)(2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2), x, algorithm="maxima")

[Out] integrate(1/((5\*x^2 + 3\*x + 2)\*(2\*x^2 - x + 3)^(5/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(2x^2 - x + 3)^{5/2} (5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2\*x^2 - x + 3)^(5/2)\*(3\*x + 5\*x^2 + 2)), x)

[Out] int(1/((2\*x^2 - x + 3)^(5/2)\*(3\*x + 5\*x^2 + 2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x\*\*2-x+3)\*\*(5/2)/(5\*x\*\*2+3\*x+2), x)

[Out] Integral(1/((2\*x\*\*2 - x + 3)\*\*(5/2)\*(5\*x\*\*2 + 3\*x + 2)), x)

$$3.95 \quad \int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=234

$$\frac{15101 - 8654x}{1035276(2x^2 - x + 3)^{3/2}} - \frac{1352542x + 3133427}{523849656\sqrt{2x^2 - x + 3}} + \frac{65x + 4}{682(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)} + \frac{625\sqrt{\frac{1}{682}(30463)}}{682(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)}$$

Rubi [A] time = 0.54, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{15101 - 8654x}{1035276(2x^2 - x + 3)^{3/2}} - \frac{1352542x + 3133427}{523849656\sqrt{2x^2 - x + 3}} + \frac{65x + 4}{682(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)} + \frac{625\sqrt{\frac{1}{682}(30463 + 23600\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31(30463 + 23600\sqrt{2})}}((687 + 445\sqrt{2}) + 242\sqrt{2} + 203)}}{\sqrt{2x^2 - x + 3}}\right)}{660176} - \frac{625\sqrt{\frac{1}{682}(23600\sqrt{2} - 30463)} \tanh^{-1}\left(\frac{\sqrt{\frac{11}{31(23600\sqrt{2} - 30463)}}((687 - 445\sqrt{2}) - 242\sqrt{2} - 203)}}{\sqrt{2x^2 - x + 3}}\right)}{660176}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^2), x]

[Out] -(15101 - 8654\*x)/(1035276\*(3 - x + 2\*x^2)^(3/2)) - (3133427 + 1352542\*x)/(523849656\*sqrt[3 - x + 2\*x^2]) + (4 + 65\*x)/(682\*(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)) + (625\*sqrt[(30463 + 23600\*sqrt[2])/682]\*ArcTan[(sqrt[11/(31\*(30463 + 23600\*sqrt[2]))])\*(203 + 242\*sqrt[2] + (687 + 445\*sqrt[2])\*x)]/sqrt[3 - x + 2\*x^2]])/660176 - (625\*sqrt[(-30463 + 23600\*sqrt[2])/682]\*ArcTanh[(sqrt[11/(31\*(-30463 + 23600\*sqrt[2]))])\*(203 - 242\*sqrt[2] + (687 - 445\*sqrt[2])\*x)]/sqrt[3 - x + 2\*x^2]])/660176

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && (!IntegerQ[p] && !LtQ[q, -1]) && !IGtQ[q, 0]

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f))*(2*
p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps



$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^{5/2} (2+3x+5x^2)^2} dx &= \frac{4+65x}{682(3-x+2x^2)^{3/2} (2+3x+5x^2)} - \frac{\int \frac{-1738+\frac{4411x}{2}-5720x^2}{(3-x+2x^2)^{5/2} (2+3x+5x^2)} dx}{7502} \\
&= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} + \frac{4+65x}{682(3-x+2x^2)^{3/2} (2+3x+5x^2)} - \frac{1}{7502} \\
&= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{1}{682(3-x+2x^2)^{3/2}} \\
&= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{1}{682(3-x+2x^2)^{3/2}} \\
&= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{1}{682(3-x+2x^2)^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 1.15, size = 296, normalized size = 1.26

$$\frac{198375\sqrt{286+22\sqrt{31}}(687\sqrt{31}+31)\sqrt{2x^2-x+3}(10x^4+x^3+16x^2+7x+6)\tanh^{-1}\left(\frac{(22-4\sqrt{31})\sqrt{2x^2-x+3}}{2\sqrt{286+22\sqrt{31}}\sqrt{2x^2-x+3}}\right)+198375\sqrt{286-22\sqrt{31}}(31+687\sqrt{31})\sqrt{2x^2-x+3}(10x^4+x^3+16x^2+7x+6)\tanh^{-1}\left(\frac{(22+4\sqrt{31})\sqrt{2x^2-x+3}}{2\sqrt{286-22\sqrt{31}}\sqrt{2x^2-x+3}}\right)+5456(13525420x^5+32686812x^4+2879479x^3+84671384x^2-5712309x+31010342)}{2858123723136(2x^2-x+3)^{3/2}(5x^2+3x+2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^2),x]

[Out] -1/2858123723136\*(5456\*(31010342 - 5712309\*x + 84671384\*x^2 + 2879479\*x^3 + 32686812\*x^4 + 13525420\*x^5) + (198375\*I)\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*(31\*I + 687\*Sqrt[31])\*Sqrt[3 - x + 2\*x^2]\*(6 + 7\*x + 16\*x^2 + x^3 + 10\*x^4)\*ArcTanh[(63 + I\*Sqrt[31] + (-22 - (4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286 + (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])] + 198375\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*(31 + (687\*I)\*Sqrt[31])\*Sqrt[3 - x + 2\*x^2]\*(6 + 7\*x + 16\*x^2 + x^3 + 10\*x^4)\*ArcTanh[(-63 + I\*Sqrt[31] + (22 - (4\*I)\*Sqrt[31])\*x)/(2\*Sqrt[286 - (22\*I)\*Sqrt[31]]\*Sqrt[3 - x + 2\*x^2])]/((3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2))

**IntegrateAlgebraic [C]** time = 1.42, size = 416, normalized size = 1.78

$$\frac{\text{RootSum}\left[-56x^5+6\sqrt{2}x^4+1791x^3-26\sqrt{2}x^2-560x+106\sqrt{2}\log\left(\frac{(-1+\sqrt{2})\sqrt{3-x+2x^2}-\sqrt{2}}{-1+\sqrt{2}}\right)+106\sqrt{2}\log\left(\frac{(-1-\sqrt{2})\sqrt{3-x+2x^2}-\sqrt{2}}{-1-\sqrt{2}}\right)\right]}{523849656(2x^2-x+3)^{3/2}(5x^2+3x+2)}, \text{RootSum}\left[-56x^5+6\sqrt{2}x^4+1791x^3-26\sqrt{2}x^2-560x+106\sqrt{2}\log\left(\frac{(-1+\sqrt{2})\sqrt{3-x+2x^2}-\sqrt{2}}{-1+\sqrt{2}}\right)+106\sqrt{2}\log\left(\frac{(-1-\sqrt{2})\sqrt{3-x+2x^2}-\sqrt{2}}{-1-\sqrt{2}}\right)\right]}{66076\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^2),x]

[Out] (-31010342 + 5712309\*x - 84671384\*x^2 - 2879479\*x^3 - 32686812\*x^4 - 13525420\*x^5)/(523849656\*(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)) + RootSum[-56 - 26\*Sqrt[2]\*#1 + 17\*#1^2 + 6\*Sqrt[2]\*#1^3 - 5\*#1^4 & , (-1376\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1] + 106\*Sqrt[2]\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1] + 95\*Log[-(Sqrt[2]\*x) + Sqrt[3 - x + 2\*x^2] - #1]\*#1^2)/

```
(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/5324 + RootSum[-56 - 2
6*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (126249*Sqrt[2]*Log[-(
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 58712*Log[-(Sqrt[2]*x) + Sqrt[3 -
x + 2*x^2] - #1]*#1 + 10095*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2]
- #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/(660176*Sq
rt[2])
```

**fricas [B]** time = 1.19, size = 2253, normalized size = 9.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/25604335602537914112*(301208632500*6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(2)
*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*sqrt(30463*sqrt(2) +
47200)*arctan(1/11117215998613*(168268*sqrt(118)*(22*6962^(3/4)*sqrt(341)*(
321084*x^7 - 1338894*x^6 + 2762802*x^5 - 4721048*x^4 + 2438224*x^3 - 131731
2*x^2 - sqrt(2)*(277258*x^7 - 994619*x^6 + 2123978*x^5 - 3198193*x^4 + 1552
680*x^3 - 621000*x^2 - 1900800*x + 1181952) - 2363904*x + 1900800) + 1829*6
962^(1/4)*sqrt(341)*(25187*x^7 - 392073*x^6 + 2114488*x^5 - 4948060*x^4 + 6
460704*x^3 - 4452768*x^2 - sqrt(2)*(20477*x^7 - 310452*x^6 + 1610140*x^5 -
3584192*x^4 + 4580640*x^3 - 2620800*x^2 - 3400704*x + 2198016) - 4396032*x
+ 3400704))*sqrt(2*x^2 - x + 3)*sqrt(30463*sqrt(2) + 47200) + 3155854864122
4*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 154
9144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104
*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 5391
36) + 1154304*x - 456192) - 2*sqrt(118/79)*(sqrt(118)*(22*6962^(3/4)*sqrt(3
41)*(1050904*x^7 - 1523916*x^6 + 5005956*x^5 - 2572736*x^4 + 3615264*x^3 +
877824*x^2 - sqrt(2)*(1065206*x^7 - 1518091*x^6 + 4815081*x^5 - 1448880*x^4
+ 1303560*x^3 + 3131136*x^2 - 3131136*x) - 877824*x) + 1829*6962^(1/4)*sq
rt(341)*(84981*x^7 - 1100084*x^6 + 4256060*x^5 - 5639616*x^4 + 7745184*x^3 +
2571264*x^2 - 242*sqrt(2)*(319*x^7 - 4124*x^6 + 15860*x^5 - 20160*x^4 + 24
480*x^3 + 20736*x^2 - 20736*x) - 2571264*x))*sqrt(2*x^2 - x + 3)*sqrt(30463
*sqrt(2) + 47200) + 187549318*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1
578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*
(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x
^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 8524969*sqrt(31)*(254591*x^8 -
4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 -
168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*
x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(6962^(1/4)*sqrt(
341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) - 277*x
+ 75)*sqrt(30463*sqrt(2) + 47200) - 219481829*x^2 - 197085724*sqrt(2)*(2*x^
2 - x + 3) + 676362371*x - 895844200)/x^2) + 358619870923*sqrt(31)*(2828123
*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 2493000
96*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 1007
0*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94
887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*
x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456) + 301208632500*
6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(2)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*
x^2 + 15*x + 18)*sqrt(30463*sqrt(2) + 47200)*arctan(1/11117215998613*(16826
8*sqrt(118)*(22*6962^(3/4)*sqrt(341)*(321084*x^7 - 1338894*x^6 + 2762802*x^
5 - 4721048*x^4 + 2438224*x^3 - 1317312*x^2 - sqrt(2)*(277258*x^7 - 994619*
x^6 + 2123978*x^5 - 3198193*x^4 + 1552680*x^3 - 621000*x^2 - 1900800*x + 11
81952) - 2363904*x + 1900800) + 1829*6962^(1/4)*sqrt(341)*(25187*x^7 - 3920
73*x^6 + 2114488*x^5 - 4948060*x^4 + 6460704*x^3 - 4452768*x^2 - sqrt(2)*(2
0477*x^7 - 310452*x^6 + 1610140*x^5 - 3584192*x^4 + 4580640*x^3 - 2620800*x
^2 - 3400704*x + 2198016) - 4396032*x + 3400704))*sqrt(2*x^2 - x + 3)*sqrt(
30463*sqrt(2) + 47200) - 31558548641224*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
```

$$\begin{aligned} & \text{qrt}(2) \cdot (8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - 2\sqrt{118/79} \cdot (\sqrt{118}) \cdot (22 \cdot 6962^{3/4} \cdot \sqrt{341}) \cdot (1050904x^7 - 1523916x^6 + 5005956x^5 - 2572736x^4 + 3615264x^3 + 877824x^2 - \sqrt{2} \cdot (1065206x^7 - 1518091x^6 + 4815081x^5 - 1448880x^4 + 1303560x^3 + 3131136x^2 - 3131136x) - 877824x) + 1829 \cdot 6962^{1/4} \cdot \sqrt{341} \cdot (84981x^7 - 1100084x^6 + 4256060x^5 - 5639616x^4 + 7745184x^3 + 2571264x^2 - 242\sqrt{2} \cdot (319x^7 - 4124x^6 + 15860x^5 - 20160x^4 + 24480x^3 + 20736x^2 - 20736x) - 2571264x) \cdot \sqrt{2x^2 - x + 3} \cdot \sqrt{30463\sqrt{2} + 47200} - 187549318\sqrt{31} \cdot \sqrt{2} \cdot (123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2} \cdot (15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) - 8524969\sqrt{31} \cdot (254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488\sqrt{2} \cdot (4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x) \cdot \sqrt{(6962^{1/4} \cdot \sqrt{341}) \cdot \sqrt{118}) \cdot \sqrt{31}) \cdot \sqrt{2x^2 - x + 3} \cdot (\sqrt{2} \cdot (101x + 176) - 277x + 75) \cdot \sqrt{30463\sqrt{2} + 47200} + 219481829x^2 + 197085724\sqrt{2} \cdot (2x^2 - x + 3) - 676362371x + 895844200) / x^2) - 358619870923\sqrt{31} \cdot (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744\sqrt{2} \cdot (1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456) + 991875 \cdot 6962^{1/4} \cdot \sqrt{341} \cdot \sqrt{118} \cdot \sqrt{31} \cdot (944000x^6 - 377600x^5 + 2879200x^4 + 47200x^3 + 2501600x^2 - 30463\sqrt{2} \cdot (20x^6 - 8x^5 + 61x^4 + x^3 + 53x^2 + 15x + 18) + 708000x + 849600) \cdot \sqrt{30463\sqrt{2} + 47200} \cdot \log(737500000000/79 \cdot (6962^{1/4} \cdot \sqrt{341}) \cdot \sqrt{118}) \cdot \sqrt{31} \cdot \sqrt{2x^2 - x + 3} \cdot (\sqrt{2} \cdot (101x + 176) - 277x + 75) \cdot \sqrt{30463\sqrt{2} + 47200} + 219481829x^2 + 197085724\sqrt{2} \cdot (2x^2 - x + 3) - 676362371x + 895844200) / x^2) - 991875 \cdot 6962^{1/4} \cdot \sqrt{341} \cdot \sqrt{118} \cdot \sqrt{31} \cdot (944000x^6 - 377600x^5 + 2879200x^4 + 47200x^3 + 2501600x^2 - 30463\sqrt{2} \cdot (20x^6 - 8x^5 + 61x^4 + x^3 + 53x^2 + 15x + 18) + 708000x + 849600) \cdot \sqrt{30463\sqrt{2} + 47200} \cdot \log(-737500000000/79 \cdot (6962^{1/4} \cdot \sqrt{341}) \cdot \sqrt{118}) \cdot \sqrt{31} \cdot \sqrt{2x^2 - x + 3} \cdot (\sqrt{2} \cdot (101x + 176) - 277x + 75) \cdot \sqrt{30463\sqrt{2} + 47200} - 219481829x^2 - 197085724\sqrt{2} \cdot (2x^2 - x + 3) + 676362371x - 895844200) / x^2) - 48877259552 \cdot (13525420x^5 + 32686812x^4 + 2879479x^3 + 84671384x^2 - 5712309x + 31010342) \cdot \sqrt{(2x^2 - x + 3)} / (20x^6 - 8x^5 + 61x^4 + x^3 + 53x^2 + 15x + 18) \end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x^2-x+3)^(5/2)/(5\*x^2+3\*x+2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity, infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity, infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity, infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity, infinity]Evaluation time: 18.73Done

**maple** [B] time = 0.10, size = 5975, normalized size = 25.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x)`

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)^2 (2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")`

[Out] `integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(5/2)), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2),x)`

[Out] `int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)`

[Out] `Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2), x)`

$$3.96 \quad \int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=269

$$\frac{1134826571 - 1504660754x}{476353953856\sqrt{2x^2 - x + 3}} + \frac{86885x + 46386}{1860496(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)} - \frac{12280939 - 19536786x}{2824232928(2x^2 - x + 3)^{3/2}} + \frac{1364(2x^2 - x + 3)^{3/2}}{1800960128}$$

Rubi [A] time = 0.59, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{1134826571 - 1504660754x}{476353953856\sqrt{2x^2 - x + 3}} + \frac{86885x + 46386}{1860496(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)} - \frac{12280939 - 19536786x}{2824232928(2x^2 - x + 3)^{3/2}} + \frac{1364(2x^2 - x + 3)^{3/2}}{1800960128}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] -(12280939 - 19536786\*x)/(2824232928\*(3 - x + 2\*x^2)^(3/2)) - (1134826571 - 1504660754\*x)/(476353953856\*sqrt[3 - x + 2\*x^2]) + (4 + 65\*x)/(1364\*(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^2) + (46386 + 86885\*x)/(1860496\*(3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)) + (35\*sqrt[(2243059557247 + 2011748500000\*sqrt[2])/682]\*ArcTan[(sqrt[11/(31\*(2243059557247 + 2011748500000\*sqrt[2]))])\*(1432939 + 2428746\*sqrt[2] + (6290431 + 3861685\*sqrt[2])\*x)]/sqrt[3 - x + 2\*x^2])/1800960128 - (35\*sqrt[(-2243059557247 + 2011748500000\*sqrt[2])/682]\*ArcTanh[(sqrt[11/(31\*(-2243059557247 + 2011748500000\*sqrt[2]))])\*(1432939 - 2428746\*sqrt[2] + (6290431 - 3861685\*sqrt[2])\*x)]/sqrt[3 - x + 2\*x^2])/1800960128

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,

0]

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_.), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^{5/2} (2+3x+5x^2)^3} dx &= \frac{4+65x}{1364(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} - \frac{\int \frac{-5687 + \frac{8635x}{2} - 8580x^2}{(3-x+2x^2)^{5/2} (2+3x+5x^2)^2} dx}{15004} \\
&= \frac{4+65x}{1364(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} + \frac{46386+86885x}{1860496(3-x+2x^2)^{3/2} (2+3x+5x^2)} \\
&= -\frac{12280939-19536786x}{2824232928(3-x+2x^2)^{3/2}} + \frac{4+65x}{1364(3-x+2x^2)^{3/2} (2+3x+5x^2)} \\
&= -\frac{12280939-19536786x}{2824232928(3-x+2x^2)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{136}{136} \\
&= -\frac{12280939-19536786x}{2824232928(3-x+2x^2)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{136}{136} \\
&= -\frac{12280939-19536786x}{2824232928(3-x+2x^2)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{136}{136} \\
&= -\frac{12280939-19536786x}{2824232928(3-x+2x^2)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{136}{136}
\end{aligned}$$

**Mathematica [C]** time = 1.96, size = 242, normalized size = 0.90

$$\frac{11109\sqrt{286+22i\sqrt{31}}(4541903-6290431i\sqrt{31})\operatorname{tanh}^{-1}\left(\frac{(22-4i\sqrt{31})\sqrt{286+22i\sqrt{31}}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2i^2-3+3}}\right)-11109\sqrt{286-22i\sqrt{31}}(6290431\sqrt{31}-4541903i)\operatorname{tanh}^{-1}\left(\frac{(22-4i\sqrt{31})\sqrt{286-22i\sqrt{31}}-63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2i^2-3+3}}\right)+\frac{5456(225699113100i^2-12234606480i^3-592923725931i^5-174241614961i^4+519223213785i^3+178650961091i^2+218659985088i+973933532)}{(i^2-i+3)^2(i^2+3i+2)}}{7796961516715008}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] ((5456\*(9739335532 + 218659985088\*x + 178650961091\*x^2 + 519223213785\*x^3 + 174241614961\*x^4 + 592923725931\*x^5 - 12234606480\*x^6 + 225699113100\*x^7)) / ((3 - x + 2\*x^2)^(3/2)\*(2 + 3\*x + 5\*x^2)^2) + 11109\*sqrt[286 + (22\*I)\*sqrt[31]]\*(4541903 - (6290431\*I)\*sqrt[31])\*ArcTanh[(63 + I\*sqrt[31] + (-22 - (4\*I)\*sqrt[31])\*x)/(2\*sqrt[286 + (22\*I)\*sqrt[31]]\*sqrt[3 - x + 2\*x^2])] - (11109\*I)\*sqrt[286 - (22\*I)\*sqrt[31]]\*(-4541903\*I + 6290431\*sqrt[31])\*ArcTanh[(-63 + I\*sqrt[31] + (22 - (4\*I)\*sqrt[31])\*x)/(2\*sqrt[286 - (22\*I)\*sqrt[31]]\*sqrt[3 - x + 2\*x^2])]) / 7796961516715008

**IntegrateAlgebraic [C]** time = 1.61, size = 601, normalized size = 2.23

$$\frac{9739335532 + 218659985088x + 178650961091x^2 + 519223213785x^3 + 174241614961x^4 + 592923725931x^5 - 12234606480x^6 + 225699113100x^7}{(142906(3-x+2x^2)^{3/2}(2+3x+5x^2)^2) + 11109\sqrt{286+(22i\sqrt{31})}\operatorname{tanh}^{-1}\left(\frac{(22-4i\sqrt{31})\sqrt{286+(22i\sqrt{31})}+63}{2\sqrt{286+(22i\sqrt{31})}\sqrt{2i^2-3+3}}\right) - 11109\sqrt{286-(22i\sqrt{31})}\operatorname{tanh}^{-1}\left(\frac{(22-4i\sqrt{31})\sqrt{286-(22i\sqrt{31})}-63}{2\sqrt{286-(22i\sqrt{31})}\sqrt{2i^2-3+3}}\right) + \frac{5456(225699113100i^2-12234606480i^3-592923725931i^5-174241614961i^4+519223213785i^3+178650961091i^2+218659985088i+973933532)}{(i^2-i+3)^2(i^2+3i+2)}}{7796961516715008}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((3 - x + 2\*x^2)^(5/2)\*(2 + 3\*x + 5\*x^2)^3), x]

[Out] (9739335532 + 218659985088\*x + 178650961091\*x^2 + 519223213785\*x^3 + 174241614961\*x^4 + 592923725931\*x^5 - 12234606480\*x^6 + 225699113100\*x^7) / (142906

```

1861568*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2) - RootSum[-56 - 26*Sqrt[
2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-26154346*Log[-(Sqrt[2]*x) +
Sqrt[3 - x + 2*x^2] - #1] + 37230166*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x
+ 2*x^2] - #1]*#1 - 1193705*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#
1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/1920782072 - Root
Sum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-3647*Log[
-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 3172*Sqrt[2]*Log[-(Sqrt[2]*x) +
Sqrt[3 - x + 2*x^2] - #1]*#1 - 485*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
#1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/234256 + (5
*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-9138
129081*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 16445754136*L
og[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 1004412885*Sqrt[2]*Log[-(S
qrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2
]*#1^2 - 10*#1^3) & ])/(952707907712*Sqrt[2])

```

**fricas [B]** time = 1.33, size = 2343, normalized size = 8.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/611377875290135815296770157063555072*(2164988593398757980*129508224872072
^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^
5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*sqrt(2243059557247*sqrt(2) + 4
023497000000)*arctan(1/452534011574628261925237033857859439*(11475013444*sq
rt(4023497)*(11*129508224872072^(3/4)*sqrt(341)*(2673027292*x^7 - 117686842
22*x^6 + 24008796626*x^5 - 42687622824*x^4 + 22428040912*x^3 - 12956821056*
x^2 - sqrt(2)*(2612082154*x^7 - 9010050347*x^6 + 19426337114*x^5 - 28170626
609*x^4 + 13394761640*x^3 - 4698131400*x^2 - 17594323200*x + 10110341376) -
20220682752*x + 17594323200) + 124728407*129508224872072^(1/4)*sqrt(341)*(
214583731*x^7 - 3372306249*x^6 + 18434388344*x^5 - 43845503580*x^4 + 576317
17152*x^3 - 41786349984*x^2 - sqrt(2)*(190078101*x^7 - 2862100476*x^6 + 146
88003420*x^5 - 32231022496*x^4 + 40927641120*x^3 - 21959568000*x^2 - 311565
03552*x + 19060075008) - 38120150016*x + 31156503552))*sqrt(2*x^2 - x + 3)*
sqrt(2243059557247*sqrt(2) + 4023497000000) + 12846126780182995822393825477
25536472*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^
5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 +
396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x
- 539136) + 1154304*x - 456192) - 2*sqrt(8046994/10139750351)*(sqrt(402349
7)*(11*129508224872072^(3/4)*sqrt(341)*(8140972152*x^7 - 11907581308*x^6 +
39777303828*x^5 - 24395365568*x^4 + 37103094432*x^3 - 1836165888*x^2 - sqrt
(2)*(10387383478*x^7 - 14753211883*x^6 + 46462095753*x^5 - 11926110640*x^4
+ 8224291080*x^3 + 34793549568*x^2 - 34793549568*x) + 1836165888*x) + 12472
8407*129508224872072^(1/4)*sqrt(341)*(692762453*x^7 - 8972954292*x^6 + 3480
3726780*x^5 - 46915651008*x^4 + 67421983392*x^3 + 10625375232*x^2 - 2*sqrt(
2)*(367903387*x^7 - 4754813452*x^6 + 18261523780*x^5 - 22991417280*x^4 + 27
054001440*x^3 + 26759248128*x^2 - 26759248128*x) - 10625375232*x))*sqrt(2*x
^2 - x + 3)*sqrt(2243059557247*sqrt(2) + 4023497000000) + 11194868609848920
9076292438*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 329307
2*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051
*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 -
1036800*x) + 3276288*x) + 5088576640840418594376929*sqrt(31)*(254591*x^8 -
4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 -
168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x
^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(129508224872072^(
1/4)*sqrt(4023497)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*
x + 2195288) - 2838501*x + 1552075)*sqrt(2243059557247*sqrt(2) + 4023497000
000) - 1921101946251381781783*x^2 - 1725071135409404048948*sqrt(2)*(2*x^2 -
x + 3) + 5920130487427727531617*x - 7841232433679109313400)/x^2) + 1459787

```



$$\begin{aligned}
& 1341117040707265710769608369*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 53385560 \\
& *x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744* \\
& \sqrt{2}*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 \\
& + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/((2585191*x^8 - 46612 \\
& 00*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296 \\
& *x^2 - 24772608*x + 18579456)) + 2164988593398757980*129508224872072^{(1/4)}* \\
& \sqrt{4023497}*\sqrt{341}*\sqrt{2}*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390 \\
& *x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*\sqrt{2243059557247*\sqrt{2} + 40234970 \\
& 00000}*\arctan(1/452534011574628261925237033857859439*(11475013444*\sqrt{4023 \\
& 497}*(11*129508224872072^{(3/4)}*\sqrt{341}*(2673027292*x^7 - 11768684222*x^6 \\
& + 24008796626*x^5 - 42687622824*x^4 + 22428040912*x^3 - 12956821056*x^2 - s \\
& \sqrt{2}*(2612082154*x^7 - 9010050347*x^6 + 19426337114*x^5 - 28170626609*x^4 \\
& + 13394761640*x^3 - 4698131400*x^2 - 17594323200*x + 10110341376) - 202206 \\
& 82752*x + 17594323200) + 124728407*129508224872072^{(1/4)}*\sqrt{341}*(2145837 \\
& 31*x^7 - 3372306249*x^6 + 18434388344*x^5 - 43845503580*x^4 + 57631717152*x \\
& ^3 - 41786349984*x^2 - \sqrt{2}*(190078101*x^7 - 2862100476*x^6 + 1468800342 \\
& 0*x^5 - 32231022496*x^4 + 40927641120*x^3 - 21959568000*x^2 - 31156503552*x \\
& + 19060075008) - 38120150016*x + 31156503552))*\sqrt{2*x^2 - x + 3}*\sqrt{22 \\
& 43059557247*\sqrt{2} + 4023497000000) - 128461267801829958223938254772553647 \\
& 2*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 154 \\
& 9144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104 \\
& *x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 5391 \\
& 36) + 1154304*x - 456192) - 2*\sqrt{8046994/10139750351}*(\sqrt{4023497}*(11* \\
& 129508224872072^{(3/4)}*\sqrt{341}*(8140972152*x^7 - 11907581308*x^6 + 3977730 \\
& 3828*x^5 - 24395365568*x^4 + 37103094432*x^3 - 1836165888*x^2 - \sqrt{2}*(10 \\
& 387383478*x^7 - 14753211883*x^6 + 46462095753*x^5 - 11926110640*x^4 + 82242 \\
& 91080*x^3 + 34793549568*x^2 - 34793549568*x) + 1836165888*x) + 124728407*12 \\
& 9508224872072^{(1/4)}*\sqrt{341}*(692762453*x^7 - 8972954292*x^6 + 34803726780 \\
& *x^5 - 46915651008*x^4 + 67421983392*x^3 + 10625375232*x^2 - 2*\sqrt{2}*(367 \\
& 903387*x^7 - 4754813452*x^6 + 18261523780*x^5 - 22991417280*x^4 + 270540014 \\
& 40*x^3 + 26759248128*x^2 - 26759248128*x) - 10625375232*x))*\sqrt{2*x^2 - x \\
& + 3}*\sqrt{2243059557247*\sqrt{2} + 4023497000000) - 111948686098489209076292 \\
& 438*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + \\
& 396480*x^4 + 798336*x^3 - 3822336*x^2 - \sqrt{2}*(15550*x^8 - 118051*x^7 + \\
& 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800 \\
& *x) + 3276288*x) - 5088576640840418594376929*\sqrt{31}*(254591*x^8 - 4815126 \\
& *x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 1689569 \\
& 28*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 36 \\
& 18*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\sqrt{((129508224872072^{(1/4)}*\sqrt{2} \\
& \sqrt{4023497}*\sqrt{341}*\sqrt{31}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(643213*x + 2195 \\
& 288) - 2838501*x + 1552075))*\sqrt{2243059557247*\sqrt{2} + 4023497000000) + 1 \\
& 921101946251381781783*x^2 + 1725071135409404048948*\sqrt{2}*(2*x^2 - x + 3) \\
& - 5920130487427727531617*x + 7841232433679109313400)/x^2) - 145978713411170 \\
& 40707265710769608369*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 1 \\
& 42835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2})* \\
& (1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x \\
& ^2 + 4320*x - 5184) + 223064064*x - 94887936))/((2585191*x^8 - 4661200*x^7 + \\
& 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 2 \\
& 4772608*x + 18579456)) + 55545*129508224872072^{(1/4)}*\sqrt{4023497}*\sqrt{341} \\
& )*\sqrt{31}*(402349700000000*x^8 + 80469940000000*x^7 + 1291542537000000*x^6 \\
& + 692041484000000*x^5 + 1569163830000000*x^4 + 949545292000000*x^3 + 96966 \\
& 2777000000*x^2 - 2243059557247*\sqrt{2}*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^ \\
& 5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) + 337973748000000*x + 14484589 \\
& 2000000)*\sqrt{2243059557247*\sqrt{2} + 4023497000000)*\log(24643919125000000 \\
& 0000/10139750351*(129508224872072^{(1/4)}*\sqrt{4023497}*\sqrt{341}*\sqrt{31}*\sqrt{2} \\
& \sqrt{2*x^2 - x + 3}*(\sqrt{2}*(643213*x + 2195288) - 2838501*x + 1552075))*\sqrt{ \\
& (2243059557247*\sqrt{2} + 4023497000000) + 1921101946251381781783*x^2 + 1725 \\
& 071135409404048948*\sqrt{2}*(2*x^2 - x + 3) - 5920130487427727531617*x + 784 \\
& 1232433679109313400)/x^2) - 55545*129508224872072^{(1/4)}*\sqrt{4023497}*\sqrt{
\end{aligned}$$

```

341)*sqrt(31)*(40234970000000*x^8 + 8046994000000*x^7 + 1291542537000000*
x^6 + 692041484000000*x^5 + 1569163830000000*x^4 + 949545292000000*x^3 + 96
9662777000000*x^2 - 2243059557247*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172
*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) + 337973748000000*x + 14484
5892000000)*sqrt(2243059557247*sqrt(2) + 4023497000000)*log(-24643919125000
00000000/10139750351*(129508224872072^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(31
)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195288) - 2838501*x + 1552075)*
sqrt(2243059557247*sqrt(2) + 4023497000000) - 1921101946251381781783*x^2 -
17250711135409404048948*sqrt(2)*(2*x^2 - x + 3) + 5920130487427727531617*x -
7841232433679109313400)/x^2) + 427817641581532204139104*(225699113100*x^7
- 12234606480*x^6 + 592923725931*x^5 + 174241614961*x^4 + 519223213785*x^3
+ 178650961091*x^2 + 218659985088*x + 9739335532)*sqrt(2*x^2 - x + 3))/(100
*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36
)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infini
ty]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity]
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinity]
]Evaluation time: 70.56Done
```

**maple** [B] time = 0.21, size = 19014, normalized size = 70.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 3x + 2)^3 (2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(5/2)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3), x)`

[Out] `int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3), x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3, x)`

[Out] `Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3), x)`

$$3.97 \quad \int \sqrt{a + bx + cx^2} (d + ex + fx^2)^2 dx$$

**Optimal.** Leaf size=436

$$\frac{(b^2 - 4ac) \tanh^{-1} \left( \frac{b+2cx}{2\sqrt{c} \sqrt{a+bx+cx^2}} \right) (8c^2 (2a^2f^2 + 12abef + 5b^2 (2df + e^2)) - 56b^2cf(af + be) - 32c^3 (a (2df + e^2) + 2d^2f))}{1024c^{11/2}}$$

**Rubi [A]** time = 0.79, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 621, 206}

$\frac{b^2 - 4ac}{2\sqrt{c}} \sqrt{a+bx+cx^2} \operatorname{arctanh} \left( \frac{b+2cx}{2\sqrt{c} \sqrt{a+bx+cx^2}} \right) + \frac{8c^2 (2a^2f^2 + 12abef + 5b^2 (2df + e^2)) - 56b^2cf(af + be) - 32c^3 (a (2df + e^2) + 2d^2f)}{1024c^{11/2}}$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]
[Out] ((128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c^3*(4*b*d*e + a*(e^2 + 2*d*f))) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]/(512*c^5) + ((640*c^3*d*e - 105*b^3*f^2 + 28*b*c*f*(10*b*e + 7*a*f) - 8*c^2*(32*a*e*f + 25*b*(e^2 + 2*d*f)))*(a + b*x + c*x^2)^(3/2))/(960*c^4) + ((21*b^2*f^2 - 4*c*f*(14*b*e + 5*a*f) + 40*c^2*(e^2 + 2*d*f))*x*(a + b*x + c*x^2)^(3/2))/(160*c^3) + (f*(8*c*e - 3*b*f)*x^2*(a + b*x + c*x^2)^(3/2))/(20*c^2) + (f^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c) - ((b^2 - 4*a*c)*(128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c^3*(4*b*d*e + a*(e^2 + 2*d*f))) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(1024*c^(11/2))
```

**Rule 206**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

**Rule 612**

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

**Rule 621**

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

**Rule 640**

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

**Rule 1661**

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^p), x]
```

$c*x^2)^{(p+1)}/(c*(q+2*p+1)), x] + \text{Dist}[1/(c*(q+2*p+1)), \text{Int}[(a+b*x+c*x^2)^p \text{ExpandToSum}[c*(q+2*p+1)*Pq - a*e*(q-1)*x^{(q-2)} - b*e*(q+p)*x^{(q-1)} - c*e*(q+2*p+1)*x^q, x], x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{LeQ}[p, -1]$

Rubi steps

$$\begin{aligned} \int \sqrt{a+bx+cx^2} (d+ex+fx^2)^2 dx &= \frac{f^2 x^3 (a+bx+cx^2)^{3/2}}{6c} + \frac{\int \sqrt{a+bx+cx^2} (6cd^2+12cdex-3(a f^2 - \dots)}{6c} \\ &= \frac{f(8ce-3bf)x^2 (a+bx+cx^2)^{3/2}}{20c^2} + \frac{f^2 x^3 (a+bx+cx^2)^{3/2}}{6c} + \frac{\int \sqrt{a+bx+cx^2} (6cd^2+12cdex-3(a f^2 - \dots)}{6c} \\ &= \frac{(21b^2 f^2 - 4cf(14be+5af) + 40c^2(e^2+2df)) x (a+bx+cx^2)^{3/2}}{160c^3} + \dots \\ &= \frac{(640c^3 de - 105b^3 f^2 + 28bcf(10be+7af) - 8c^2(32aef+25b(e^2+2df)))}{960c^4} \\ &= \frac{(128c^4 d^2 + 21b^4 f^2 - 56b^2 cf(be+af) - 32c^3(4bde+a(e^2+2df)))}{512c^5} + \dots \\ &= \frac{(128c^4 d^2 + 21b^4 f^2 - 56b^2 cf(be+af) - 32c^3(4bde+a(e^2+2df)))}{512c^5} + \dots \\ &= \frac{(128c^4 d^2 + 21b^4 f^2 - 56b^2 cf(be+af) - 32c^3(4bde+a(e^2+2df)))}{512c^5} + \dots \end{aligned}$$

**Mathematica [A]** time = 0.93, size = 657, normalized size = 1.51

---

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)^2,x]

[Out] (3840\*c^(9/2)\*d^2\*(b + 2\*c\*x)\*Sqrt[a + x\*(b + c\*x)] + 10240\*c^(9/2)\*d\*e\*(a + x\*(b + c\*x))^(3/2) + 3840\*c^(9/2)\*(e^2 + 2\*d\*f)\*x\*(a + x\*(b + c\*x))^(3/2) + 6144\*c^(9/2)\*e\*f\*x^2\*(a + x\*(b + c\*x))^(3/2) + 2560\*c^(9/2)\*f^2\*x^3\*(a + x\*(b + c\*x))^(3/2) - 1920\*c^4\*(b^2 - 4\*a\*c)\*d^2\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])] - 1920\*b\*c^3\*d\*e\*(2\*Sqrt[c]\*(b + 2\*c\*x)\*Sqrt[a + x\*(b + c\*x)] - (b^2 - 4\*a\*c)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])]) + 8\*c\*e\*f\*(-16\*c^(3/2)\*(-35\*b^2 + 32\*a\*c + 42\*b\*c\*x)\*(a + x\*(b + c\*x))^(3/2) - 15\*b\*(7\*b^2 - 12\*a\*c)\*(2\*Sqrt[c]\*(b + 2\*c\*x)\*Sqrt[a + x\*(b + c\*x)] - (b^2 - 4\*a\*c)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])) - 40\*c^2\*(e^2 + 2\*d\*f)\*(80\*b\*c^(3/2)\*(a + x\*(b + c\*x))^(3/2) - 3\*(5\*b^2 - 4\*a\*c)\*(2\*Sqrt[c]\*(b + 2\*c\*x)\*Sqrt[a + x\*(b + c\*x)] - (b^2 - 4\*a\*c)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])) - f^2\*(2304\*b\*c^(7/2)\*x^2\*(a + x\*(b + c\*x))^(3/2) + 16\*c^(3/2)\*(105\*b^3 - 196\*a\*b\*c - 126\*b^2\*c\*x + 120\*a\*c^2\*x)\*(a + x\*(b + c\*x))^(3/2) - 15\*(21\*b^4 - 56\*a\*b^2\*c + 16\*a^2\*c^2)\*(2\*Sqrt[c]\*(b + 2\*c\*x)\*Sqrt[a + x\*(b + c\*x)] - (b^2 - 4\*a\*c)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])))/(15360\*c^(11/2))

**IntegrateAlgebraic [A]** time = 2.82, size = 655, normalized size = 1.50

---

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)^2,x]

[Out] (Sqrt[a + b\*x + c\*x^2]\*(1920\*b\*c^4\*d^2 - 1920\*b^2\*c^3\*d\*e + 5120\*a\*c^4\*d\*e + 600\*b^3\*c^2\*e^2 - 2080\*a\*b\*c^3\*e^2 + 1200\*b^3\*c^2\*d\*f - 4160\*a\*b\*c^3\*d\*f - 840\*b^4\*c\*e\*f + 3680\*a\*b^2\*c^2\*e\*f - 2048\*a^2\*c^3\*e\*f + 315\*b^5\*f^2 - 1680\*a\*b^3\*c\*f^2 + 1808\*a^2\*b\*c^2\*f^2 + 3840\*c^5\*d^2\*x + 1280\*b\*c^4\*d\*e\*x - 400\*b^2\*c^3\*e^2\*x + 960\*a\*c^4\*e^2\*x - 800\*b^2\*c^3\*d\*f\*x + 1920\*a\*c^4\*d\*f\*x + 560\*b^3\*c^2\*e\*f\*x - 1856\*a\*b\*c^3\*e\*f\*x - 210\*b^4\*c\*f^2\*x + 896\*a\*b^2\*c^2\*f^2\*x - 480\*a^2\*c^3\*f^2\*x + 5120\*c^5\*d\*e\*x^2 + 320\*b\*c^4\*e^2\*x^2 + 640\*b\*c^4\*d\*f\*x^2 - 448\*b^2\*c^3\*e\*f\*x^2 + 1024\*a\*c^4\*e\*f\*x^2 + 168\*b^3\*c^2\*f^2\*x^2 - 544\*a\*b\*c^3\*f^2\*x^2 + 1920\*c^5\*d^2\*x^3 + 3840\*c^5\*d\*f\*x^3 + 384\*b\*c^4\*e\*f\*x^3 - 144\*b^2\*c^3\*f^2\*x^3 + 320\*a\*c^4\*f^2\*x^3 + 3072\*c^5\*e\*f\*x^4 + 128\*b\*c^4\*f^2\*x^4 + 1280\*c^5\*f^2\*x^5))/(7680\*c^5) + ((128\*b^2\*c^4\*d^2 - 512\*a\*c^5\*d^2 - 128\*b^3\*c^3\*d\*e + 512\*a\*b\*c^4\*d\*e + 40\*b^4\*c^2\*e^2 - 192\*a\*b^2\*c^3\*e^2 + 128\*a^2\*c^4\*e^2 + 80\*b^4\*c^2\*d\*f - 384\*a\*b^2\*c^3\*d\*f + 256\*a^2\*c^4\*d\*f - 56\*b^5\*c\*e\*f + 320\*a\*b^3\*c^2\*e\*f - 384\*a^2\*b\*c^3\*e\*f + 21\*b^6\*f^2 - 140\*a\*b^4\*c\*f^2 + 240\*a^2\*b^2\*c^2\*f^2 - 64\*a^3\*c^3\*f^2)\*Log[b + 2\*c\*x - 2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2]])/(1024\*c^(11/2))

**fricas** [A] time = 0.74, size = 1269, normalized size = 2.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)\*(f\*x^2+e\*x+d)^2,x, algorithm="fricas")

[Out] [-1/30720\*(15\*(128\*(b^2\*c^4 - 4\*a\*c^5)\*d^2 - 128\*(b^3\*c^3 - 4\*a\*b\*c^4)\*d\*e + 8\*(5\*b^4\*c^2 - 24\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*e^2 + (21\*b^6 - 140\*a\*b^4\*c + 240\*a^2\*b^2\*c^2 - 64\*a^3\*c^3)\*f^2 + 8\*(2\*(5\*b^4\*c^2 - 24\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*d - (7\*b^5\*c - 40\*a\*b^3\*c^2 + 48\*a^2\*b\*c^3)\*e)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(1280\*c^6\*f^2\*x^5 + 1920\*b\*c^5\*d^2 + 128\*(24\*c^6\*e\*f + b\*c^5\*f^2)\*x^4 + 16\*(120\*c^6\*e^2 - (9\*b^2\*c^4 - 20\*a\*c^5)\*f^2 + 24\*(10\*c^6\*d + b\*c^5\*e)\*f)\*x^3 - 640\*(3\*b^2\*c^4 - 8\*a\*c^5)\*d\*e + 40\*(15\*b^3\*c^3 - 52\*a\*b\*c^4)\*e^2 + (315\*b^5\*c - 1680\*a\*b^3\*c^2 + 1808\*a^2\*b\*c^3)\*f^2 + 8\*(640\*c^6\*d\*e + 40\*b\*c^5\*e^2 + (21\*b^3\*c^3 - 68\*a\*b\*c^4)\*f^2 + 8\*(10\*b\*c^5\*d - (7\*b^2\*c^4 - 16\*a\*c^5)\*e)\*f)\*x^2 + 8\*(10\*(15\*b^3\*c^3 - 52\*a\*b\*c^4)\*d - (105\*b^4\*c^2 - 460\*a\*b^2\*c^3 + 256\*a^2\*c^4)\*e)\*f + 2\*(1920\*c^6\*d^2 + 640\*b\*c^5\*d\*e - 40\*(5\*b^2\*c^4 - 12\*a\*c^5)\*e^2 - (105\*b^4\*c^2 - 448\*a\*b^2\*c^3 + 240\*a^2\*c^4)\*f^2 - 8\*(10\*(5\*b^2\*c^4 - 12\*a\*c^5)\*d - (35\*b^3\*c^3 - 116\*a\*b\*c^4)\*e)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^6, 1/15360\*(15\*(128\*(b^2\*c^4 - 4\*a\*c^5)\*d^2 - 128\*(b^3\*c^3 - 4\*a\*b\*c^4)\*d\*e + 8\*(5\*b^4\*c^2 - 24\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*e^2 + (21\*b^6 - 140\*a\*b^4\*c + 240\*a^2\*b^2\*c^2 - 64\*a^3\*c^3)\*f^2 + 8\*(2\*(5\*b^4\*c^2 - 24\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*d - (7\*b^5\*c - 40\*a\*b^3\*c^2 + 48\*a^2\*b\*c^3)\*e)\*f)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) + 2\*(1280\*c^6\*f^2\*x^5 + 1920\*b\*c^5\*d^2 + 128\*(24\*c^6\*e\*f + b\*c^5\*f^2)\*x^4 + 16\*(120\*c^6\*e^2 - (9\*b^2\*c^4 - 20\*a\*c^5)\*f^2 + 24\*(10\*c^6\*d + b\*c^5\*e)\*f)\*x^3 - 640\*(3\*b^2\*c^4 - 8\*a\*c^5)\*d\*e + 40\*(15\*b^3\*c^3 - 52\*a\*b\*c^4)\*e^2 + (315\*b^5\*c - 1680\*a\*b^3\*c^2 + 1808\*a^2\*b\*c^3)\*f^2 + 8\*(640\*c^6\*d\*e + 40\*b\*c^5\*e^2 + (21\*b^3\*c^3 - 68\*a\*b\*c^4)\*f^2 + 8\*(10\*b\*c^5\*d - (7\*b^2\*c^4 - 16\*a\*c^5)\*e)\*f)\*x^2 + 8\*(10\*(15\*b^3\*c^3 - 52\*a\*b\*c^4)\*d - (105\*b^4\*c^2 - 460\*a\*b^2\*c^3 + 256\*a^2\*c^4)\*e)\*f + 2\*(1920\*c^6\*d^2 + 640\*b\*c^5\*d\*e - 40\*(5\*b^2\*c^4 - 12\*a\*c^5)\*e^2 - (105\*b^4\*c^2 - 448\*a\*b^2\*c^3 + 240\*a^2\*c^4)\*f^2 - 8\*(10\*(5\*b^2\*c^4 - 12\*a\*c^5)\*d - (35\*b^3\*c^3 - 116\*a\*b\*c^4)\*e)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^6]

**giac** [A] time = 0.36, size = 638, normalized size = 1.46

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)\*(f\*x^2+e\*x+d)^2,x, algorithm="giac")

[Out]  $\frac{1}{7680}\sqrt{c x^2 + b x + a} \left( 2 \left( 4 \left( 2 \left( 8 \left( 10 f^2 x + (b c^4 f^2 + 24 c^5 f e) / c^5 \right) x + (240 c^5 d f - 9 b^2 c^3 f^2 + 20 a c^4 f^2 + 24 b c^4 f e + 120 c^5 e^2) / c^5 \right) x + (80 b c^4 d f + 21 b^3 c^2 f^2 - 68 a b c^3 f^2 + 640 c^5 d e - 56 b^2 c^3 f e + 128 a c^4 f e + 40 b c^4 e^2) / c^5 \right) x + (1920 c^5 d^2 - 400 b^2 c^3 d f + 960 a c^4 d f - 105 b^4 c f^2 + 448 a b^2 c^2 f^2 - 240 a^2 c^3 f^2 + 640 b c^4 d e + 280 b^3 c^2 f e - 928 a b c^3 f e - 200 b^2 c^3 e^2 + 480 a c^4 e^2) / c^5 \right) x + (1920 b c^4 d^2 + 1200 b^3 c^2 d f - 4160 a b c^3 d f + 315 b^5 f^2 - 1680 a b^3 c f^2 + 1808 a^2 b c^2 f^2 - 1920 b^2 c^3 d e + 5120 a c^4 d e - 840 b^4 c f e + 3680 a b^2 c^2 f e - 2048 a^2 c^3 f e + 600 b^3 c^2 e^2 - 2080 a b c^3 e^2) / c^5 \right) + \frac{1}{1024} \left( 128 b^2 c^4 d^2 - 512 a c^5 d^2 + 80 b^4 c^2 d f - 384 a b^2 c^3 d f + 256 a^2 c^4 d f + 21 b^6 f^2 - 140 a b^4 c f^2 + 240 a^2 b^2 c^2 f^2 - 64 a^3 c^3 f^2 - 128 b^3 c^3 d e + 512 a b c^4 d e - 56 b^5 c f e + 320 a b^3 c^2 f e - 384 a^2 b c^3 f e + 40 b^4 c^2 e^2 - 192 a b^2 c^3 e^2 + 128 a^2 c^4 e^2 \right) \log(a b s(-2(\sqrt{c} x - \sqrt{c x^2 + b x + a})) \sqrt{c} - b) / c^{11/2}$

**maple [B]** time = 0.02, size = 1429, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(1/2)\*(f\*x^2+e\*x+d)^2,x)

[Out]  $\frac{1}{6} f^2 x^3 (c x^2 + b x + a)^{3/2} / c - \frac{7}{32} f^2 b^2 / c^3 a x x (c x^2 + b x + a)^{1/2} - \frac{7}{20} e f b / c^2 x x (c x^2 + b x + a)^{3/2} - \frac{7}{32} e f b^3 / c^3 x x (c x^2 + b x + a)^{1/2} + \frac{2}{3} d e (c x^2 + b x + a)^{3/2} / c + \frac{1}{4} d^2 / c (c x^2 + b x + a)^{1/2} * b + \frac{1}{2} d^2 / c^{1/2} * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) * a - \frac{1}{8} d^2 / c^{3/2} * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) * b^2 + \frac{1}{4} x x (c x^2 + b x + a)^{3/2} / c * e^2 - \frac{5}{24} b / c^2 * (c x^2 + b x + a)^{3/2} * e^2 + \frac{5}{64} b^3 / c^3 * (c x^2 + b x + a)^{1/2} * e^2 - \frac{5}{128} b^4 / c^{7/2} * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) * e^2 - \frac{1}{8} a^2 / c^{3/2} * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) * e^2 + \frac{21}{512} f^2 b^5 / c^5 * (c x^2 + b x + a)^{1/2} - \frac{21}{1024} f^2 b^6 / c^{11/2} * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) + \frac{1}{16} f^2 a^3 / c^{5/2} * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) - \frac{7}{64} f^2 b^3 / c^4 * (c x^2 + b x + a)^{3/2} + \frac{3}{8} e f b / c^2 a x x (c x^2 + b x + a)^{1/2} - \frac{5}{16} e f b^3 / c^{7/2} * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) * a + \frac{3}{16} e f b^2 / c^3 a * (c x^2 + b x + a)^{1/2} + \frac{3}{8} e f b / c^{5/2} a^2 * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) + \frac{5}{16} b^2 / c^2 x x (c x^2 + b x + a)^{1/2} * d f + \frac{3}{8} b^2 / c^{5/2} * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) * a * d f - \frac{1}{4} a / c x x (c x^2 + b x + a)^{1/2} * d f - \frac{1}{8} a / c^2 * (c x^2 + b x + a)^{1/2} * b * d f - \frac{1}{2} d e b / c x x (c x^2 + b x + a)^{1/2} - \frac{1}{2} d e b / c^{3/2} * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) * a + \frac{1}{2} d^2 x x (c x^2 + b x + a)^{1/2} + \frac{21}{256} f^2 b^4 / c^4 x x (c x^2 + b x + a)^{1/2} - \frac{5}{12} b / c^2 * (c x^2 + b x + a)^{3/2} * d f + \frac{5}{32} b^2 / c^2 x x (c x^2 + b x + a)^{1/2} * e^2 + \frac{5}{32} b^3 / c^3 * (c x^2 + b x + a)^{1/2} * d f + \frac{3}{16} b^2 / c^{5/2} * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) * a * e^2 - \frac{5}{64} b^4 / c^{7/2} * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) * d f - \frac{1}{8} a / c x x (c x^2 + b x + a)^{1/2} * e^2 - \frac{1}{16} a / c^2 * (c x^2 + b x + a)^{1/2} * b * e^2 - \frac{1}{4} a^2 / c^{3/2} * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) * d f - \frac{1}{4} d e b^2 / c^2 * (c x^2 + b x + a)^{1/2} + \frac{35}{256} f^2 b^4 / c^9 * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) * a - \frac{7}{64} f^2 b^3 / c^4 a * (c x^2 + b x + a)^{1/2} - \frac{15}{64} f^2 b^2 / c^{7/2} a^2 * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) + \frac{49}{240} f^2 b / c^3 a * (c x^2 + b x + a)^{3/2} - \frac{1}{8} f^2 a / c^2 x x (c x^2 + b x + a)^{3/2} + \frac{1}{16} f^2 a^2 / c^2 x x (c x^2 + b x + a)^{1/2} + \frac{1}{32} f^2 a^2 / c^3 * (c x^2 + b x + a)^{1/2} * b + \frac{2}{5} e f x x^2 * (c x^2 + b x + a)^{3/2} / c + \frac{7}{24} e f b^2 / c^3 * (c x^2 + b x + a)^{3/2} - \frac{7}{64} e f b^4 / c^4 * (c x^2 + b x + a)^{1/2} + \frac{7}{128} e f b^5 / c^9 * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2}) - \frac{4}{15} e f a / c^2 * (c x^2 + b x + a)^{3/2} - \frac{3}{20} f^2 b / c^2 x x^2 * (c x^2 + b x + a)^{3/2} + \frac{21}{160} f^2 b^2 / c^3 x x (c x^2 + b x + a)^{3/2} + \frac{1}{2} x x (c x^2 + b x + a)^{3/2} / c * d f + \frac{1}{8} d e b^3 / c^{5/2} * \ln((1/2 * b + c x) / c^{1/2} + (c x^2 + b x + a)^{1/2})$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [B]** time = 5.31, size = 1299, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2,x)
```

```
[Out] d^2*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (e^2*x*(a + b*x + c*x^2)^(3/2)
)/ (4*c) + (a*f^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2)
))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a
+ b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c)
+ (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) +
(a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c))/(2*c) - (3*b
*f^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2)
))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a
+ b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*
(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b
*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c))/(10*c) - (2*a*((log
((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5
/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^
2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c))/(4*c) + (f^2*x^3*(a + b*
x + c*x^2)^(3/2))/(6*c) - (a*e^2*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) +
(log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3
/2))))/(4*c) + (d^2*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c
- b^2/4))/(2*c^(3/2)) - (5*b*e^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x +
c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2
*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (4*a*e*f*((log((b + 2*c
*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((
8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c
) - (5*b*d*f*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(4*c) + (d*e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a
+ b*x + c*x^2)^(1/2))/(12*c^2) + (d*f*x*(a + b*x + c*x^2)^(3/2))/(2*c) + (7
*b*e*f*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2)
))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a
+ b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2
+ b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x +
c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c))/(5*c) + (2*e*f*x^2*(a +
b*x + c*x^2)^(3/2))/(5*c) - (a*d*f*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2)
) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c
^(3/2))))/(2*c) + (d*e*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2)
)*(b^3 - 4*a*b*c))/(8*c^(5/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx + cx^2} (d + ex + fx^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)**2, x)
```



### 3.98 $\int \sqrt{a + bx + cx^2} (d + ex + fx^2) dx$

**Optimal.** Leaf size=175

$$\frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af + 2be) + 5b^2f + 16c^2d)}{128c^{7/2}} + \frac{(b + 2cx)\sqrt{a + bx + cx^2}(-4acf + 5b^2f)}{64c^3}$$

**Rubi [A]** time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b + 2cx)\sqrt{a + bx + cx^2}(-4acf + 5b^2f - 8bce + 16c^2d)}{64c^3} - \frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af + 2be) + 5b^2f + 16c^2d)}{128c^{7/2}} + \frac{(a + bx + cx^2)^{3/2}(8ce - 5bf)}{24c^2} + \frac{fx(a + bx + cx^2)^{3/2}}{4c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] ((16\*c^2\*d - 8\*b\*c\*e + 5\*b^2\*f - 4\*a\*c\*f)\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(64\*c^3) + ((8\*c\*e - 5\*b\*f)\*(a + b\*x + c\*x^2)^(3/2))/(24\*c^2) + (f\*x\*(a + b\*x + c\*x^2)^(3/2))/(4\*c) - ((b^2 - 4\*a\*c)\*(16\*c^2\*d + 5\*b^2\*f - 4\*c\*(2\*b\*e + a\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(128\*c^(7/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \sqrt{a+bx+cx^2} (d+ex+fx^2) dx &= \frac{fx(a+bx+cx^2)^{3/2}}{4c} + \frac{\int (4cd-af + \frac{1}{2}(8ce-5bf)x) \sqrt{a+bx+cx^2} dx}{4c} \\ &= \frac{(8ce-5bf)(a+bx+cx^2)^{3/2}}{24c^2} + \frac{fx(a+bx+cx^2)^{3/2}}{4c} + \frac{(16c^2d-8bce+5b^2f-4acf)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} \\ &= \frac{(16c^2d-8bce+5b^2f-4acf)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)(a+bx+cx^2)^{3/2}}{24c^2} \\ &= \frac{(16c^2d-8bce+5b^2f-4acf)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)(a+bx+cx^2)^{3/2}}{24c^2} \\ &= \frac{(16c^2d-8bce+5b^2f-4acf)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)(a+bx+cx^2)^{3/2}}{24c^2} \end{aligned}$$

**Mathematica [A]** time = 0.27, size = 173, normalized size = 0.99

$$\frac{2\sqrt{c}\sqrt{a+(b+cx)}(4bc(2c(6d+2ex+fx^2)-13af)+8c^2(a(8e+3fx)+2cx(6d+4ex+3fx^2))+15b^3f-2b^2c(12e+5fx))-3(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+(b+cx)}}\right)(-4c(af+2be)+5b^2f+16c^2d)}{384c^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] (2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)]\*(15\*b^3\*f - 2\*b^2\*c\*(12\*e + 5\*f\*x) + 4\*b\*c\*(-13\*a\*f + 2\*c\*(6\*d + 2\*e\*x + f\*x^2)) + 8\*c^2\*(a\*(8\*e + 3\*f\*x) + 2\*c\*x\*(6\*d + 4\*e\*x + 3\*f\*x^2))) - 3\*(b^2 - 4\*a\*c)\*(16\*c^2\*d + 5\*b^2\*f - 4\*c\*(2\*b\*e + a\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])]/(384\*c^(7/2))

**IntegrateAlgebraic [A]** time = 0.77, size = 207, normalized size = 1.18

$$\frac{\log\left(-2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)\left(16a^2c^2f-24ab^2cf+32abc^2e-64ac^3d+5b^4f-8b^3ce+16b^2c^2d\right)+\sqrt{a+bx+cx^2}\left(-52abcf+64ac^2e+24ac^2fx+15b^3f-24b^2ce-10b^2cfx+48b^2d+16b^2cx+8bc^2fx^2+96c^3dx+64c^3ex^2+48c^3fx^3\right)}{128c^{7/2}+192c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] (Sqrt[a + b\*x + c\*x^2]\*(48\*b\*c^2\*d - 24\*b^2\*c\*e + 64\*a\*c^2\*e + 15\*b^3\*f - 5\*2\*a\*b\*c\*f + 96\*c^3\*d\*x + 16\*b\*c^2\*e\*x - 10\*b^2\*c\*f\*x + 24\*a\*c^2\*f\*x + 64\*c^3\*e\*x^2 + 8\*b\*c^2\*f\*x^2 + 48\*c^3\*f\*x^3))/(192\*c^3) + ((16\*b^2\*c^2\*d - 64\*a\*c^3\*d - 8\*b^3\*c\*e + 32\*a\*b\*c^2\*e + 5\*b^4\*f - 24\*a\*b^2\*c\*f + 16\*a^2\*c^2\*f)\*Log[b + 2\*c\*x - 2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2]])/(128\*c^(7/2))

**fricas [A]** time = 0.47, size = 465, normalized size = 2.66

$$\frac{1}{192c^3} \left( (16b^2c^2d - 64ac^3d - 8b^3ce + 32abc^2e + 5b^4f - 24a^2c^2f) \sqrt{a+bx+cx^2} + \log\left(\frac{b+2cx-\sqrt{a+bx+cx^2}}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (48b^2c^2d - 24b^2ce + 64ac^2e + 15b^3f - 5(2abcf - 96c^3dx + 64c^3ex^2 + 48c^3fx^3)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)\*(f\*x^2+e\*x+d), x, algorithm="fricas")

[Out] [1/768\*(3\*(16\*(b^2\*c^2 - 4\*a\*c^3)\*d - 8\*(b^3\*c - 4\*a\*b\*c^2)\*e + (5\*b^4 - 24\*a\*b^2\*c + 16\*a^2\*c^2)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 + 4\*sqrt(c)\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) + 4\*(48\*c^4\*f\*x^3 + 48\*b\*c^3\*d + 8\*(8\*c^4\*e + b\*c^3\*f)\*x^2 - 8\*(3\*b^2\*c^2 - 8\*a\*c^3)\*e + (15\*b^3\*c - 52\*a\*b\*c^2)\*f + 2\*(48\*c^4\*d + 8\*b\*c^3\*e - (5\*b^2\*c^2 - 12\*a\*c^3)\*f)\*x)\*sqrt(c\*x

$(^2 + b*x + a)/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c^2)*e + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*f*x^3 + 48*b*c^3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f)*x)*sqrt(c*x^2 + b*x + a))/c^4]$

**giac [A]** time = 0.30, size = 212, normalized size = 1.21

$$\frac{1}{192} \sqrt{cx^2 + bx + a} \left( 2 \left( 4 \left( 6fx + \frac{bc^2 + 8c^3e}{c^3} \right) x + \frac{48c^3d - 5b^2cf + 12ac^2f + 8bc^2e}{c^3} \right) x + \frac{48bc^2d + 15b^3f - 52abcf - 24b^2ce + 64a^2e}{c^3} \right) + \frac{(16b^2c^2d - 64a^3d + 5b^4f - 24ab^2cf + 16a^2c^2f - 8b^3ce + 32abc^2e) \log \left( \left| -2 \left( \sqrt{cx^2 + bx + a} \right) \sqrt{c-b} \right| \right)}{128c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)\*(f\*x^2+e\*x+d),x, algorithm="giac")

[Out]  $1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (b*c^2*f + 8*c^3*e)/c^3)*x + (48*c^3*d - 5*b^2*c*f + 12*a*c^2*f + 8*b*c^2*e)/c^3)*x + (48*b*c^2*d + 15*b^3*f - 52*a*b*c*f - 24*b^2*c*e + 64*a*c^2*e)/c^3 + 1/128*(16*b^2*c^2*d - 64*a*c^3*d + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2*f - 8*b^3*c*e + 32*a*b*c^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)$

**maple [B]** time = 0.01, size = 453, normalized size = 2.59

$$\frac{d \ln \left( \frac{\sqrt{cx^2 + bx + a}}{c} \right)}{c^3} + \frac{5bf \ln \left( \frac{\sqrt{cx^2 + bx + a}}{c} \right)}{16c^3} + \frac{5bf \ln \left( \frac{\sqrt{cx^2 + bx + a}}{c} \right)}{16c^3} + \frac{5bf \ln \left( \frac{\sqrt{cx^2 + bx + a}}{c} \right)}{16c^3} + \frac{5bf \ln \left( \frac{\sqrt{cx^2 + bx + a}}{c} \right)}{16c^3} + \frac{5bf \ln \left( \frac{\sqrt{cx^2 + bx + a}}{c} \right)}{16c^3} + \frac{5bf \ln \left( \frac{\sqrt{cx^2 + bx + a}}{c} \right)}{16c^3} + \frac{5bf \ln \left( \frac{\sqrt{cx^2 + bx + a}}{c} \right)}{16c^3} + \frac{5bf \ln \left( \frac{\sqrt{cx^2 + bx + a}}{c} \right)}{16c^3} + \frac{5bf \ln \left( \frac{\sqrt{cx^2 + bx + a}}{c} \right)}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(1/2)\*(f\*x^2+e\*x+d),x)

[Out]  $1/4*f*x*(c*x^2+b*x+a)^(3/2)/c - 5/24*f*b/c^2*(c*x^2+b*x+a)^(3/2) + 5/32*f*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x + 5/64*f*b^3/c^3*(c*x^2+b*x+a)^(1/2) + 3/16*f*b^2/c^2*(5/2)*ln((c*x+1/2*b)/c^(1/2) + (c*x^2+b*x+a)^(1/2))*a - 5/128*f*b^4/c^(7/2)*ln((c*x+1/2*b)/c^(1/2) + (c*x^2+b*x+a)^(1/2)) - 1/8*f*a/c*(c*x^2+b*x+a)^(1/2)*x - 1/16*f*a/c^2*(c*x^2+b*x+a)^(1/2)*b - 1/8*f*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2) + (c*x^2+b*x+a)^(1/2)) + 1/3*e*(c*x^2+b*x+a)^(3/2)/c - 1/4*e*b/c*(c*x^2+b*x+a)^(1/2)*x - 1/8*e*b^2/c^2*(c*x^2+b*x+a)^(1/2) - 1/4*e*b/c^2*(3/2)*ln((c*x+1/2*b)/c^(1/2) + (c*x^2+b*x+a)^(1/2))*a + 1/16*e*b^3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2) + (c*x^2+b*x+a)^(1/2)) + 1/2*d*(c*x^2+b*x+a)^(1/2)*x + 1/4*d/c*(c*x^2+b*x+a)^(1/2)*b + 1/2*d/c^(1/2)*ln((c*x+1/2*b)/c^(1/2) + (c*x^2+b*x+a)^(1/2))*a - 1/8*d/c^(3/2)*ln((c*x+1/2*b)/c^(1/2) + (c*x^2+b*x+a)^(1/2))*b^2$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)\*(f\*x^2+e\*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is 4\*a\*c-b^2 positive, negative or zero?

**mupad [B]** time = 3.91, size = 320, normalized size = 1.83

$$d \left( \frac{x}{2} + \frac{b}{4c} \right) \sqrt{cx^2 + bx + a} - \frac{d \left( \frac{x}{2} + \frac{b}{4c} \right) \sqrt{cx^2 + bx + a} + \frac{d \left( \frac{x}{2} + \frac{b}{4c} \right) \sqrt{cx^2 + bx + a}}{2c}}{4c} + \frac{d \ln \left( \frac{\sqrt{cx^2 + bx + a}}{c} \right) (c - \frac{d}{c})}{2c^2} + \frac{e \ln \left( \frac{\sqrt{cx^2 + bx + a}}{c} \right) (b^3 - 4abd)}{16c^2} - \frac{5bf \left( \frac{\sqrt{cx^2 + bx + a}}{c} \right) (b^3 - 4abd)}{8c} + \frac{(-3b^2d + 2cxb + 8c(c^2 + a)) \sqrt{cx^2 + bx + a}}{24c^2} + \frac{f x (cx^2 + bx + a)^{3/2}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2),x)

```
[Out] d*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (a*f*((x/2 + b/(4*c))*(a + b*x
+ c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c -
b^2/4))/(2*c^(3/2))))/(4*c) + (d*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^
2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*log((b + 2*c*x)/c^(1/2) + 2*(a +
b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (5*b*f*((log((b + 2*c*x
)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*
c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c)
+ (e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)
+ (f*x*(a + b*x + c*x^2)^(3/2))/(4*c)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx + cx^2} (d + ex + fx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d), x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

$$3.99 \quad \int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$$

**Optimal.** Leaf size=431

$$\frac{\sqrt{f(2af - b(e - \sqrt{e^2 - 4df})) + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1} \left( \frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef}} \right)}{\sqrt{2}f\sqrt{e^2 - 4df}}$$

**Rubi [A]** time = 1.05, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {989, 621, 206, 1032, 724}

$$\frac{\sqrt{f(2af - b(e - \sqrt{e^2 - 4df})) + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1} \left( \frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef}} \right)}{\sqrt{2}f\sqrt{e^2 - 4df}} + \frac{\sqrt{f(2af - b(\sqrt{e^2 - 4df} + e)) + c(e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1} \left( \frac{4af + 2x(bf - c(\sqrt{e^2 - 4df} + e)) - b(\sqrt{e^2 - 4df} + e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef}} \right)}{\sqrt{2}f\sqrt{e^2 - 4df}} + \frac{\sqrt{c} \tanh^{-1} \left( \frac{bx}{2\sqrt{a+bx+cx^2}} \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]/(d + e\*x + f\*x^2), x]

[Out] (Sqrt[c]\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/f - (Sqrt[c\*(e^2 - 2\*d\*f - e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e - Sqrt[e^2 - 4\*d\*f]))]\*ArcTanh[(4\*a\*f - b\*(e - Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e - Sqrt[e^2 - 4\*d\*f]))\*x]/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 - (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2]))/(Sqrt[2]\*f\*Sqrt[e^2 - 4\*d\*f]) + (Sqrt[c\*(e^2 - 2\*d\*f + e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]))]\*ArcTanh[(4\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e + Sqrt[e^2 - 4\*d\*f]))\*x]/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 + (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2]))/(Sqrt[2]\*f\*Sqrt[e^2 - 4\*d\*f])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] :> Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] :> Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 989

Int[Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]/((d\_) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2), x\_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b\*x + c\*x^2], x], x] - Dist[1/f, Int[(c\*d - a\*f + (c\*e - b\*f)\*x)/(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0]

#### Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx = -\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{f} + \frac{c \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f}$$

$$= \frac{(2c) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right) - (2f(cd-af) - (ce-bf)(e - \sqrt{e^2-4df})) \int \frac{1}{(e-\sqrt{e^2-4df})\sqrt{a+bx+cx^2}} dx}{f}$$

$$= \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{(2(2f(cd-af) - (ce-bf)(e - \sqrt{e^2-4df}))) \text{Subst}\left(\int \frac{1}{16c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\sqrt{c}(e^2 - 2df - e\sqrt{e^2-4df}) + f(2af - b(e - \sqrt{e^2-4df}))}{\sqrt{2}f\sqrt{e^2-4df}}$$

**Mathematica [A]** time = 1.22, size = 417, normalized size = 0.97

$$\frac{\sqrt{f(2af - b(\sqrt{e^2-4df} + e)) + c(e\sqrt{e^2-4df} - 2df + e^2)} \tanh^{-1}\left(\frac{4af + (\sqrt{e^2-4df} + 2f) - 2c(\sqrt{e^2-4df} + e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{(2af - b(\sqrt{e^2-4df} + e)) + c(e\sqrt{e^2-4df} - 2df + e^2)}}\right) - \sqrt{f(2af + b\sqrt{e^2-4df} + b(-e) + c(-e\sqrt{e^2-4df} - 2df + e^2))} \tanh^{-1}\left(\frac{4af + (\sqrt{e^2-4df} + 2f) - 2c(\sqrt{e^2-4df} + e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{(2af + b\sqrt{e^2-4df} + b(-e) + c(-e\sqrt{e^2-4df} - 2df + e^2))}}\right) + \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}}{\sqrt{2}f\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2), x]
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/f + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])*Sqrt[a + x*(b + c*x)]] - Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])*Sqrt[a + x*(b + c*x)]])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
```

**IntegrateAlgebraic [C]** time = 0.60, size = 402, normalized size = 0.93

$$\frac{\text{RootSum}\left[\#1^4 f^3 - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + \#1^2 b e + 4\#1^2 c d + 2\#1 a \sqrt{c} e - 4\#1 b \sqrt{c} d + a^2 f - a b e + b^2 d e, \frac{\#1^2 \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}) + \#1^2 \log(-\#1 + \sqrt{a+bx+cx^2} + \sqrt{c}) - 2\#1^2 d \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}) + 2\#1^2 d \log(-\#1 + \sqrt{a+bx+cx^2} + \sqrt{c}) - a c \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}) - 2\#1 a \sqrt{c} \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{c})}{-2\#1^3 \sqrt{c}^2 \sqrt{e^2-4df} - \#1 b e - 4\#1 c d + 2\#1 a \sqrt{c} e}\right]}{f} - \frac{\sqrt{c} \log(-2\sqrt{c} f \sqrt{a+bx+cx^2} + b f + 2c f)}{f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2), x]
[Out] -((Sqrt[c]*Log[b*f + 2*c*f*x - 2*Sqrt[c]*f*Sqrt[a + b*x + c*x^2]])/f) + RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 +
```

$c*e*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 - b*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2)/(2*b*\text{Sqrt}[c]*d - a*\text{Sqrt}[c]*e - 4*c*d*\#1 - b*e*\#1 + 2*a*f*\#1 + 3*\text{Sqrt}[c]*e*\#1^2 - 2*f*\#1^3) \& ]/f$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d),x, algorithm="fricas")

[Out] Timed out

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [B] time = 0.05, size = 6019, normalized size = 13.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d),x)

[Out] result too large to display

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*d\*f-e^2>0)', see `assume?` for more details)Is 4\*d\*f-e^2 positive, negative or zero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x + c\*x^2)^(1/2)/(d + e\*x + f\*x^2),x)

[Out] int((a + b\*x + c\*x^2)^(1/2)/(d + e\*x + f\*x^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d), x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```



$$3.100 \quad \int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2} dx$$

**Optimal.** Leaf size=488

$$\frac{(f(be - 4af) - (e - \sqrt{e^2 - 4df})(ce - bf)) \tanh^{-1} \left( \frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} \right) (f(be - 4af) - (e - \sqrt{e^2 - 4df})(ce - bf))}{\sqrt{2}(e^2 - 4df)^{3/2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} +$$

**Rubi [A]** time = 2.93, antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, number of rules / integrand size = 0.148, Rules used = {971, 1032, 724, 206}

$$\frac{(f(be - 4af) - (e - \sqrt{e^2 - 4df})(ce - bf)) \tanh^{-1} \left( \frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} \right) (f(be - 4af) - (e - \sqrt{e^2 - 4df})(ce - bf))}{\sqrt{2}(e^2 - 4df)^{3/2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} + \frac{(f(be - 4af) - (\sqrt{e^2 - 4df} + e)(ce - bf)) \tanh^{-1} \left( \frac{4af + 2x(bf - c(\sqrt{e^2 - 4df} + e)) - b(\sqrt{e^2 - 4df} + e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 + \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} \right) (e + 2fx)\sqrt{a + bx + cx^2}}{\sqrt{2}(e^2 - 4df)^{3/2}\sqrt{2af^2 + \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} - \frac{(e + 2fx)\sqrt{a + bx + cx^2}}{(e^2 - 4df)(d + ex + fx^2)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]/(d + e\*x + f\*x^2)^2, x]

[Out] -(((e + 2\*f\*x)\*Sqrt[a + b\*x + c\*x^2])/((e^2 - 4\*d\*f)\*(d + e\*x + f\*x^2))) - ((f\*(b\*e - 4\*a\*f) - (c\*e - b\*f)\*(e - Sqrt[e^2 - 4\*d\*f]))\*ArcTanh[(4\*a\*f - b\*(e - Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e - Sqrt[e^2 - 4\*d\*f]))\*x]/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 - (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2]))/(Sqrt[2]\*(e^2 - 4\*d\*f)^(3/2)\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 - (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]) + ((f\*(b\*e - 4\*a\*f) - (c\*e - b\*f)\*(e + Sqrt[e^2 - 4\*d\*f]))\*ArcTanh[(4\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e + Sqrt[e^2 - 4\*d\*f]))\*x]/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 + (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2]))/(Sqrt[2]\*(e^2 - 4\*d\*f)^(3/2)\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 + (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]])

**Rule 206**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 724**

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

**Rule 971**

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/((b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[2\*c\*d\*(2\*p + 3) + b\*e\*q + (2\*b\*f\*q + 2\*c\*e\*(2\*p + q + 3))\*x + 2\*c\*f\*(2\*p + 2\*q + 3)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

**Rule 1032**

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dis

t[(2\*c\*g - h\*(b - q))/q, Int[1/((b - q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[(2\*c\*g - h\*(b + q))/q, Int[1/((b + q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && PosQ[b^2 - 4\*a\*c]

Rubi steps

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex + fx^2)^2} dx = -\frac{(e + 2fx)\sqrt{a + bx + cx^2}}{(e^2 - 4df)(d + ex + fx^2)} - \frac{\int \frac{\frac{1}{2}(be - 4af) + (ce - bf)x}{\sqrt{a + bx + cx^2}(d + ex + fx^2)} dx}{-e^2 + 4df}$$

$$= -\frac{(e + 2fx)\sqrt{a + bx + cx^2}}{(e^2 - 4df)(d + ex + fx^2)} - \frac{(ce(e - \sqrt{e^2 - 4df}) + f(4af - b(2e - \sqrt{e^2 - 4df}))) \int \frac{1}{(e^2 - 4df)^{3/2}}}{(e^2 - 4df)^{3/2}}$$

$$= -\frac{(e + 2fx)\sqrt{a + bx + cx^2}}{(e^2 - 4df)(d + ex + fx^2)} + \frac{(2(ce(e - \sqrt{e^2 - 4df}) + f(4af - b(2e - \sqrt{e^2 - 4df}))))}{(e^2 - 4df)^{3/2}}$$

$$= -\frac{(e + 2fx)\sqrt{a + bx + cx^2}}{(e^2 - 4df)(d + ex + fx^2)} + \frac{(ce(e - \sqrt{e^2 - 4df}) + f(4af - b(2e - \sqrt{e^2 - 4df}))) \operatorname{tanh}^{-1}\left(\frac{\sqrt{2}(e^2 - 4df)^{3/2} \sqrt{ce^2 - 2cdf - bef}}{\dots}\right)}{\sqrt{2}(e^2 - 4df)^{3/2} \sqrt{ce^2 - 2cdf - bef}}$$

**Mathematica [A]** time = 5.09, size = 555, normalized size = 1.14

$$\frac{4f(e + 2fx)\sqrt{a + bx + cx^2}}{(e^2 - 4df)(\sqrt{e^2 - 4df} - e - 2fx)(\sqrt{e^2 - 4df} + e + 2fx)} + \frac{(ce(\sqrt{e^2 - 4df} - e) - f(4af + b(\sqrt{e^2 - 4df} - 2))) \operatorname{tanh}^{-1}\left(\frac{-4af + b(\sqrt{e^2 - 4df} + 2) + 2c(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{ce^2 - 2cdf - bef}\sqrt{(2af + b(\sqrt{e^2 - 4df} - e)) + c(\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}(e^2 - 4df)^{3/2} \sqrt{f(2af + b(\sqrt{e^2 - 4df} - e)) + c(\sqrt{e^2 - 4df} - 2df + e^2)}} - \frac{(f(4af - b(\sqrt{e^2 - 4df} + 2)) + ce(\sqrt{e^2 - 4df} + e)) \operatorname{tanh}^{-1}\left(\frac{4af - b(\sqrt{e^2 - 4df} + 2) + 2c(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{ce^2 - 2cdf - bef}\sqrt{(2af - b(\sqrt{e^2 - 4df} + e)) + c(\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}(e^2 - 4df)^{3/2} \sqrt{f(2af - b(\sqrt{e^2 - 4df} + e)) + c(\sqrt{e^2 - 4df} - 2df + e^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x + c\*x^2]/(d + e\*x + f\*x^2)^2, x]

[Out] (4\*f\*(e + 2\*f\*x)\*Sqrt[a + x\*(b + c\*x)]/((e^2 - 4\*d\*f)\*(-e + Sqrt[e^2 - 4\*d\*f] - 2\*f\*x)\*(e + Sqrt[e^2 - 4\*d\*f] + 2\*f\*x)) + ((c\*e\*(-e + Sqrt[e^2 - 4\*d\*f]) - f\*(4\*a\*f + b\*(-2\*e + Sqrt[e^2 - 4\*d\*f]))) \* ArcTanh[(-4\*a\*f + 2\*c\*(e - Sqrt[e^2 - 4\*d\*f])\*x + b\*(e - Sqrt[e^2 - 4\*d\*f] - 2\*f\*x))/(2\*Sqrt[2]\*Sqrt[c\*(e^2 - 2\*d\*f - e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f + b\*(-e + Sqrt[e^2 - 4\*d\*f]))])\*Sqrt[a + x\*(b + c\*x)])/((Sqrt[2]\*(e^2 - 4\*d\*f)^(3/2)\*Sqrt[c\*(e^2 - 2\*d\*f - e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f + b\*(-e + Sqrt[e^2 - 4\*d\*f]))]) - ((c\*e\*(e + Sqrt[e^2 - 4\*d\*f]) + f\*(4\*a\*f - b\*(2\*e + Sqrt[e^2 - 4\*d\*f]))) \* ArcTanh[(4\*a\*f - 2\*c\*(e + Sqrt[e^2 - 4\*d\*f])\*x - b\*(e + Sqrt[e^2 - 4\*d\*f] - 2\*f\*x))/(2\*Sqrt[2]\*Sqrt[c\*(e^2 - 2\*d\*f + e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]))])\*Sqrt[a + x\*(b + c\*x)])/((Sqrt[2]\*(e^2 - 4\*d\*f)^(3/2)\*Sqrt[c\*(e^2 - 2\*d\*f + e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]))])

**IntegrateAlgebraic [C]** time = 2.55, size = 946, normalized size = 1.94



Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b\*x + c\*x^2]/(d + e\*x + f\*x^2)^2, x]

[Out] ((-e - 2\*f\*x)\*Sqrt[a + b\*x + c\*x^2])/((e^2 - 4\*d\*f)\*(d + e\*x + f\*x^2)) - (c\*RootSum[b^2\*d - a\*b\*e + a^2\*f - 4\*b\*Sqrt[c]\*d\*#1 + 2\*a\*Sqrt[c]\*e\*#1 + 4\*c\*

```

d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (4*c*e*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 5*b*f*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1] + 2*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1]*#1)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 -
3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/f^2 - RootSum[b^2*d - a*b*e + a^2*f - 4*b
*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*S
qrt[c]*e*#1^3 + f*#1^4 & , (8*c^2*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1] - 32*c^2*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 1
0*b*c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 40*b*c*d*f^2*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^2*e*f^2*Log[-(Sqrt[c]*x)
+ Sqrt[a + b*x + c*x^2] - #1] - 2*a*c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1] - 2*a*b*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
+ 4*c^(3/2)*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 16*c^
(3/2)*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]
*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 8*a*Sqrt[c]*f^3*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*c*e*f^2*Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 2*b*f^3*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1]*#1^2)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#
1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/(2*f^2*(-e^2 + 4*d*f))

```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [B] time = 0.04, size = 22287, normalized size = 45.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x)
```

```
[Out] result too large to display
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(f*x^2 + e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x + c\*x^2)^(1/2)/(d + e\*x + f\*x^2)^2,x)

[Out] int((a + b\*x + c\*x^2)^(1/2)/(d + e\*x + f\*x^2)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(1/2)/(f\*x\*\*2+e\*x+d)\*\*2,x)

[Out] Timed out

$$3.101 \quad \int (a + bx + cx^2)^{3/2} (d + ex + fx^2)^2 dx$$

**Optimal.** Leaf size=564

$$\frac{(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (16c^2(3a^2f^2 + 24abef + 14b^2(df + e^2)) - 72b^2cf(3af + 4be) - 128c^3(a + bx + cx^2))}{32768c^{13/2}}$$

**Rubi [A]** time = 0.93, antiderivative size = 564, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1661, 640, 612, 621, 206}

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2)^2,x]

[Out]  $-\frac{(b^2 - 4ac)(768c^4d^2 + 99b^4f^2 - 72b^2cf(4be + 3af) - 128c^3(6bde + a(e^2 + 2df)) + 16c^2(24abef + 3a^2f^2 + 14b^2(e^2 + 2df)))\sqrt{a + bx + cx^2}}{(16384c^6)} + \frac{(768c^4d^2 + 99b^4f^2 - 72b^2cf(4be + 3af) - 128c^3(6bde + a(e^2 + 2df)) + 16c^2(24abef + 3a^2f^2 + 14b^2(e^2 + 2df)))\sqrt{a + bx + cx^2}}{(6144c^5)} + \frac{(5376c^3de - 693b^3f^2 + 36b^2cf(56be + 31af) - 32c^2(48aef + 49b(e^2 + 2df)))\sqrt{a + bx + cx^2}}{(13440c^4)} + \frac{(99b^2f^2 - 12cf(24be + 7af) + 224c^2(e^2 + 2df))x\sqrt{a + bx + cx^2}}{(1344c^3)} + \frac{(f(32ce - 11bf)x^2\sqrt{a + bx + cx^2}}{(112c^2)} + \frac{(f^2x^3\sqrt{a + bx + cx^2}}{(8c)} + \frac{(b^2 - 4ac)^2(768c^4d^2 + 99b^4f^2 - 72b^2cf(4be + 3af) - 128c^3(6bde + a(e^2 + 2df)) + 16c^2(24abef + 3a^2f^2 + 14b^2(e^2 + 2df)))\text{ArcTanh}\left[\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right]}{(32768c^{13/2})}$

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661



$$\begin{aligned} & /2) + 256*c^{(5/2)}*(231*b^3 - 372*a*b*c - 330*b^2*c*x + 280*a*c^2*x)*(a + x \\ & (b + c*x))^{(5/2)} - 35*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(16*c^{(3/2)}*(b + 2 \\ & *c*x)*(a + x*(b + c*x))^{(3/2)} - 3*(b^2 - 4*a*c)*(2*sqrt[c]*(b + 2*c*x)*sqrt \\ & [a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x \\ & *(b + c*x)])))/c^{(11/2)})/(3440640*c) \end{aligned}$$

**IntegrateAlgebraic [B]** time = 7.79, size = 1210, normalized size = 2.15

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2)^2,x]

[Out] (sqrt[a + b\*x + c\*x^2]\*(-80640\*b^3\*c^4\*d^2 + 537600\*a\*b\*c^5\*d^2 + 80640\*b^4\*c^3\*d\*e - 537600\*a\*b^2\*c^4\*d\*e + 688128\*a^2\*c^5\*d\*e - 23520\*b^5\*c^2\*e^2 + 170240\*a\*b^3\*c^3\*e^2 - 290304\*a^2\*b\*c^4\*e^2 - 47040\*b^5\*c^2\*d\*f + 340480\*a\*b^3\*c^3\*d\*f - 580608\*a^2\*b\*c^4\*d\*f + 30240\*b^6\*c\*e\*f - 241920\*a\*b^4\*c^2\*e\*f + 526848\*a^2\*b^2\*c^3\*e\*f - 196608\*a^3\*c^4\*e\*f - 10395\*b^7\*f^2 + 91980\*a\*b^5\*c\*f^2 - 244944\*a^2\*b^3\*c^2\*f^2 + 176448\*a^3\*b\*c^3\*f^2 + 53760\*b^2\*c^5\*d^2\*x + 1075200\*a\*c^6\*d^2\*x - 53760\*b^3\*c^4\*d\*e\*x + 301056\*a\*b\*c^5\*d\*e\*x + 15680\*b^4\*c^3\*d\*f\*x - 96768\*a\*b^2\*c^4\*d\*f\*x + 107520\*a^2\*c^5\*d\*f\*x + 31360\*b^4\*c^3\*d\*f\*x - 193536\*a\*b^2\*c^4\*d\*f\*x + 215040\*a^2\*c^5\*d\*f\*x - 20160\*b^5\*c^2\*e\*f\*x + 139776\*a\*b^3\*c^3\*e\*f\*x - 224256\*a^2\*b\*c^4\*e\*f\*x + 6930\*b^6\*c\*f^2\*x - 53928\*a\*b^4\*c^2\*f^2\*x + 113376\*a^2\*b^2\*c^3\*f^2\*x - 40320\*a^3\*c^4\*f^2\*x + 645120\*b\*c^6\*d^2\*x^2 + 43008\*b^2\*c^5\*d\*e\*x^2 + 1376256\*a\*c^6\*d\*e\*x^2 - 12544\*b^3\*c^4\*d\*f\*x^2 + 64512\*a\*b\*c^5\*d\*f\*x^2 - 25088\*b^3\*c^4\*d\*f\*x^2 + 129024\*a\*b\*c^5\*d\*f\*x^2 + 16128\*b^4\*c^3\*e\*f\*x^2 - 95232\*a\*b^2\*c^4\*e\*f\*x^2 + 98304\*a^2\*c^5\*e\*f\*x^2 - 5544\*b^5\*c^2\*f^2\*x^2 + 37440\*a\*b^3\*c^3\*f^2\*x^2 - 57984\*a^2\*b\*c^4\*f^2\*x^2 + 430080\*c^7\*d^2\*x^3 + 946176\*b\*c^6\*d\*e\*x^3 + 10752\*b^2\*c^5\*d\*f\*x^3 + 501760\*a\*c^6\*d\*f\*x^3 + 21504\*b^2\*c^5\*d\*f\*x^3 + 1003520\*a\*c^6\*d\*f\*x^3 - 13824\*b^3\*c^4\*d\*f\*x^3 + 67584\*a\*b\*c^5\*d\*f\*x^3 + 4752\*b^4\*c^3\*f^2\*x^3 - 27264\*a\*b^2\*c^4\*f^2\*x^3 + 26880\*a^2\*c^5\*f^2\*x^3 + 688128\*c^7\*d\*e\*x^4 + 372736\*b\*c^6\*d\*f\*x^4 + 745472\*b\*c^6\*d\*f\*x^4 + 12288\*b^2\*c^5\*d\*f\*x^4 + 786432\*a\*c^6\*d\*f\*x^4 - 4224\*b^3\*c^4\*f^2\*x^4 + 19968\*a\*b\*c^5\*f^2\*x^4 + 286720\*c^7\*d\*f\*x^5 + 573440\*c^7\*d\*f\*x^5 + 614400\*b\*c^6\*d\*f\*x^5 + 3840\*b^2\*c^5\*d\*f\*x^5 + 322560\*a\*c^6\*d\*f\*x^5 + 491520\*c^7\*d\*f\*x^6 + 261120\*b\*c^6\*d\*f\*x^6 + 215040\*c^7\*d\*f\*x^7))/((1720320\*c^6) + ((-768\*b^4\*c^4\*d^2 + 6144\*a\*b^2\*c^5\*d^2 - 12288\*a^2\*c^6\*d^2 + 768\*b^5\*c^3\*d\*e - 6144\*a\*b^3\*c^4\*d\*e + 12288\*a^2\*b\*c^5\*d\*e - 224\*b^6\*c^2\*e^2 + 1920\*a\*b^4\*c^3\*e^2 - 4608\*a^2\*b^2\*c^4\*e^2 + 2048\*a^3\*c^5\*e^2 - 448\*b^6\*c^2\*d\*f + 3840\*a\*b^4\*c^3\*d\*f - 9216\*a^2\*b^2\*c^4\*d\*f + 4096\*a^3\*c^5\*d\*f + 288\*b^7\*c\*e\*f - 2688\*a\*b^5\*c^2\*e\*f + 7680\*a^2\*b^3\*c^3\*e\*f - 6144\*a^3\*b\*c^4\*e\*f - 99\*b^8\*f^2 + 1008\*a\*b^6\*c\*f^2 - 3360\*a^2\*b^4\*c^2\*f^2 + 3840\*a^3\*b^2\*c^3\*f^2 - 768\*a^4\*c^4\*f^2)\*Log[b + 2\*c\*x - 2\*sqrt[c]\*sqrt[a + b\*x + c\*x^2]])/(32768\*c^(13/2))

**fricas [B]** time = 1.94, size = 2179, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)^2,x, algorithm="fricas")

[Out] [1/6881280\*(105\*(768\*(b^4\*c^4 - 8\*a\*b^2\*c^5 + 16\*a^2\*c^6)\*d^2 - 768\*(b^5\*c^3 - 8\*a\*b^3\*c^4 + 16\*a^2\*b\*c^5)\*d\*e + 32\*(7\*b^6\*c^2 - 60\*a\*b^4\*c^3 + 144\*a^2\*b^2\*c^4 - 64\*a^3\*c^5)\*e^2 + 3\*(33\*b^8 - 336\*a\*b^6\*c + 1120\*a^2\*b^4\*c^2 - 1280\*a^3\*b^2\*c^3 + 256\*a^4\*c^4)\*f^2 + 32\*(2\*(7\*b^6\*c^2 - 60\*a\*b^4\*c^3 + 144\*a^2\*b^2\*c^4 - 64\*a^3\*c^5)\*d - 3\*(3\*b^7\*c - 28\*a\*b^5\*c^2 + 80\*a^2\*b^3\*c^3 - 64\*a^3\*b\*c^4)\*e)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) + 4\*(215040\*c^8\*f^2\*x^7 + 15360\*(32\*c^8\*e\*f + 17\*b\*c^7\*f^2)\*x^6 + 1280\*(224\*c^8\*e^2 + 3\*(b^2\*c^6 + 84\*a\*c^7)\*f

$$\begin{aligned} &^2 + 32*(14*c^8*d + 15*b*c^7*e)*f)*x^5 + 128*(5376*c^8*d*e + 2912*b*c^7*e^2 \\ &- 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2 + 32*(182*b*c^7*d + 3*(b^2*c^6 + 64*a*c^7) \\ &)*e)*f)*x^4 + 16*(26880*c^8*d^2 + 59136*b*c^7*d*e + 224*(3*b^2*c^6 + 140*a \\ &*c^7)*e^2 + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f^2 + 32*(14*(3*b^2 \\ &*c^6 + 140*a*c^7)*d - 3*(9*b^3*c^5 - 44*a*b*c^6)*e)*f)*x^3 - 26880*(3*b^3*c^5 \\ &- 20*a*b*c^6)*d^2 + 5376*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d*e \\ &- 224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*e^2 - 3*(3465*b^7*c - \\ &30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f^2 + 8*(80640*b*c^7 \\ &*d^2 + 5376*(b^2*c^6 + 32*a*c^7)*d*e - 224*(7*b^3*c^5 - 36*a*b*c^6)*e^2 - \\ &3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2 - 32*(14*(7*b^3*c^5 - \\ &36*a*b*c^6)*d - 3*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*e)*f)*x^2 - 3 \\ &2*(14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 3*(315*b^6*c^2 - 2 \\ &520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e)*f + 2*(26880*(b^2*c^6 + \\ &20*a*c^7)*d^2 - 5376*(5*b^3*c^5 - 28*a*b*c^6)*d*e + 224*(35*b^4*c^4 - 216* \\ &a*b^2*c^5 + 240*a^2*c^6)*e^2 + 3*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2 \\ &*b^2*c^4 - 6720*a^3*c^5)*f^2 + 32*(14*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2 \\ &*c^6)*d - 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e)*f)*x)*sqrt(c*x \\ &x^2 + b*x + a))/c^7, -1/3440640*(105*(768*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6) \\ &*d^2 - 768*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d*e + 32*(7*b^6*c^2 - \\ &60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e^2 + 3*(33*b^8 - 336*a*b^6*c \\ &+ 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f^2 + 32*(2*(7*b^6*c^2 \\ &- 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 3*(3*b^7*c - 28*a*b^5*c^2 \\ &+ 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + \\ &b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(215040*c^8*f^2* \\ &x^7 + 15360*(32*c^8*e*f + 17*b*c^7*f^2)*x^6 + 1280*(224*c^8*e^2 + 3*(b^2*c^6 \\ &+ 84*a*c^7)*f^2 + 32*(14*c^8*d + 15*b*c^7*e)*f)*x^5 + 128*(5376*c^8*d*e + \\ &2912*b*c^7*e^2 - 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2 + 32*(182*b*c^7*d + 3*(b^2 \\ &*c^6 + 64*a*c^7)*e)*f)*x^4 + 16*(26880*c^8*d^2 + 59136*b*c^7*d*e + 224*(3* \\ &b^2*c^6 + 140*a*c^7)*e^2 + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f^2 \\ &+ 32*(14*(3*b^2*c^6 + 140*a*c^7)*d - 3*(9*b^3*c^5 - 44*a*b*c^6)*e)*f)*x^3 \\ &- 26880*(3*b^3*c^5 - 20*a*b*c^6)*d^2 + 5376*(15*b^4*c^4 - 100*a*b^2*c^5 + 1 \\ &28*a^2*c^6)*d*e - 224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*e^2 - \\ &3*(3465*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f^2 \\ &+ 8*(80640*b*c^7*d^2 + 5376*(b^2*c^6 + 32*a*c^7)*d*e - 224*(7*b^3*c^5 - 36* \\ &a*b*c^6)*e^2 - 3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2 - 32*( \\ &14*(7*b^3*c^5 - 36*a*b*c^6)*d - 3*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6 \\ &)*e)*f)*x^2 - 32*(14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 3*( \\ &315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e)*f + 2*(2 \\ &6880*(b^2*c^6 + 20*a*c^7)*d^2 - 5376*(5*b^3*c^5 - 28*a*b*c^6)*d*e + 224*(35 \\ &*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*e^2 + 3*(1155*b^6*c^2 - 8988*a*b^4* \\ &c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f^2 + 32*(14*(35*b^4*c^4 - 216*a*b^2 \\ &*c^5 + 240*a^2*c^6)*d - 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e \\ &)*f)*x)*sqrt(c*x^2 + b*x + a))/c^7] \end{aligned}$$

**giac [B]** time = 0.60, size = 1150, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)^2,x, algorithm="giac")

[Out] 1/1720320\*sqrt(c\*x^2 + b\*x + a)\*(2\*(4\*(2\*(8\*(10\*(12\*(14\*c\*f^2\*x + (17\*b\*c^7 \\ \*f^2 + 32\*c^8\*f\*e)/c^7)\*x + (448\*c^8\*d\*f + 3\*b^2\*c^6\*f^2 + 252\*a\*c^7\*f^2 + \\ 480\*b\*c^7\*f\*e + 224\*c^8\*e^2)/c^7)\*x + (5824\*b\*c^7\*d\*f - 33\*b^3\*c^5\*f^2 + 15 \\ 6\*a\*b\*c^6\*f^2 + 5376\*c^8\*d\*e + 96\*b^2\*c^6\*f\*e + 6144\*a\*c^7\*f\*e + 2912\*b\*c^7 \\ \*e^2)/c^7)\*x + (26880\*c^8\*d^2 + 1344\*b^2\*c^6\*d\*f + 62720\*a\*c^7\*d\*f + 297\*b^ \\ 4\*c^4\*f^2 - 1704\*a\*b^2\*c^5\*f^2 + 1680\*a^2\*c^6\*f^2 + 59136\*b\*c^7\*d\*e - 864\*b \\ ^3\*c^5\*f\*e + 4224\*a\*b\*c^6\*f\*e + 672\*b^2\*c^6\*e^2 + 31360\*a\*c^7\*e^2)/c^7)\*x + \\ (80640\*b\*c^7\*d^2 - 3136\*b^3\*c^5\*d\*f + 16128\*a\*b\*c^6\*d\*f - 693\*b^5\*c^3\*f^2 \\ + 4680\*a\*b^3\*c^4\*f^2 - 7248\*a^2\*b\*c^5\*f^2 + 5376\*b^2\*c^6\*d\*e + 172032\*a\*c^7



$$\begin{aligned} & *d*e + 2016*b^4*c^4*f*e - 11904*a*b^2*c^5*f*e + 12288*a^2*c^6*f*e - 1568*b^3*c^5*e^2 + 8064*a*b*c^6*e^2)/c^7)*x + (26880*b^2*c^6*d^2 + 537600*a*c^7*d^2 + 15680*b^4*c^4*d*f - 96768*a*b^2*c^5*d*f + 107520*a^2*c^6*d*f + 3465*b^6*c^2*f^2 - 26964*a*b^4*c^3*f^2 + 56688*a^2*b^2*c^4*f^2 - 20160*a^3*c^5*f^2 - 26880*b^3*c^5*d*e + 150528*a*b*c^6*d*e - 10080*b^5*c^3*f*e + 69888*a*b^3*c^4*f*e - 112128*a^2*b*c^5*f*e + 7840*b^4*c^4*e^2 - 48384*a*b^2*c^5*e^2 + 53760*a^2*c^6*e^2)/c^7)*x - (80640*b^3*c^5*d^2 - 537600*a*b*c^6*d^2 + 47040*b^5*c^3*d*f - 340480*a*b^3*c^4*d*f + 580608*a^2*b*c^5*d*f + 10395*b^7*c*f^2 - 91980*a*b^5*c^2*f^2 + 244944*a^2*b^3*c^3*f^2 - 176448*a^3*b*c^4*f^2 - 80640*b^4*c^4*d*e + 537600*a*b^2*c^5*d*e - 688128*a^2*c^6*d*e - 30240*b^6*c^2*f*e + 241920*a*b^4*c^3*f*e - 526848*a^2*b^2*c^4*f*e + 196608*a^3*c^5*f*e + 23520*b^5*c^3*e^2 - 170240*a*b^3*c^4*e^2 + 290304*a^2*b*c^5*e^2)/c^7) - 1/32768*(768*b^4*c^4*d^2 - 6144*a*b^2*c^5*d^2 + 12288*a^2*c^6*d^2 + 448*b^6*c^2*d*f - 3840*a*b^4*c^3*d*f + 9216*a^2*b^2*c^4*d*f - 4096*a^3*c^5*d*f + 99*b^8*f^2 - 1008*a*b^6*c*f^2 + 3360*a^2*b^4*c^2*f^2 - 3840*a^3*b^2*c^3*f^2 + 768*a^4*c^4*f^2 - 768*b^5*c^3*d*e + 6144*a*b^3*c^4*d*e - 12288*a^2*b*c^5*d*e - 288*b^7*c*f*e + 2688*a*b^5*c^2*f*e - 7680*a^2*b^3*c^3*f*e + 6144*a^3*b*c^4*f*e + 224*b^6*c^2*e^2 - 1920*a*b^4*c^3*e^2 + 4608*a^2*b^2*c^4*e^2 - 2048*a^3*c^5*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2) \end{aligned}$$

**maple [B]** time = 0.02, size = 2458, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)^2,x)

[Out]  $\frac{1}{8}f^2x^3(c^2x^2+bx+a)^{5/2}/c+1/8d^2/c*(c^2x^2+bx+a)^{3/2}+3/8d^2*(c^2x^2+bx+a)^{1/2}+x^2a-3/64d^2/c^2*(c^2x^2+bx+a)^{1/2}+b^3+3/8d^2/c^{1/2}*\ln((c^2x^2+bx+a)/c^{1/2}+(c^2x^2+bx+a)^{1/2})*a^2+3/128d^2/c^{5/2}*\ln((c^2x^2+bx+a)/c^{1/2}+(c^2x^2+bx+a)^{1/2}))*b^4+2/5d*e*(c^2x^2+bx+a)^{5/2}/c+1/6*x*(c^2x^2+bx+a)^{5/2}/c^2-7/60*b/c^2*(c^2x^2+bx+a)^{5/2}*e^2+7/192*b^3/c^3*(c^2x^2+bx+a)^{3/2}*e^2-7/512*b^5/c^4*(c^2x^2+bx+a)^{1/2}*e^2+7/1024*b^6/c^{9/2}*\ln((c^2x^2+bx+a)/c^{1/2}+(c^2x^2+bx+a)^{1/2}))*e^2-1/16*a^3/c^{3/2}*\ln((c^2x^2+bx+a)/c^{1/2}+(c^2x^2+bx+a)^{1/2}))*e^2+3/128*f^2*a^4/c^{5/2}*\ln((c^2x^2+bx+a)/c^{1/2}+(c^2x^2+bx+a)^{1/2}))+99/32768*f^2*b^8/c^{13/2}*\ln((c^2x^2+bx+a)/c^{1/2}+(c^2x^2+bx+a)^{1/2}))-33/640*f^2*b^3/c^4*(c^2x^2+bx+a)^{5/2}+33/2048*f^2*b^5/c^5*(c^2x^2+bx+a)^{3/2}-99/16384*f^2*b^7/c^6*(c^2x^2+bx+a)^{1/2}-3/16*e*f*b^3/c^3*(c^2x^2+bx+a)^{1/2}+x^2a+1/8*e*f*b/c^2*a*x*(c^2x^2+bx+a)^{3/2}+3/16*e*f*b/c^2*a^2*(c^2x^2+bx+a)^{1/2}+x+1/4*b^2/c^2*(c^2x^2+bx+a)^{1/2}+x^2a*d*f-3/8*d*e*b/c*(c^2x^2+bx+a)^{1/2}+x^2a+3/16*d*e*b^3/c^{5/2}*\ln((c^2x^2+bx+a)/c^{1/2}+(c^2x^2+bx+a)^{1/2}))*a+1/16*e*f*b^2/c^3*a*(c^2x^2+bx+a)^{3/2}+3/32*e*f*b^2/c^3*a^2*(c^2x^2+bx+a)^{1/2}-3/14*e*f*b/c^2*x*(c^2x^2+bx+a)^{5/2}-3/32*e*f*b^3/c^3*x*(c^2x^2+bx+a)^{3/2}+9/256*e*f*b^5/c^4*(c^2x^2+bx+a)^{1/2}+x-3/32*e*f*b^4/c^4*(c^2x^2+bx+a)^{1/2}+a-15/64*e*f*b^3/c^{7/2}*\ln((c^2x^2+bx+a)/c^{1/2}+(c^2x^2+bx+a)^{1/2}))*a^2+21/256*e*f*b^5/c^{9/2}*\ln((c^2x^2+bx+a)/c^{1/2}+(c^2x^2+bx+a)^{1/2}))*a+3/16*e*f*b/c^{5/2}+a^3*\ln((c^2x^2+bx+a)/c^{1/2}+(c^2x^2+bx+a)^{1/2}))+7/48*b^2/c^2*x*(c^2x^2+bx+a)^{3/2}+d*f+1/8*b^2/c^2*(c^2x^2+bx+a)^{1/2}+x^2a*e^2-7/128*b^4/c^3*(c^2x^2+bx+a)^{1/2}+x*d*f-57/512*f^2*b^2/c^3*a^2*(c^2x^2+bx+a)^{1/2}+x+153/2048*f^2*b^4/c^4*(c^2x^2+bx+a)^{1/2}+x^2a-9/128*f^2*b^2/c^3*a*x*(c^2x^2+bx+a)^{3/2}+1/8*b^3/c^3*(c^2x^2+bx+a)^{1/2}+a*d*f+9/32*b^2/c^{5/2}*\ln((c^2x^2+bx+a)/c^{1/2}+(c^2x^2+bx+a)^{1/2}))*a^2*d*f-15/128*b^4/c^{7/2}*\ln((c^2x^2+bx+a)/c^{1/2}+(c^2x^2+bx+a)^{1/2}))*a*d*f-1/12*a/c*x*(c^2x^2+bx+a)^{3/2}+d*f-1/24*a/c^2*(c^2x^2+bx+a)^{3/2}+b*d*f-1/8*a^2/c*(c^2x^2+bx+a)^{1/2}+x*d*f-1/16*a^2/c^2*(c^2x^2+bx+a)^{1/2}+b*d*f-1/4*d*e*b/c*x*(c^2x^2+bx+a)^{3/2}+3/32*d*e*b^3/c^2*(c^2x^2+bx+a)^{1/2}+x-3/16*d*e*b^2/c^2*(c^2x^2+bx+a)^{1/2}+a-3/8*d*e*b/c^{3/2}*\ln((c^2x^2+bx+a)/c^{1/2}+(c^2x^2+bx+a)^{1/2}))*a^2+1/4*d^2*x*(c^2x^2+bx+a)^{3/2}+153/4096*f^2*b^5/c^5*(c^2x^2+bx+a)^{1/2}+a+93/1120*f^2*b/c^3*a*(c^2x^2+bx+a)^{5/2}-11/11$

```

2*f^2*b/c^2*x^2*(c*x^2+b*x+a)^(5/2)+105/1024*f^2*b^4/c^(9/2)*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-1/8*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))*d*f-7/30*b/c^2*(c*x^2+b*x+a)^(5/2)*d*f+7/96*b^2/c^2*x*(c
*x^2+b*x+a)^(3/2)*e^2+7/96*b^3/c^3*(c*x^2+b*x+a)^(3/2)*d*f-7/256*b^4/c^3*(c
*x^2+b*x+a)^(1/2)*x*e^2+1/16*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a*e^2-9/1024*e*f*b
^7/c^(11/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/35*e*f*a/c^2*(c*x
^2+b*x+a)^(5/2)+3/20*e*f*b^2/c^3*(c*x^2+b*x+a)^(5/2)-3/64*e*f*b^4/c^4*(c*x
^2+b*x+a)^(3/2)+9/512*e*f*b^6/c^5*(c*x^2+b*x+a)^(1/2)+2/7*e*f*x^2*(c*x^2+b*x
+a)^(5/2)/c-9/256*f^2*b^3/c^4*a*(c*x^2+b*x+a)^(3/2)-57/1024*f^2*b^3/c^4*a^2
*(c*x^2+b*x+a)^(1/2)+33/448*f^2*b^2/c^3*x*(c*x^2+b*x+a)^(5/2)-63/2048*f^2*b
^6/c^(11/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-15/128*f^2*b^2/c
^(7/2)*a^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/64*f^2*a^2/c^2*x*(c
*x^2+b*x+a)^(3/2)+1/128*f^2*a^2/c^3*(c*x^2+b*x+a)^(3/2)*b+3/128*f^2*a^3/c^2
*(c*x^2+b*x+a)^(1/2)*x+3/256*f^2*a^3/c^3*(c*x^2+b*x+a)^(1/2)*b-1/16*f^2*a/c
^2*x*(c*x^2+b*x+a)^(5/2)+33/1024*f^2*b^4/c^4*x*(c*x^2+b*x+a)^(3/2)-99/8192*
f^2*b^6/c^5*(c*x^2+b*x+a)^(1/2)*x-15/256*b^4/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*a*e^2+7/512*b^6/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*d*f-1/24*a/c*x*(c*x^2+b*x+a)^(3/2)*e^2-1/48*a/c^2*(c*x^2+b*x
+a)^(3/2)*b*e^2-1/16*a^2/c*(c*x^2+b*x+a)^(1/2)*x*e^2+1/3*x*(c*x^2+b*x+a)^(5
/2)/c*d*f-1/8*d*e*b^2/c^2*(c*x^2+b*x+a)^(3/2)+3/64*d*e*b^4/c^3*(c*x^2+b*x+a
)^(1/2)-3/128*d*e*b^5/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3
/32*d^2/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/16*d^2/c*(c*x^2+b*x+a)^(1/2)*b*a-3/16
*d^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a-1/32*a^2/c^2
*(c*x^2+b*x+a)^(1/2)*b*e^2-7/256*b^5/c^4*(c*x^2+b*x+a)^(1/2)*d*f+9/64*b^2/c
^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*e^2

```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (cx^2 + bx + a)^{3/2} (fx^2 + ex + d)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2)^{3/2} (d + ex + fx^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)**2, x)
```

### 3.102 $\int (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx$

**Optimal.** Leaf size=236

$$\frac{(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-4c(af + 3be) + 7b^2f + 24c^2d) (b^2 - 4ac) (b + 2cx)\sqrt{a + bx + cx^2} (-4c)}{1024c^{9/2} \cdot 512c^4}$$

**Rubi [A]** time = 0.23, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, number of rules / integrand size = 0.200, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)(a+bx+cx^2)^{3/2}(-4acf+7b^2f-12bc+24c^2d)}{192c^3} - \frac{(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}(-4c(af+3be)+7b^2f+24c^2d)}{512c^4} + \frac{(b^2-4ac)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+3be)+7b^2f+24c^2d)}{1024c^{9/2}} + \frac{(a+bx+cx^2)^{5/2}(12cx-7bf)}{60c^2} + \frac{fx(a+bx+cx^2)^{5/2}}{6c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2), x]

[Out] -((b^2 - 4\*a\*c)\*(24\*c^2\*d + 7\*b^2\*f - 4\*c\*(3\*b\*e + a\*f))\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(512\*c^4) + ((24\*c^2\*d - 12\*b\*c\*e + 7\*b^2\*f - 4\*a\*c\*f)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(3/2))/(192\*c^3) + ((12\*c\*e - 7\*b\*f)\*(a + b\*x + c\*x^2)^(5/2))/(60\*c^2) + (f\*x\*(a + b\*x + c\*x^2)^(5/2))/(6\*c) + ((b^2 - 4\*a\*c)^2\*(24\*c^2\*d + 7\*b^2\*f - 4\*c\*(3\*b\*e + a\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(1024\*c^(9/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx &= \frac{fx(a + bx + cx^2)^{5/2}}{6c} + \frac{\int (6cd - af + \frac{1}{2}(12ce - 7bf)x)(a + bx + cx^2)^{3/2}}{6c} \\
 &= \frac{(12ce - 7bf)(a + bx + cx^2)^{5/2}}{60c^2} + \frac{fx(a + bx + cx^2)^{5/2}}{6c} + \frac{(2c(6cd - af))}{192c^3} \\
 &= \frac{(24c^2d - 12bce + 7b^2f - 4acf)(b + 2cx)(a + bx + cx^2)^{3/2}}{192c^3} + \frac{(12ce - 7b)}{192c^3} \\
 &= -\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \dots \\
 &= -\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \dots \\
 &= -\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \dots
 \end{aligned}$$

**Mathematica [A]** time = 0.61, size = 392, normalized size = 1.66

$$\frac{1920d^2 - 4a \left( (b^2 - 4ac) \operatorname{tanh}^{-1} \left( \frac{bx + 2c}{\sqrt{a + bx + cx^2}} \right) - 2\sqrt{b + 2cx} \sqrt{a + bx + cx^2} \right) - 60bc \left( \frac{(b^2 - 4ac) \operatorname{tanh}^{-1} \left( \frac{bx + 2c}{\sqrt{a + bx + cx^2}} \right) - 2\sqrt{b + 2cx} \sqrt{a + bx + cx^2}}{2c} + \frac{162b^2c + (b + 2cx)^2}{4c} \right) + \frac{\int \left( \frac{(b^2 - 4ac) \operatorname{tanh}^{-1} \left( \frac{bx + 2c}{\sqrt{a + bx + cx^2}} \right) + \sqrt{b + 2cx} \sqrt{a + bx + cx^2}}{2c} \right) \sqrt{a + bx + cx^2}}{15360c} + 1920d(b + 2cx)(a + bx + cx)^{3/2} + 3072e(a + bx + cx)^{5/2} + 2560f(a + bx + cx)^{5/2}}{15360c}$$

Antiderivative was successfully verified.

```

[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x]
[Out] (1920*d*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 3072*e*(a + x*(b + c*x))^(5/2)
) + 2560*f*x*(a + x*(b + c*x))^(5/2) + (360*(b^2 - 4*a*c)*d*(-2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - 60*b*e*((16*(b + 2*c*x)*(a + x*(b +
c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c
*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)
]))/c^(5/2)) + (f*(-1792*b*(a + x*(b + c*x))^(5/2) + 5*(7*b^2 - 4*a*c)*((16*
(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b +
2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + x*(b + c*x)])]))/c^(5/2))))/c/(15360*c)
    
```

**IntegrateAlgebraic [A]** time = 1.70, size = 408, normalized size = 1.73

$$\frac{\sqrt{c} \sqrt{a + bx + cx^2} (-1296b^3c^2d + 2400a^2b^3c^2d + 180b^4c^2e - 1200a^2b^2c^2e + 1536a^2c^3e - 105b^5f + 760a^2b^3c^2f - 1296a^2b^2c^2f + 240b^2c^3d^2x + 4800a^2c^4d^2x - 120b^3c^2e^2x + 672a^2b^3c^3e^2x + 70b^4c^4f^2x - 432a^2b^2c^2f^2x + 480a^2c^3f^2x + 2880b^2c^4d^2x^2 + 96b^2c^3e^2x^2 + 3072a^2c^4e^2x^2 - 56b^3c^2f^2x^2 + 288a^2b^2c^3f^2x^2 + 1920c^5d^2x^3 + 2112b^2c^4e^2x^3 + 48b^2c^3f^2x^3 + 2240a^2c^4f^2x^3 + 1536c^5e^2x^4 + 1664b^2c^4f^2x^4 + 1280c^5f^2x^5)/(7680c^4) + ((-24b^4c^2d + 192a^2b^2c^3d - 384a^2c^4d + 12b^5c^2e - 96a^2b^3c^2e + 192a^2b^2c^3e - 7b^6f + 60a^2b^4c^2f - 144a^2b^2c^2f + 64a^3c^3f)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(1024*c^(9/2))$$

Antiderivative was successfully verified.

```

[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x]
[Out] (Sqrt[a + b*x + c*x^2]*(-360*b^3*c^2*d + 2400*a*b*c^3*d + 180*b^4*c^2*e - 120
0*a*b^2*c^2*e + 1536*a^2*c^3*e - 105*b^5*f + 760*a*b^3*c^2*f - 1296*a^2*b^2*c^2
*f + 240*b^2*c^3*d^2*x + 4800*a^2*c^4*d^2*x - 120*b^3*c^2*e^2*x + 672*a*b^3*c^3
e^2*x + 70*b^4*c^4*f^2*x - 432*a*b^2*c^2*f^2*x + 480*a^2*c^3*f^2*x + 2880*b^2
c^4*d^2*x^2 + 96*b^2*c^3*e^2*x^2 + 3072*a^2*c^4*e^2*x^2 - 56*b^3*c^2*f^2*x^2 +
288*a*b^2*c^3*f^2*x^2 + 1920*c^5*d^2*x^3 + 2112*b^2*c^4*e^2*x^3 + 48*b^2*c^3
f^2*x^3 + 2240*a^2*c^4*f^2*x^3 + 1536*c^5*e^2*x^4 + 1664*b^2*c^4*f^2*x^4 +
1280*c^5*f^2*x^5)/(7680*c^4) + ((-24*b^4*c^2*d + 192*a^2*b^2*c^3*d - 384*a^2
c^4*d + 12*b^5*c^2*e - 96*a*b^3*c^2*e + 192*a^2*b^2*c^3*e - 7*b^6*f + 60*a^2
b^4*c^2*f - 144*a^2*b^2*c^2*f + 64*a^3*c^3*f)*Log[b + 2*c*x - 2*Sqrt[c]*Sqr
t[a + b*x + c*x^2]])/(1024*c^(9/2))
    
```

**fricas** [A] time = 0.84, size = 839, normalized size = 3.56

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="fricas")

[Out] [-1/30720\*(15\*(24\*(b^4\*c^2 - 8\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*d - 12\*(b^5\*c - 8\*a\*b^3\*c^2 + 16\*a^2\*b\*c^3)\*e + (7\*b^6 - 60\*a\*b^4\*c + 144\*a^2\*b^2\*c^2 - 64\*a^3\*c^3)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 + 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(1280\*c^6\*f\*x^5 + 128\*(12\*c^6\*e + 13\*b\*c^5\*f)\*x^4 + 16\*(120\*c^6\*d + 132\*b\*c^5\*e + (3\*b^2\*c^4 + 140\*a\*c^5)\*f)\*x^3 + 8\*(360\*b\*c^5\*d + 12\*(b^2\*c^4 + 32\*a\*c^5)\*e - (7\*b^3\*c^3 - 36\*a\*b\*c^4)\*f)\*x^2 - 120\*(3\*b^3\*c^3 - 20\*a\*b\*c^4)\*d + 12\*(15\*b^4\*c^2 - 100\*a\*b^2\*c^3 + 128\*a^2\*c^4)\*e - (105\*b^5\*c - 760\*a\*b^3\*c^2 + 1296\*a^2\*b\*c^3)\*f + 2\*(120\*(b^2\*c^4 + 20\*a\*c^5)\*d - 12\*(5\*b^3\*c^3 - 28\*a\*b\*c^4)\*e + (35\*b^4\*c^2 - 216\*a\*b^2\*c^3 + 240\*a^2\*c^4)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^5, -1/15360\*(15\*(24\*(b^4\*c^2 - 8\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*d - 12\*(b^5\*c - 8\*a\*b^3\*c^2 + 16\*a^2\*b\*c^3)\*e + (7\*b^6 - 60\*a\*b^4\*c + 144\*a^2\*b^2\*c^2 - 64\*a^3\*c^3)\*f)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) - 2\*(1280\*c^6\*f\*x^5 + 128\*(12\*c^6\*e + 13\*b\*c^5\*f)\*x^4 + 16\*(120\*c^6\*d + 132\*b\*c^5\*e + (3\*b^2\*c^4 + 140\*a\*c^5)\*f)\*x^3 + 8\*(360\*b\*c^5\*d + 12\*(b^2\*c^4 + 32\*a\*c^5)\*e - (7\*b^3\*c^3 - 36\*a\*b\*c^4)\*f)\*x^2 - 120\*(3\*b^3\*c^3 - 20\*a\*b\*c^4)\*d + 12\*(15\*b^4\*c^2 - 100\*a\*b^2\*c^3 + 128\*a^2\*c^4)\*e - (105\*b^5\*c - 760\*a\*b^3\*c^2 + 1296\*a^2\*b\*c^3)\*f + 2\*(120\*(b^2\*c^4 + 20\*a\*c^5)\*d - 12\*(5\*b^3\*c^3 - 28\*a\*b\*c^4)\*e + (35\*b^4\*c^2 - 216\*a\*b^2\*c^3 + 240\*a^2\*c^4)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^5]

**giac** [A] time = 0.61, size = 417, normalized size = 1.77

$\frac{1}{320} \sqrt{c^2 + b^2} \left( \left( \left( \left( \frac{133b^7 + 12c^2}{120c^2 + 3b^2} + \frac{120c^2 + 3b^2}{120c^2 + 3b^2} + \frac{120c^2 + 3b^2}{120c^2 + 3b^2} \right) \right) \right) \right) \dots$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="giac")

[Out] 1/7680\*sqrt(c\*x^2 + b\*x + a)\*(2\*(4\*(2\*(8\*(10\*c\*f\*x + (13\*b\*c^5\*f + 12\*c^6\*e))/c^5)\*x + (120\*c^6\*d + 3\*b^2\*c^4\*f + 140\*a\*c^5\*f + 132\*b\*c^5\*e)/c^5)\*x + (360\*b\*c^5\*d - 7\*b^3\*c^3\*f + 36\*a\*b\*c^4\*f + 12\*b^2\*c^4\*e + 384\*a\*c^5\*e)/c^5)\*x + (120\*b^2\*c^4\*d + 2400\*a\*c^5\*d + 35\*b^4\*c^2\*f - 216\*a\*b^2\*c^3\*f + 240\*a^2\*c^4\*f - 60\*b^3\*c^3\*e + 336\*a\*b\*c^4\*e)/c^5)\*x - (360\*b^3\*c^3\*d - 2400\*a\*b\*c^4\*d + 105\*b^5\*c\*f - 760\*a\*b^3\*c^2\*f + 1296\*a^2\*b\*c^3\*f - 180\*b^4\*c^2\*e + 1200\*a\*b^2\*c^3\*e - 1536\*a^2\*c^4\*e)/c^5) - 1/1024\*(24\*b^4\*c^2\*d - 192\*a\*b^2\*c^3\*d + 384\*a^2\*c^4\*d + 7\*b^6\*f - 60\*a\*b^4\*c\*f + 144\*a^2\*b^2\*c^2\*f - 64\*a^3\*c^3\*f - 12\*b^5\*c\*e + 96\*a\*b^3\*c^2\*e - 192\*a^2\*b\*c^3\*e)\*log(abs(-2\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*sqrt(c) - b))/c^(9/2)

**maple** [B] time = 0.01, size = 862, normalized size = 3.65

$\frac{1}{320} \sqrt{c^2 + b^2} \left( \left( \left( \left( \frac{133b^7 + 12c^2}{120c^2 + 3b^2} + \frac{120c^2 + 3b^2}{120c^2 + 3b^2} + \frac{120c^2 + 3b^2}{120c^2 + 3b^2} \right) \right) \right) \right) \dots$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d),x)

[Out] 1/8\*f\*b^2/c^2\*(c\*x^2+b\*x+a)^(1/2)\*x\*a-3/16\*e\*b/c\*(c\*x^2+b\*x+a)^(1/2)\*x\*a+1/4\*d\*(c\*x^2+b\*x+a)^(3/2)\*x+1/5\*e\*(c\*x^2+b\*x+a)^(5/2)/c+1/6\*f\*x\*(c\*x^2+b\*x+a)^(5/2)/c-3/32\*d/c\*(c\*x^2+b\*x+a)^(1/2)\*x\*b^2-3/16\*e\*b/c^(3/2)\*ln((c\*x+1/2\*b)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))\*a^2+3/128\*d/c^(5/2)\*ln((c\*x+1/2\*b)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))\*b^4+1/8\*d/c\*(c\*x^2+b\*x+a)^(3/2)\*b-1/16\*e\*b^2/c^2\*(c\*x^2+b\*x+a)^(3/2)+3/128\*e\*b^4/c^3\*(c\*x^2+b\*x+a)^(1/2)-3/256\*e\*b^5/c^(7/2)\*ln((c

```

x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-7/60*f*b/c^2*(c*x^2+b*x+a)^(5/2)+7/19
2*f*b^3/c^3*(c*x^2+b*x+a)^(3/2)-7/512*f*b^5/c^4*(c*x^2+b*x+a)^(1/2)+7/1024*
f*b^6/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/16*f*a^3/c^(3/2)
)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/8*d/c^(1/2)*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/8*d*(c*x^2+b*x+a)^(1/2)*x*a-3/64*d/c^2*(
c*x^2+b*x+a)^(1/2)*b^3-1/24*f*a/c*(c*x^2+b*x+a)^(3/2)*x-1/48*f*a/c^2*(c*x^2
+b*x+a)^(3/2)*b-1/16*f*a^2/c*(c*x^2+b*x+a)^(1/2)*x+7/96*f*b^2/c^2*(c*x^2+b*
x+a)^(3/2)*x-7/256*f*b^4/c^3*(c*x^2+b*x+a)^(1/2)*x+1/16*f*b^3/c^3*(c*x^2+b*
x+a)^(1/2)*a+9/64*f*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a^2-15/256*f*b^4/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-3/1
6*d/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a+3/16*d/c*(c*x
^2+b*x+a)^(1/2)*b*a+3/64*e*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x-3/32*e*b^2/c^2*(c*
x^2+b*x+a)^(1/2)*a-1/8*e*b/c*(c*x^2+b*x+a)^(3/2)*x-1/32*f*a^2/c^2*(c*x^2+b*
x+a)^(1/2)*b+3/32*e*b^3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a

```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (cx^2 + bx + a)^{3/2} (fx^2 + ex + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

$$3.103 \quad \int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2} dx$$

**Optimal.** Leaf size=679

$$\frac{\left( (e - \sqrt{e^2 - 4df})(ce - bf)(f(be - 2af) - c(e^2 - 2df)) - 2f(-f^2(b^2d - a^2f) + 2cdf(be - af) + c^2(-d)(e^2 - 2df)) \right) \sqrt{2} f^3 \sqrt{e^2 - 4df} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cd}}{\dots}$$

**Rubi [A]** time = 11.03, antiderivative size = 678, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {977, 1076, 621, 206, 1032, 724}

$$\frac{\left( (e - \sqrt{e^2 - 4df})(ce - bf)(f(be - 2af) - c(e^2 - 2df)) - 2f(-f^2(b^2d - a^2f) + 2cdf(be - af) + c^2(-d)(e^2 - 2df)) \right) \sqrt{2} f^3 \sqrt{e^2 - 4df} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cd}}{\dots}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2), x]

[Out] -((4\*c\*e - 5\*b\*f - 2\*c\*f\*x)\*Sqrt[a + b\*x + c\*x^2])/(4\*f^2) + ((3\*b^2\*f^2 - 12\*c\*f\*(b\*e - a\*f) + 8\*c^2\*(e^2 - d\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2]])/(8\*Sqrt[c]\*f^3) + (((c\*e - b\*f)\*(e - Sqrt[e^2 - 4\*d\*f])\*(f\*(b\*e - 2\*a\*f) - c\*(e^2 - 2\*d\*f)) - 2\*f\*(2\*c\*d\*f\*(b\*e - a\*f) - f^2\*(b^2\*d - a^2\*f) - c^2\*d\*(e^2 - d\*f)))\*ArcTanh[(4\*a\*f - b\*(e - Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e - Sqrt[e^2 - 4\*d\*f]))\*x]/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 - (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2]))/(Sqrt[2]\*f^3\*Sqrt[e^2 - 4\*d\*f]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 - (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]) + (((4\*c\*d\*f^2\*(b\*e - a\*f) - 2\*f^3\*(b^2\*d - a^2\*f) - 2\*c^2\*d\*f\*(e^2 - d\*f) - (c\*e - b\*f)\*(e + Sqrt[e^2 - 4\*d\*f]))\*(f\*(b\*e - 2\*a\*f) - c\*(e^2 - 2\*d\*f)))\*ArcTanh[(4\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e + Sqrt[e^2 - 4\*d\*f]))\*x]/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 + (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2]))/(Sqrt[2]\*f^3\*Sqrt[e^2 - 4\*d\*f]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 + (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 977

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b\*f\*(3\*p + 2\*q) - c\*e\*(2\*p + q) + 2\*c\*f\*(p + q)\*x)\*(a + b\*x + c\*x^2)^(p - 1)\*(d + e\*x + f\*x^2)^(q + 1))/(2\*f^2\*(p + q)

```

*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*
x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q* Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)
*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
+ c*(2*d*f - e^2*(2*p + q)))] + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q)
- (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(
2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1)))]*x + ((c*e - b*f)^2*(1 - p)*p + c*(p
+ q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
- 1)))]*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*
q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

```

Rule 1032

```

Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

```

Rule 1076

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx + cx^2)^{3/2}}{d + ex + fx^2} dx &= -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} - \frac{\int \frac{\frac{1}{4}(-4bcde + 5b^2df + 4af(cd - 2af)) - \frac{1}{4}(8c^2de - 4acef - bf(5be - 16e^2))}{\sqrt{a + bx + cx^2}} dx}{2} \\
 &= -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} - \frac{\int \frac{\frac{1}{4}f(-4bcde + 5b^2df + 4af(cd - 2af)) - \frac{1}{4}d(-3b^2f^2 + 12cf(be - af))}{\sqrt{a + bx + cx^2}} dx}{2} \\
 &= -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} + \frac{(3b^2f^2 - 12cf(be - af) + 8c^2(e^2 - df)) \text{Subst}\left(\int \frac{1}{\sqrt{u}} du, u = a + bx + cx^2\right)}{4f^3} \\
 &= -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} + \frac{(3b^2f^2 - 12cf(be - af) + 8c^2(e^2 - df)) \tanh^{-1}\left(\frac{\sqrt{a + bx + cx^2}}{\sqrt{c}}\right)}{8\sqrt{c}f^3} \\
 &= -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} + \frac{(3b^2f^2 - 12cf(be - af) + 8c^2(e^2 - df)) \tanh^{-1}\left(\frac{\sqrt{a + bx + cx^2}}{\sqrt{c}}\right)}{8\sqrt{c}f^3}
 \end{aligned}$$

**Mathematica [A]** time = 4.58, size = 1232, normalized size = 1.81



Warning: Unable to verify antiderivative.



[In] Integrate[(a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2),x]

[Out] 
$$\begin{aligned} &((-2*b^2*f^2 - 16*a*c*f^2 + 10*b*c*f*(e + \sqrt{e^2 - 4*d*f})) - 4*c^2*(e + \sqrt{e^2 - 4*d*f}))^2 - 4*b*c*f^2*x + 4*c^2*f*(e + \sqrt{e^2 - 4*d*f})*x)*\sqrt{a + x*(b + c*x)} \\ &+ 2*\sqrt{a + x*(b + c*x)}*(b^2*f^2 - 2*c^2*(-2*e^2 + 4*d*f + 2*e*\sqrt{e^2 - 4*d*f} + e*f*x - f*\sqrt{e^2 - 4*d*f})*x) + c*f*(8*a*f + b*(-5*e + 5*\sqrt{e^2 - 4*d*f} + 2*f*x)) \\ &- ((b*f + c*(-e + \sqrt{e^2 - 4*d*f}))*(b^2*f^2 + 4*c^2*(-e^2 + 2*d*f + e*\sqrt{e^2 - 4*d*f})) - 4*c*f*(3*a*f + b*(-e + \sqrt{e^2 - 4*d*f}))) \\ &)*\text{ArcTanh}[(b + 2*c*x)/(2*\sqrt{c}*\sqrt{a + x*(b + c*x)})]/(\sqrt{c}*f) + ((-(b*f) + c*(e + \sqrt{e^2 - 4*d*f}))*(-(b^2*f^2) + 4*c^2*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})) - 4*c*f*(-3*a*f + b*(e + \sqrt{e^2 - 4*d*f}))) \\ &)*\text{ArcTanh}[(b + 2*c*x)/(2*\sqrt{c}*\sqrt{a + x*(b + c*x)})]/(\sqrt{c}*f) + (8*\sqrt{2}*c*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*\sqrt{e^2 - 4*d*f}) - 2*d*e*f*\sqrt{e^2 - 4*d*f})) + f^2*(2*a^2*f^2 - 2*a*b*f*(e + \sqrt{e^2 - 4*d*f}) + b^2*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})) + 2*c*f*(a*f*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}) - b*(e^3 - 3*d*e*f + e^2*\sqrt{e^2 - 4*d*f}) - d*f*\sqrt{e^2 - 4*d*f})) \\ &)*\text{ArcTanh}[(4*a*f - 2*c*(e + \sqrt{e^2 - 4*d*f})*x - b*(e + \sqrt{e^2 - 4*d*f} - 2*f*x))/(2*\sqrt{2}*\sqrt{c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e + \sqrt{e^2 - 4*d*f}))})*\sqrt{a + x*(b + c*x)})]/(f*\sqrt{c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f - b*(e + \sqrt{e^2 - 4*d*f}))}) + (8*\sqrt{2}*c*(c^2*(-e^4 + 4*d*e^2*f - 2*d^2*f^2 + e^3*\sqrt{e^2 - 4*d*f}) - 2*d*e*f*\sqrt{e^2 - 4*d*f})) + f^2*(-2*a^2*f^2 + 2*a*b*f*(e - \sqrt{e^2 - 4*d*f}) + b^2*(-e^2 + 2*d*f + e*\sqrt{e^2 - 4*d*f})) + 2*c*f*(a*f*(-e^2 + 2*d*f + e*\sqrt{e^2 - 4*d*f}) + b*(e^3 - 3*d*e*f - e^2*\sqrt{e^2 - 4*d*f}) + d*f*\sqrt{e^2 - 4*d*f})) \\ &)*\text{ArcTanh}[(4*a*f + 2*c*(-e + \sqrt{e^2 - 4*d*f})*x + b*(-e + \sqrt{e^2 - 4*d*f} + 2*f*x))/(2*\sqrt{2}*\sqrt{c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f + b*(-e + \sqrt{e^2 - 4*d*f}))})*\sqrt{a + x*(b + c*x)})]/(f*\sqrt{c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f}) + f*(2*a*f + b*(-e + \sqrt{e^2 - 4*d*f}))}) \\ &))/((16*c*f^2*\sqrt{e^2 - 4*d*f}) \end{aligned}$$

**IntegrateAlgebraic [C]** time = 1.67, size = 1160, normalized size = 1.71

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2),x]

[Out] 
$$\begin{aligned} &((-4*c*e + 5*b*f + 2*c*f*x)*\sqrt{a + b*x + c*x^2})/(4*f^2) + ((-8*c^2*e^2 + 8*c^2*d*f + 12*b*c*e*f - 3*b^2*f^2 - 12*a*c*f^2)*\text{Log}[b + 2*c*x - 2*\sqrt{c}*\sqrt{a + b*x + c*x^2}])/(8*\sqrt{c}*f^3) + \text{RootSum}[b^2*d - a*b*e + a^2*f - 4*b*\sqrt{c}*d*\#1 + 2*a*\sqrt{c}*e*\#1 + 4*c*d*\#1^2 + b*e*\#1^2 - 2*a*f*\#1^2 - 2*\sqrt{c}*e*\#1^3 + f*\#1^4 \& , (b*c^2*d*e^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] - a*c^2*e^3*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] - b*c^2*d^2*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] - 2*b^2*c*d*e*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] + 2*a*c^2*d*e*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] + 2*a*b*c*e^2*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] + b^3*d*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] - a*b^2*e*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] - 2*a^2*c*e*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] + a^2*b*f^3*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] - 2*c^(5/2)*d*e^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1 + 2*c^(5/2)*d^2*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1 + 4*b*c^(3/2)*d*e*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1 - 2*b^2*\sqrt{c}*d*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1 - 4*a*c^(3/2)*d*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1 + 2*a^2*\sqrt{c}*f^3*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1 + c^2*e^3*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1^2 - 2*c^2*d*e*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1^2 - 2*b*c*e^2*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1^2 + 2*b*c*d*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1^2 + b^2*e*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1^2 + 2*a*c*e*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1^2 \end{aligned}$$

$$\frac{b*x + c*x^2 - \#1*\#1^2 - 2*a*b*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1*\#1^2)/(2*b*\text{Sqrt}[c]*d - a*\text{Sqrt}[c]*e - 4*c*d*\#1 - b*e*\#1 + 2*a*f*\#1 + 3*\text{Sqrt}[c]*e*\#1^2 - 2*f*\#1^3) \& ]}{f^3}$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d),x, algorithm="fricas")

[Out] Timed out

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x)::OUTPUT:sym2poly/r2sym(const gen & e,const index\_m &  
i,const vecteur & l) Error: Bad Argument Value

**maple** [B] time = 0.03, size = 22523, normalized size = 33.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d),x)

[Out] result too large to display

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a  
dditional constraints; using the 'assume' command before evaluation \*may\* h  
elp (example of legal syntax is 'assume(4\*d\*f-e^2>0)', see `assume?` for mo  
re details)Is 4\*d\*f-e^2 positive, negative or zero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2),x)

[Out] int((a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)/(f\*x\*\*2+e\*x+d),x)

[Out] Timed out

$$3.104 \quad \int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2} dx$$

**Optimal.** Leaf size=704

$$\left( (e - \sqrt{e^2 - 4df})(ce - bf)(f(be - 2af) + 2c(e^2 - 5df)) - 2f(f(-be(3af + cd) + 4af(af + cd) + 2b^2df) + \right. \\ \left. 2\sqrt{2}f^2(e^2 - 4df)^{3/2}\sqrt{2af^2 - \sqrt{e^2 - 4df}}(ce - bf) - \right.$$

**Rubi [A]** time = 11.95, antiderivative size = 704, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, number of rules / integrand size = 0.259, Rules used = {971, 1066, 1076, 621, 206, 1032, 724}

$$\frac{((e - \sqrt{e^2 - 4df})(ce - bf)(f(be - 2af) + 2c(e^2 - 5df)) - 2f(f(-be(3af + cd) + 4af(af + cd) + 2b^2df) + 2\sqrt{2}f^2(e^2 - 4df)^{3/2}\sqrt{2af^2 - \sqrt{e^2 - 4df}}(ce - bf) - \dots)}{2\sqrt{2}f^2(e^2 - 4df)^{3/2}\sqrt{2af^2 - \sqrt{e^2 - 4df}}(ce - bf) - \dots}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^2,x]

[Out] -(((c\*e - 2\*b\*f - 2\*c\*f\*x)\*Sqrt[a + b\*x + c\*x^2])/(f\*(e^2 - 4\*d\*f))) - ((e + 2\*f\*x)\*(a + b\*x + c\*x^2)^(3/2))/((e^2 - 4\*d\*f)\*(d + e\*x + f\*x^2)) + (c^(3/2)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/f^2 - (((c\*e - b\*f)\*(f\*(b\*e - 2\*a\*f) + 2\*c\*(e^2 - 5\*d\*f))\*(e - Sqrt[e^2 - 4\*d\*f]) - 2\*f\*(2\*c^2\*d\*(e^2 - 4\*d\*f) + f\*(2\*b^2\*d\*f + 4\*a\*f\*(c\*d + a\*f) - b\*e\*(c\*d + 3\*a\*f))) \* ArcTanh[(4\*a\*f - b\*(e - Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e - Sqrt[e^2 - 4\*d\*f]))\*x]/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 - (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2]))/(2\*Sqrt[2]\*f^2\*(e^2 - 4\*d\*f)^(3/2)\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 - (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]) + (((c\*e - b\*f)\*(f\*(b\*e - 2\*a\*f) + 2\*c\*(e^2 - 5\*d\*f))\*(e + Sqrt[e^2 - 4\*d\*f]) - 2\*f\*(2\*c^2\*d\*(e^2 - 4\*d\*f) + f\*(2\*b^2\*d\*f + 4\*a\*f\*(c\*d + a\*f) - b\*e\*(c\*d + 3\*a\*f))) \* ArcTanh[(4\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e + Sqrt[e^2 - 4\*d\*f]))\*x]/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 + (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2]))/(2\*Sqrt[2]\*f^2\*(e^2 - 4\*d\*f)^(3/2)\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 + (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]])

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 971

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e

```
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]
```

### Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

### Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

### Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2)^{3/2}}{(d + ex + fx^2)^2} dx &= \frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{(e^2 - 4df)(d + ex + fx^2)} - \frac{\int \frac{\sqrt{a+bx+cx^2} \left( \frac{1}{2}(3be-4af) + (3ce+bf)x + 4cfx^2 \right)}{d+ex+fx^2} dx}{-e^2 + 4df} \\
&= -\frac{(ce - 2bf - 2cfx)\sqrt{a + bx + cx^2}}{f(e^2 - 4df)} - \frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{(e^2 - 4df)(d + ex + fx^2)} - \frac{\int \frac{cf(2b^2df + 4af(cd + ex + fx^2))}{(d + ex + fx^2)^2} dx}{f^2} \\
&= -\frac{(ce - 2bf - 2cfx)\sqrt{a + bx + cx^2}}{f(e^2 - 4df)} - \frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{(e^2 - 4df)(d + ex + fx^2)} + \frac{c^2 \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f^2} \\
&= -\frac{(ce - 2bf - 2cfx)\sqrt{a + bx + cx^2}}{f(e^2 - 4df)} - \frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{(e^2 - 4df)(d + ex + fx^2)} + \frac{(2c^2) \text{Subst} \left( \int \frac{1}{\sqrt{a+bx+cx^2}} dx \right)}{f^2} \\
&= -\frac{(ce - 2bf - 2cfx)\sqrt{a + bx + cx^2}}{f(e^2 - 4df)} - \frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{(e^2 - 4df)(d + ex + fx^2)} + \frac{c^{3/2} \tanh^{-1} \left( \frac{\sqrt{a+bx+cx^2}}{2\sqrt{c}} \right)}{f^2} \\
&= -\frac{(ce - 2bf - 2cfx)\sqrt{a + bx + cx^2}}{f(e^2 - 4df)} - \frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{(e^2 - 4df)(d + ex + fx^2)} + \frac{c^{3/2} \tanh^{-1} \left( \frac{\sqrt{a+bx+cx^2}}{2\sqrt{c}} \right)}{f^2}
\end{aligned}$$

**Mathematica [B]** time = 6.83, size = 2843, normalized size = 4.04

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^2,x]

[Out] 
$$\begin{aligned}
&(-2*f*(a + x*(b + c*x))^{(3/2)})/((e^2 - 4*d*f)*(e - \text{Sqrt}[e^2 - 4*d*f] + 2*f*x)) - (2*f*(a + x*(b + c*x))^{(3/2)})/((e^2 - 4*d*f)*(e + \text{Sqrt}[e^2 - 4*d*f] + 2*f*x)) - (3*f*(a + x*(b + c*x))^{(3/2)}*((( -4*b*c*f - 2*c*(b*f + 2*c*(-e + \text{Sqrt}[e^2 - 4*d*f])) - 4*c^2*f*x)*\text{Sqrt}[a + b*x + c*x^2]))/(8*c*f^2) - ((2*\text{Sqrt}[c]*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]))))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/f + (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\text{Sqrt}[e^2 - 4*d*f] + b*f*\text{Sqrt}[e^2 - 4*d*f]]*(4*c*f*(8*a*b*f^2 - 3*b^2*f*(e - \text{Sqrt}[e^2 - 4*d*f]) - 4*a*c*f*(e - \text{Sqrt}[e^2 - 4*d*f]) + 4*b*c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) + 4*c*(-e + \text{Sqrt}[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]))))*\text{ArcTanh}[( -4*a*f - b*(-e + \text{Sqrt}[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + \text{Sqrt}[e^2 - 4*d*f])))*x]/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\text{Sqrt}[e^2 - 4*d*f] + b*f*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2]))/(f*(16*a*f^2 + 8*b*f*(-e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*(-e + \text{Sqrt}[e^2 - 4*d*f])^2))/(16*c*f^2)))/((e^2 - 4*d*f)*(a + b*x + c*x^2)^(3/2)) + (f*(a + x*(b + c*x))^{(3/2)}*((( -4*c*f*(4*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f])) - 2*(b*f - c*(e - \text{Sqrt}[e^2 - 4*d*f]))*(b*f + 2*c*(-e + \text{Sqrt}[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e - \text{Sqrt}[e^2 - 4*d*f])))*x)*\text{Sqrt}[a + b*x + c*x^2]))/(8*c*f^2) - ((-2*(b*f - c*(e - \text{Sqrt}[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]))))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/( \text{Sqrt}[c]*f) + (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\text{Sqrt}[e^2 - 4*d*f] + b*f*\text{Sqrt}[e^2 - 4*d*f]]*(-4*(-e + \text{Sqrt}[e^2 - 4*d*f])*(b*f - c*(e - \text{Sqrt}[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*
\end{aligned}$$

$$\begin{aligned} & \text{Sqrt}[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f])) + 4*f*(2*c*f \\ & * (4*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]))^2 - (e - \text{Sqrt}[e^2 - 4*d*f])*(b*f - c* \\ & (e - \text{Sqrt}[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e - \text{Sqrt}[e^2 - 4*d*f])) \\ & )*\text{ArcTanh}[(-4*a*f - b*(-e + \text{Sqrt}[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + \text{Sqrt}[e^2 \\ & 2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\text{Sqr} \\ & \text{t}[e^2 - 4*d*f] + b*f*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2))]/(f*(16*a*f \\ & ^2 + 8*b*f*(-e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*(-e + \text{Sqrt}[e^2 - 4*d*f])^2))/((16 \\ & *c*f^2))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) - (f*(a + x*(b + c* \\ & x))^(3/2)*(((4*c*f*(-4*a*f + b*(e + \text{Sqrt}[e^2 - 4*d*f])) + 2*(b*f - c*(e + \text{S} \\ & \text{qrt}[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f])) - 4*c*f*(b*f - c* \\ & (e + \text{Sqrt}[e^2 - 4*d*f]))*x)*\text{Sqrt}[a + b*x + c*x^2))/(8*c*f^2) - ((-2*(b*f - \\ & c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d* \\ & *f]) - 4*c*f*(3*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f])))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sq} \\ & \text{rt}[c]*\text{Sqrt}[a + b*x + c*x^2))]/(\text{Sqrt}[c]*f) - (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d* \\ & f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*(4*(e \\ & + \text{Sqrt}[e^2 - 4*d*f])*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 \\ & - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) \\ & )) + 4*f*(2*c*f*(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))^2 - (e + \text{Sqrt}[e^2 - 4*d* \\ & *f])*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e + \text{Sqrt}[e \\ & ^2 - 4*d*f])))*\text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) - (-2*b*f + 2*c* \\ & (e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f \\ & ^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2)) \\ & )/(f*(16*a*f^2 - 8*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*(e + \text{Sqrt}[e^2 - 4*d*f] \\ & )^2))/((16*c*f^2))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) + (3*f*(a \\ & + x*(b + c*x))^(3/2)*(((4*b*c*f - 2*c*(-(b*f) + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f] \\ & )) + 4*c^2*f*x)*\text{Sqrt}[a + b*x + c*x^2))/(8*c*f^2) - ((-2*\text{Sqrt}[c]*(b^2*f^2 + \\ & 4*c^2*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + \text{Sqrt}[e^2 - \\ & 4*d*f])))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2)))/f - (2*\text{Sq} \\ & \text{rt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f* \\ & \text{Sqrt}[e^2 - 4*d*f]]*(4*c*(e + \text{Sqrt}[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d* \\ & *f + e*\text{Sqrt}[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f])) + 4*c* \\ & f*(3*b^2*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*a*c*f*(e + \text{Sqrt}[e^2 - 4*d*f]) - 4*b* \\ & (2*a*f^2 + c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])))*\text{ArcTanh}[(4*a*f - b*(e + \\ & \text{Sqrt}[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]* \\ & \text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e \\ & ^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2)))/(f*(16*a*f^2 - 8*b*f*(e + \text{Sqrt}[e^2 - \\ & 4*d*f]) + 4*c*(e + \text{Sqrt}[e^2 - 4*d*f])^2))/((16*c*f^2))/((e^2 - 4*d*f)*(a + \\ & b*x + c*x^2)^(3/2)) \end{aligned}$$

**IntegrateAlgebraic [C]** time = 5.18, size = 2427, normalized size = 3.45

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^2,x]

[Out] ((c\*d\*e - 2\*b\*d\*f + a\*e\*f + c\*e^2\*x - 2\*c\*d\*f\*x - b\*e\*f\*x + 2\*a\*f^2\*x)\*\text{Sqrt}[a + b\*x + c\*x^2))/(f\*(-e^2 + 4\*d\*f)\*(d + e\*x + f\*x^2)) - (c^(3/2)\*\text{Log}[b + 2\*c\*x - 2\*\text{Sqrt}[c]\*\text{Sqrt}[a + b\*x + c\*x^2)])/f^2 - \text{RootSum}[b^2\*d - a\*b\*e + a^2\*f - 4\*b\*\text{Sqrt}[c]\*d\*#1 + 2\*a\*\text{Sqrt}[c]\*e\*#1 + 4\*c\*d\*#1^2 + b\*e\*#1^2 - 2\*a\*f\*#1^2 - 2\*\text{Sqrt}[c]\*e\*#1^3 + f\*#1^4 & , (4\*c^3\*e^3\*\text{Log}[-(\text{Sqrt}[c]\*x) + \text{Sqrt}[a + b\*x + c\*x^2] - #1] - 8\*c^3\*d\*e\*f\*\text{Log}[-(\text{Sqrt}[c]\*x) + \text{Sqrt}[a + b\*x + c\*x^2] - #1] - 9\*b\*c^2\*e^2\*f\*\text{Log}[-(\text{Sqrt}[c]\*x) + \text{Sqrt}[a + b\*x + c\*x^2] - #1] + 10\*b\*c^2\*d\*f^2\*\text{Log}[-(\text{Sqrt}[c]\*x) + \text{Sqrt}[a + b\*x + c\*x^2] - #1] + 6\*b^2\*c\*e\*f^2\*\text{Log}[-(\text{Sqrt}[c]\*x) + \text{Sqrt}[a + b\*x + c\*x^2] - #1] + 6\*a\*c^2\*e\*f^2\*\text{Log}[-(\text{Sqrt}[c]\*x) + \text{Sqrt}[a + b\*x + c\*x^2] - #1] - b^3\*f^3\*\text{Log}[-(\text{Sqrt}[c]\*x) + \text{Sqrt}[a + b\*x + c\*x^2] - #1] - 8\*a\*b\*c\*f^3\*\text{Log}[-(\text{Sqrt}[c]\*x) + \text{Sqrt}[a + b\*x + c\*x^2] - #1] + 2\*c^(5/2)\*e^2\*f\*\text{Log}[-(\text{Sqrt}[c]\*x) + \text{Sqrt}[a + b\*x + c\*x^2] - #1]\*#1 - 4\*c^(5/2)\*d\*f^2\*\text{Log}[-(\text{Sqrt}[c]\*x) + \text{Sqrt}[a + b\*x + c\*x^2] - #1]\*#1 - 4\*b\*c^(3/2)\*e\*f^2\*\text{Log}[-(\text{Sqrt}[c]\*x) + \text{Sqrt}[a + b\*x + c\*x^2] - #1]\*#1 + 2\*b^2\*\text{Sqrt}[c]\*f^3

$$\begin{aligned} & * \text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 + 4*a*c^{(3/2)}*f^3*\text{Log}[-(\text{Sqrt}[c]*x) \\ & + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 + 2*c^2*e*f^2*\text{Log}[-(\text{Sqrt}[c]*x) \\ & + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 - 2*b*c*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + \\ & b*x + c*x^2] - \#1]*\#1^2)/(-2*b*\text{Sqrt}[c]*d + a*\text{Sqrt}[c]*e + 4*c*d*\#1 + b*e*\#1 \\ & - 2*a*f*\#1 - 3*\text{Sqrt}[c]*e*\#1^2 + 2*f*\#1^3) \& ]/f^4 - \text{RootSum}[b^2*d - a*b*e + \\ & a^2*f - 4*b*\text{Sqrt}[c]*d*\#1 + 2*a*\text{Sqrt}[c]*e*\#1 + 4*c*d*\#1^2 + b*e*\#1^2 - 2*a* \\ & f*\#1^2 - 2*\text{Sqrt}[c]*e*\#1^3 + f*\#1^4 \& , (8*c^3*e^5*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a \\ & + b*x + c*x^2] - \#1] - 48*c^3*d*e^3*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c* \\ & x^2] - \#1] - 18*b*c^2*e^4*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] \\ & + 64*c^3*d^2*e*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + 90*b*c^2 \\ & *d*e^2*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + 12*b^2*c*e^3*f \\ & ^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + 14*a*c^2*e^3*f^2*\text{Log}[-(\text{Sqrt}[c]*x) \\ & + \text{Sqrt}[a + b*x + c*x^2] - \#1] - 72*b*c^2*d^2*f^3*\text{Log}[-(\text{Sqrt}[c]*x) \\ & ) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - 47*b^2*c*d*e*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[ \\ & a + b*x + c*x^2] - \#1] - 58*a*c^2*d*e*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + \\ & c*x^2] - \#1] - 2*b^3*e^2*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1 \\ & ] - 17*a*b*c*e^2*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + 6*b^3 \\ & *d*f^4*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + 70*a*b*c*d*f^4*\text{Log} \\ & [-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + 2*a*b^2*e*f^4*\text{Log}[-(\text{Sqrt}[c]*x) \\ & + \text{Sqrt}[a + b*x + c*x^2] - \#1] - 2*a^2*c*e*f^4*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + \\ & b*x + c*x^2] - \#1] - 2*a^2*b*f^5*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \\ & \#1] + 4*c^{(5/2)}*e^4*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 - \\ & 20*c^{(5/2)}*d*e^2*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 - 8* \\ & b*c^{(3/2)}*e^3*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 + 16*c^{(5/2)} \\ & *d^2*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 + 30*b*c^{(3/2)} \\ & *d*e*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 + 4*b^2*\text{Sqrt}[ \\ & c]*e^2*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 + 8*a*c^{(3/2)}* \\ & e^2*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 - 12*b^2*\text{Sqrt}[c]* \\ & d*f^4*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 - 24*a*c^{(3/2)}*d*f^4 \\ & *\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 - 6*a*b*\text{Sqrt}[c]*e*f^4*L \\ & \text{og}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 + 8*a^2*\text{Sqrt}[c]*f^5*\text{Log}[-(\text{Sqrt}[c]*x) \\ & + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 + 2*c^2*e^3*f^2*\text{Log}[-(\text{Sqrt}[c]*x) \\ & ) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 - 6*c^2*d*e*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqr} \\ & \text{t}[a + b*x + c*x^2] - \#1]*\#1^2 - 3*b*c*e^2*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b \\ & *x + c*x^2] - \#1]*\#1^2 + 6*b*c*d*f^4*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^ \\ & 2] - \#1]*\#1^2 + b^2*e*f^4*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 \\ & ^2 + 2*a*c*e*f^4*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 - 2*a* \\ & b*f^5*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2)/(-2*b*\text{Sqrt}[c]*d \\ & + a*\text{Sqrt}[c]*e + 4*c*d*\#1 + b*e*\#1 - 2*a*f*\#1 - 3*\text{Sqrt}[c]*e*\#1^2 + 2*f*\#1^3) \\ & \& ]/(2*f^4*(-e^2 + 4*d*f)) \end{aligned}$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er

$\text{ror}\{\{-1, 0\} : [1, 0, \{-1, [1]\}]\}, [8, 4, 8, 0, 0, 0]\} + \{\{16, [1]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [8, 4, 6, 0, 1, 0]\} + \{\{-96, [2]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [8, 4, 4, 0, 2, 0]\} + \{\{256, [3]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [8, 4, 2, 0, 3, 0]\} + \{\{-256, [4]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [8, 4, 0, 0, 4, 0]\} + \{\{4, [1]\}\}, [7, 3, 8, 1, 0, 0]\} + \{\{-64, [2]\}\}, [7, 3, 6, 1, 1, 0]\} + \{\{384, [3]\}\}, [7, 3, 4, 1, 2, 0]\} + \{\{-1024, [4]\}\}, [7, 3, 2, 1, 3, 0]\} + \{\{1024, [5]\}\}, [7, 3, 0, 1, 4, 0]\} + \{\{4, 0\} : [1, 0, \{-1, [1]\}\}, [6, 4, 8, 0, 1, 0]\} + \{\{-64, [1]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 4, 6, 0, 2, 0]\} + \{\{-384, [2]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 4, 4, 0, 3, 0]\} + \{\{-1024, [3]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 4, 2, 0, 4, 0]\} + \{\{1024, [4]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 4, 0, 0, 5, 0]\} + \{\{-2, 0\} : [1, 0, \{-1, [1]\}\}, [6, 3, 9, 1, 0, 0]\} + \{\{-8, [1]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 3, 8, 0, 0, 1]\} + \{\{-32, [1]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 3, 7, 1, 1, 0]\} + \{\{-128, [2]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 3, 6, 0, 1, 1]\} + \{\{-192, [2]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 3, 5, 1, 2, 0]\} + \{\{-768, [3]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 3, 4, 0, 2, 1]\} + \{\{-512, [3]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 3, 3, 1, 3, 0]\} + \{\{-2048, [4]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 3, 2, 0, 3, 1]\} + \{\{-512, [4]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 3, 1, 1, 4, 0]\} + \{\{-2048, [5]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 3, 0, 0, 4, 1]\} + \{\{-4, [1]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 2, 8, 2, 0, 0]\} + \{\{-64, [2]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 2, 6, 2, 1, 0]\} + \{\{-384, [3]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 2, 4, 2, 2, 0]\} + \{\{1024, [4]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 2, 2, 2, 3, 0]\} + \{\{-1024, [5]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [6, 2, 0, 2, 4, 0]\} + \{\{8, [1]\}\}, [5, 3, 9, 0, 0, 1]\} + \{\{-12, [1]\}\}, [5, 3, 8, 1, 1, 0]\} + \{\{-128, [2]\}\}, [5, 3, 7, 0, 1, 1]\} + \{\{-192, [2]\}\}, [5, 3, 6, 1, 2, 0]\} + \{\{768, [3]\}\}, [5, 3, 5, 0, 2, 1]\} + \{\{-1152, [3]\}\}, [5, 3, 4, 1, 3, 0]\} + \{\{-2048, [4]\}\}, [5, 3, 3, 0, 3, 1]\} + \{\{3072, [4]\}\}, [5, 3, 2, 1, 4, 0]\} + \{\{2048, [5]\}\}, [5, 3, 1, 0, 4, 1]\} + \{\{-3072, [5]\}\}, [5, 3, 0, 1, 5, 0]\} + \{\{4, [1]\}\}, [5, 2, 9, 2, 0, 0]\} + \{\{16, [2]\}\}, [5, 2, 8, 1, 0, 1]\} + \{\{-64, [2]\}\}, [5, 2, 7, 2, 1, 0]\} + \{\{-256, [3]\}\}, [5, 2, 6, 1, 1, 1]\} + \{\{384, [3]\}\}, [5, 2, 5, 2, 2, 0]\} + \{\{1536, [4]\}\}, [5, 2, 4, 1, 2, 1]\} + \{\{-1024, [4]\}\}, [5, 2, 3, 2, 3, 0]\} + \{\{-4096, [5]\}\}, [5, 2, 2, 1, 3, 1]\} + \{\{1024, [5]\}\}, [5, 2, 1, 2, 4, 0]\} + \{\{4096, [6]\}\}, [5, 2, 0, 1, 4, 1]\} + \{\{-6, 0\} : [1, 0, \{-1, [1]\}\}, [4, 4, 8, 0, 2, 0]\} + \{\{96, [1]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 4, 6, 0, 3, 0]\} + \{\{-576, [2]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 4, 4, 0, 4, 0]\} + \{\{-1536, [3]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 4, 2, 0, 5, 0]\} + \{\{-1536, [4]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 4, 0, 0, 6, 0]\} + \{\{-2, 0\} : [1, 0, \{-1, [1]\}\}, [4, 3, 10, 0, 0, 1]\} + \{\{6, 0\} : [1, 0, \{-1, [1]\}\}, [4, 3, 9, 1, 1, 0]\} + \{\{48, [1]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 3, 8, 0, 1, 1]\} + \{\{-96, [1]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 3, 7, 1, 2, 0]\} + \{\{-448, [2]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 3, 6, 0, 2, 1]\} + \{\{576, [2]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 3, 5, 1, 3, 0]\} + \{\{2048, [3]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 3, 4, 0, 3, 1]\} + \{\{-1536, [3]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 3, 3, 1, 4, 0]\} + \{\{-4608, [4]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 3, 2, 0, 4, 1]\} + \{\{1536, [4]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 3, 1, 1, 5, 0]\} + \{\{-4096, [5]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 3, 0, 0, 5, 1]\} + \{\{-1, 0\} : [1, 0, \{-1, [1]\}\}, [4, 2, 10, 2, 0, 0]\} + \{\{-24, [1]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 2, 9, 1, 0, 1]\} + \{\{24, [1]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 2, 8, 2, 1, 0]\} + \{\{-16, [2]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 2, 8, 0, 0, 2]\} + \{\{384, [2]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 2, 7, 1, 1, 1]\} + \{\{-224, [2]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 2, 6, 2, 2, 0]\} + \{\{256, [3]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 2, 6, 0, 1, 2]\} + \{\{-2304, [3]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 2, 5, 1, 2, 1]\} + \{\{1024, [3]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 2, 4, 2, 3, 0]\} + \{\{-1536, [4]\}\}, 0 : [1, 0, \{-1, [1]\}\}, [4, 2, 4, 0, 2, 2]\} + \{\{$



$[6144, [4]] : [1, 0, -1, [1]]$ ,  $[4, 2, 3, 1, 3, 1] + [6144, [4]]$ ,  $[-2304, [4]] : [1, 0, -1, [1]]$ ,  $[4, 2, 2, 2, 4, 0] + [4096, [5]]$ ,  $[5] : [1, 0, -1, [1]]$ ,  $[4, 2, 2, 0, 3, 2] + [6144, [5]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[4, 2, 1, 1, 4, 1] + [2048, [5]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[4, 2, 0, 2, 5, 0] + [4096, [6]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[4, 2, 0, 0, 4, 2] + [16, [1]]$ ,  $[3, 3, 9, 0, 1, 1] + [12, [1]]$ ,  $[3, 3, 8, 1, 2, 0] + [256, [2]]$ ,  $[3, 3, 7, 0, 2, 1] + [192, [2]]$ ,  $[3, 3, 6, 1, 3, 0] + [1536, [3]]$ ,  $[3, 3, 5, 0, 3, 1] + [1152, [3]]$ ,  $[3, 3, 4, 1, 4, 0] + [4096, [4]]$ ,  $[3, 3, 3, 0, 4, 1] + [3072, [4]]$ ,  $[3, 3, 2, 1, 5, 0] + [4096, [5]]$ ,  $[3, 3, 1, 0, 5, 1] + [3072, [5]]$ ,  $[3, 3, 0, 1, 6, 0] + [12, [1]]$ ,  $[3, 2, 10, 1, 0, 1] + [8, [1]]$ ,  $[3, 2, 9, 2, 1, 0] + [32, [2]]$ ,  $[3, 2, 9, 0, 0, 2] + [208, [2]]$ ,  $[3, 2, 8, 1, 1, 1] + [128, [2]]$ ,  $[3, 2, 7, 2, 2, 0] + [512, [3]]$ ,  $[3, 2, 7, 0, 1, 2] + [1408, [3]]$ ,  $[3, 2, 6, 1, 2, 1] + [768, [3]]$ ,  $[3, 2, 5, 2, 3, 0] + [3072, [4]]$ ,  $[3, 2, 5, 0, 2, 2] + [4608, [4]]$ ,  $[3, 2, 4, 1, 3, 1] + [2048, [4]]$ ,  $[3, 2, 3, 2, 4, 0] + [8192, [5]]$ ,  $[3, 2, 3, 0, 3, 2] + [7168, [5]]$ ,  $[3, 2, 2, 1, 4, 1] + [2048, [5]]$ ,  $[3, 2, 1, 2, 5, 0] + [8192, [6]]$ ,  $[3, 2, 1, 0, 4, 2] + [4096, [6]]$ ,  $[3, 2, 0, 1, 5, 1] + [4, 0] : [1, 0, -1, [1]]$ ,  $[2, 4, 8, 0, 3, 0] + [64, [1]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 4, 6, 0, 4, 0] + [384, [2]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 4, 4, 0, 5, 0] + [1024, [3]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 4, 2, 0, 6, 0] + [1024, [4]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 4, 0, 0, 7, 0] + [4, 0] : [1, 0, -1, [1]]$ ,  $[2, 3, 10, 0, 1, 1] + [6, 0] : [1, 0, -1, [1]]$ ,  $[2, 3, 9, 1, 2, 0] + [72, [1]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 3, 8, 0, 2, 1] + [96, [1]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 3, 7, 1, 3, 0] + [512, [2]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 3, 6, 0, 3, 1] + [576, [2]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 3, 5, 1, 4, 0] + [1792, [3]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 3, 4, 0, 4, 1] + [1536, [3]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 3, 3, 1, 5, 0] + [3072, [4]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 3, 2, 0, 5, 1] + [1536, [4]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 3, 1, 1, 6, 0] + [2048, [5]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 3, 0, 0, 6, 1] + [2, 0] : [1, 0, -1, [1]]$ ,  $[2, 2, 11, 1, 0, 1] + [2, 0] : [1, 0, -1, [1]]$ ,  $[2, 2, 10, 2, 1, 0] + [24, [1]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 10, 0, 0, 2] + [56, [1]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 9, 1, 1, 1] + [36, [1]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 8, 2, 2, 0] + [384, [2]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 8, 0, 1, 2] + [576, [2]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 7, 1, 2, 1] + [256, [2]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 6, 2, 3, 0] + [2304, [3]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 6, 0, 2, 2] + [2816, [3]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 5, 1, 3, 1] + [896, [3]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 4, 2, 4, 0] + [6144, [4]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 4, 0, 3, 2] + [6656, [4]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 3, 1, 4, 1] + [1536, [4]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 2, 2, 5, 0] + [6144, [5]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 2, 0, 4, 2] + [6144, [5]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 1, 1, 5, 1] + [1024, [5]]$ ,  $[0] : [1, 0, -1, [1]]$ ,  $[2, 2, 0, 2, 6, 0] + [8, [1]]$ ,  $[1, 3, 9, 0, 2, 1] + [4, [1]]$ ,  $[1, 3, 8, 1, 3, 0] + [128, [2]]$ ,  $[1, 3, 7, 0, 3, 1] + [64, [2]]$ ,  $[1, 3, 6, 1, 4, 0] + [768, [3]]$ ,  $[1, 3, 5, 0, 4, 1] + [384, [3]]$ ,  $[1, 3, 4, 1, 5, 0] + [2048, [4]]$ ,  $[1, 3, 3, 0, 5, 1] + [1024, [4]]$ ,  $[1, 3, 2, 1, 6, 0] + [2048, [5]]$ ,  $[1, 3, 1, 0, 6, 1] + [1024, [5]]$ ,  $[1, 3, 0, 1, 7, 0] + [8, [1]]$ ,  $[1, 2, 11, 0, 0, 2] + [12, [1]]$ ,  $[1, 2, 10, 1, 1, 1] + [4, [1]]$ ,  $[1, 2, 9, 2, 2, 0] + [128, [2]]$ ,  $[1, 2, 9, 0, 1, 2] + [192, [2]]$ ,  $[1, 2, 8, 1, 2, 1] + [64, [2]]$ ,  $[1, 2, 7, 2, 3, 0] + [768, [3]]$ ,  $[1, 2, 7, 0, 2, 2] + [1152, [3]]$ ,  $[1, 2, 6, 1, 3, 1] + [384, [3]]$ ,  $[1, 2, 5, 2, 4, 0] + [2048, [4]]$ ,  $[1, 2, 5, 0, 3, 2$

$\} + \{3072, [4]\}$ ,  $[1, 2, 4, 1, 4, 1]\} + \{-1024, [4]\}$ ,  $[1, 2, 3, 2, 5, 0]\} + \{2048, [5]\}$ ,  $[1, 2, 3, 0, 4, 2]\} + \{-3072, [5]\}$ ,  $[1, 2, 2, 1, 5, 1]\} + \{1024, [5]\}$ ,  $[1, 2, 1, 2, 6, 0]\} + \{-1, 0\} : [1, 0, \{-1, [1]\}]$ ,  $[0, 4, 8, 0, 4, 0]\} + \{16, [1]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 4, 6, 0, 5, 0]\} + \{-96, [2]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 4, 4, 0, 6, 0]\} + \{256, [3]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 4, 2, 0, 7, 0]\} + \{-256, [4]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 4, 0, 0, 8, 0]\} + \{-2, 0\} : [1, 0, \{-1, [1]\}]$ ,  $[0, 3, 10, 0, 2, 1]\} + \{2, 0\} : [1, 0, \{-1, [1]\}]$ ,  $[0, 3, 9, 1, 3, 0]\} + \{32, [1]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 3, 8, 0, 3, 1]\} + \{-32, [1]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 3, 7, 1, 4, 0]\} + \{-192, [2]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 3, 6, 0, 4, 1]\} + \{192, [2]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 3, 5, 1, 5, 0]\} + \{512, [3]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 3, 4, 0, 5, 1]\} + \{-512, [3]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 3, 3, 1, 6, 0]\} + \{-512, [4]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 3, 2, 0, 6, 1]\} + \{512, [4]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 3, 1, 1, 7, 0]\} + \{-1, 0\} : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 12, 0, 0, 2]\} + \{2, 0\} : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 11, 1, 1, 1]\} + \{-1, 0\} : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 10, 2, 2, 0]\} + \{16, [1]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 10, 0, 1, 2]\} + \{-32, [1]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 9, 1, 2, 1]\} + \{16, [1]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 8, 2, 3, 0]\} + \{-96, [2]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 8, 0, 2, 2]\} + \{192, [2]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 7, 1, 3, 1]\} + \{-96, [2]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 6, 2, 4, 0]\} + \{256, [3]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 6, 0, 3, 2]\} + \{-512, [3]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 5, 1, 4, 1]\} + \{256, [3]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 4, 2, 5, 0]\} + \{-256, [4]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 4, 0, 4, 2]\} + \{512, [4]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 3, 1, 5, 1]\} + \{-256, [4]\}, 0 : [1, 0, \{-1, [1]\}]$ ,  $[0, 2, 2, 2, 6, 0]\} / \{1, [1]\}, [8, 2, 0, 0, 0, 0]\} + \{poly 1[-4, [1]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[7, 1, 0, 1, 0, 0]\} + \{-4, [1]\}, [6, 2, 0, 0, 1, 0]\} + \{2, [1]\}, [6, 1, 1, 1, 0, 0]\} + \{8, [2]\}, [6, 1, 0, 0, 0, 1]\} + \{4, [2]\}, [6, 0, 0, 2, 0, 0]\} + \{poly 1[-8, [1]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[5, 1, 1, 0, 0, 1]\} + \{poly 1[12, [1]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[5, 1, 0, 1, 1, 0]\} + \{poly 1[-4, [1]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[5, 0, 1, 2, 0, 0]\} + \{poly 1[-16, [2]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[5, 0, 0, 1, 0, 1]\} + \{6, [1]\}, [4, 2, 0, 0, 2, 0]\} + \{2, [1]\}, [4, 1, 2, 0, 0, 1]\} + \{-6, [1]\}, [4, 1, 1, 1, 1, 0]\} + \{-16, [2]\}, [4, 1, 0, 0, 1, 1]\} + \{1, [1]\}, [4, 0, 2, 2, 0, 0]\} + \{24, [2]\}, [4, 0, 1, 1, 0, 1]\} + \{-8, [2]\}, [4, 0, 0, 2, 1, 0]\} + \{16, [3]\}, [4, 0, 0, 0, 0, 2]\} + \{poly 1[16, [1]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[3, 1, 1, 0, 1, 1]\} + \{poly 1[-12, [1]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[3, 1, 0, 1, 2, 0]\} + \{poly 1[-12, [1]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[3, 0, 2, 1, 0, 1]\} + \{poly 1[8, [1]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[3, 0, 1, 2, 1, 0]\} + \{poly 1[-32, [2]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[3, 0, 1, 0, 0, 2]\} + \{poly 1[16, [2]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[3, 0, 0, 1, 1, 1]\} + \{-4, [1]\}, [2, 2, 0, 0, 3, 0]\} + \{-4, [1]\}, [2, 1, 2, 0, 1, 1]\} + \{6, [1]\}, [2, 1, 1, 1, 2, 0]\} + \{8, [2]\}, [2, 1, 0, 0, 2, 1]\} + \{2, [1]\}, [2, 0, 3, 1, 0, 1]\} + \{-2, [1]\}, [2, 0, 2, 2, 1, 0]\} + \{24, [2]\}, [2, 0, 2, 0, 0, 2]\} + \{-24, [2]\}, [2, 0, 1, 1, 1, 1]\} + \{4, [2]\}, [2, 0, 0, 2, 2, 0]\} + \{poly 1[-8, [1]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[1, 1, 1, 0, 2, 1]\} + \{poly 1[4, [1]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[1, 1, 0, 1, 3, 0]\} + \{poly 1[-8, [1]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[1, 0, 3, 0, 0, 2]\} + \{poly 1[12, [1]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[1, 0, 2, 1, 1, 1]\} + \{poly 1[-4, [1]], 0\} : [1, 0, \{-1, [1]\}]$ ,  $[1, 0, 1, 2, 2, 0]\} + \{1, [1]\}, [0, 2, 0, 0, 4, 0]\} + \{2, [1]\}, [0, 1, 2, 0, 2, 1]\} + \{-2, [1]\}, [0, 1, 1, 1, 3, 0]\} + \{1, [1]\}, [0, 0, 4, 0, 0, 2]\} + \{-2, [1]\}, [0, 0, 3, 1, 1, 1]\} + \{1, [1]\}, [0, 0, 2, 2, 2, 0]\} Error: Bad Argument Value$

**maple** [B] time = 0.04, size = 72576, normalized size = 103.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d)^2,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d)^2,x, algorithm="maxima")

[Out] integrate((c\*x^2 + b\*x + a)^(3/2)/(f\*x^2 + e\*x + d)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^2,x)

[Out] int((a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)/(f\*x\*\*2+e\*x+d)\*\*2,x)

[Out] Timed out

**3.105**  $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3} dx$

**Optimal.** Leaf size=671

$$3 \left( 2 \left( e - \sqrt{e^2 - 4df} \right) (ce - bf)(2af - be + 2cd) - f \left( 4be(3af + cd) - 4a(4af^2 + ce^2) - (b^2(4df + e^2)) \right) \right) \tanh^{-1} \frac{4\sqrt{2} \left( e^2 - 4df \right)^{5/2} \sqrt{2af^2 - \sqrt{e^2 - 4df} (ce - bf) - bef - 2cdf + c}}$$

**Rubi [A]** time = 11.60, antiderivative size = 669, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {971, 1013, 1032, 724, 206}

$$\frac{3(-2(-\sqrt{e^2-4df})(a-bf)(2af-be+2cd)+4bf(3af+cd)-4f(4af^2+ce^2)+b^2(4df+e^2))\tanh^{-1}\left(\frac{4\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+c}}{2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+c}}\right)+3(-2(\sqrt{e^2-4df}+e)(a-bf)(2af-be+2cd)+4bf(3af+cd)-4f(4af^2+ce^2)+b^2(4df+e^2))\tanh^{-1}\left(\frac{4\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+c}}{2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+c}}\right)}{4\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
[Out] -((e + 2*f*x)*(a + b*x + c*x^2)^(3/2))/(2*(e^2 - 4*d*f)*(d + e*x + f*x^2)^2) + (3*(4*c*d*e + 4*a*e*f - b*(e^2 + 4*d*f) + 2*(c*e^2 - 2*b*e*f + 4*a*f^2)*x)*Sqrt[a + b*x + c*x^2]/(4*(e^2 - 4*d*f)^2*(d + e*x + f*x^2)) + (3*(4*b*e*f*(c*d + 3*a*f) - b^2*f*(e^2 + 4*d*f) - 4*a*f*(c*e^2 + 4*a*f^2) - 2*(2*c*d - b*e + 2*a*f)*(c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (3*(4*b*e*f*(c*d + 3*a*f) - b^2*f*(e^2 + 4*d*f) - 4*a*f*(c*e^2 + 4*a*f^2) - 2*(2*c*d - b*e + 2*a*f)*(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

**Rule 206**

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

**Rule 724**

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

**Rule 971**

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p + 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]
```

Rule 1013

Int[((g\_.) + (h\_.)\*(x\_.))\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_)^2)^(p\_.)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_.), x\_Symbol] := Simp[((g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/((b^2 - 4\*a\*c)^(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)^(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[e\*q\*(g\*b - 2\*a\*h) - d\*(b\*h - 2\*g\*c)\*(2\*p + 3) + (2\*f\*q\*(g\*b - 2\*a\*h) - e\*(b\*h - 2\*g\*c)\*(2\*p + q + 3))\*x - f\*(b\*h - 2\*g\*c)\*(2\*p + 2\*q + 3)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0]

Rule 1032

Int[((g\_.) + (h\_.)\*(x\_.))/(((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c\*g - h\*(b - q))/q, Int[1/((b - q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[(2\*c\*g - h\*(b + q))/q, Int[1/((b + q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && PosQ[b^2 - 4\*a\*c]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx + cx^2)^{3/2}}{(d + ex + fx^2)^3} dx &= -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{\int \frac{\left(\frac{3}{2}(be - 4af) + 3(ce - bf)x\right)\sqrt{a + bx + cx^2}}{(d + ex + fx^2)^2} dx}{2(e^2 - 4df)} \\ &= -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{3(4cde + 4aef - b(e^2 + 4df) + 2(ce^2 - 2bef + 4af^2))}{4(e^2 - 4df)^2(d + ex + fx^2)} \\ &= -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{3(4cde + 4aef - b(e^2 + 4df) + 2(ce^2 - 2bef + 4af^2))}{4(e^2 - 4df)^2(d + ex + fx^2)} \\ &= -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{3(4cde + 4aef - b(e^2 + 4df) + 2(ce^2 - 2bef + 4af^2))}{4(e^2 - 4df)^2(d + ex + fx^2)} \\ &= -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{3(4cde + 4aef - b(e^2 + 4df) + 2(ce^2 - 2bef + 4af^2))}{4(e^2 - 4df)^2(d + ex + fx^2)} \end{aligned}$$

**Mathematica [B]** time = 7.23, size = 4727, normalized size = 7.04

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^3,x]

[Out] (-2\*f^2\*(a + x\*(b + c\*x))^(3/2))/((e^2 - 4\*d\*f)^(3/2)\*(e - Sqrt[e^2 - 4\*d\*f] + 2\*f\*x)^2) + (6\*f^2\*(a + x\*(b + c\*x))^(3/2))/((e^2 - 4\*d\*f)^2\*(e - Sqrt[e^2 - 4\*d\*f] + 2\*f\*x)) + (2\*f^2\*(a + x\*(b + c\*x))^(3/2))/((e^2 - 4\*d\*f)^(3/2)\*(e + Sqrt[e^2 - 4\*d\*f] + 2\*f\*x)^2) + (6\*f^2\*(a + x\*(b + c\*x))^(3/2))/((e^2 - 4\*d\*f)^2\*(e + Sqrt[e^2 - 4\*d\*f] + 2\*f\*x)) + (9\*f^2\*(a + x\*(b + c\*x))^(3/2))/((e^2 - 4\*d\*f)^(3/2)\*(e - Sqrt[e^2 - 4\*d\*f] + 2\*f\*x)^2)

$$\begin{aligned}
& 3/2)*((( -4*b*c*f - 2*c*(b*f + 2*c*(-e + \sqrt{e^2 - 4*d*f})) - 4*c^2*f*x)*\sqrt{a + b*x + c*x^2})/(8*c*f^2) - ((2*\sqrt{c}*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})) + 4*c*f*(a*f - b*(e - \sqrt{e^2 - 4*d*f}))))*\text{ArcTanh}[(b + 2*c*x)/(2*\sqrt{c}*\sqrt{a + b*x + c*x^2}))/f + (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\sqrt{e^2 - 4*d*f} + b*f*\sqrt{e^2 - 4*d*f}})* \\
& (4*c*f*(8*a*b*f^2 - 3*b^2*f*(e - \sqrt{e^2 - 4*d*f})) - 4*a*c*f*(e - \sqrt{e^2 - 4*d*f})) + 4*b*c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})) + 4*c*(-e + \sqrt{e^2 - 4*d*f})*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})) + 4*c*f*(a*f - b*(e - \sqrt{e^2 - 4*d*f}))))*\text{ArcTanh}[( -4*a*f - b*(-e + \sqrt{e^2 - 4*d*f})) - (2*b*f + 2*c*(-e + \sqrt{e^2 - 4*d*f}))]*x)/(2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\sqrt{e^2 - 4*d*f} + b*f*\sqrt{e^2 - 4*d*f}})*\sqrt{a + b*x + c*x^2}))/ \\
& (f*(16*a*f^2 + 8*b*f*(-e + \sqrt{e^2 - 4*d*f})) + 4*c*(-e + \sqrt{e^2 - 4*d*f})^2))/((16*c*f^2))/((e^2 - 4*d*f)^2*(a + b*x + c*x^2)^{(3/2)}) - (3*f^2*(a + x*(b + c*x))^{(3/2)})*((( -4*c*f*(4*a*f - b*(e - \sqrt{e^2 - 4*d*f})) - 2*(b*f - c*(e - \sqrt{e^2 - 4*d*f})))*(b*f + 2*c*(-e + \sqrt{e^2 - 4*d*f})) - 4*c*f*(b*f - c*(e - \sqrt{e^2 - 4*d*f})))*x)*\sqrt{a + b*x + c*x^2})/(8*c*f^2) - (( -2*(b*f - c*(e - \sqrt{e^2 - 4*d*f})))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})) - 4*c*f*(3*a*f - b*(e - \sqrt{e^2 - 4*d*f}))))*\text{ArcTanh}[(b + 2*c*x)/(2*\sqrt{c}*\sqrt{a + b*x + c*x^2}))/(\sqrt{c}*f) + \\
& (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\sqrt{e^2 - 4*d*f} + b*f*\sqrt{e^2 - 4*d*f}})*(-4*(-e + \sqrt{e^2 - 4*d*f})*(b*f - c*(e - \sqrt{e^2 - 4*d*f})))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})) - 4*c*f*(3*a*f - b*(e - \sqrt{e^2 - 4*d*f})))) + 4*f*(2*c*f*(4*a*f - b*(e - \sqrt{e^2 - 4*d*f}))^2 - (e - \sqrt{e^2 - 4*d*f})*(b*f - c*(e - \sqrt{e^2 - 4*d*f}))*(b^2*f + 4*a*c*f - 2*b*c*(e - \sqrt{e^2 - 4*d*f}))))*\text{ArcTanh}[( -4*a*f - b*(-e + \sqrt{e^2 - 4*d*f})) - (2*b*f + 2*c*(-e + \sqrt{e^2 - 4*d*f}))]*x)/(2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\sqrt{e^2 - 4*d*f} + b*f*\sqrt{e^2 - 4*d*f}})*\sqrt{a + b*x + c*x^2}))/ \\
& (f*(16*a*f^2 + 8*b*f*(-e + \sqrt{e^2 - 4*d*f})) + 4*c*(-e + \sqrt{e^2 - 4*d*f})^2))/((16*c*f^2))/((e^2 - 4*d*f)^{(5/2)}*(a + b*x + c*x^2)^{(3/2)}) + (3*f^2*(a + x*(b + c*x))^{(3/2)})*((( -2*b*f - 2*c*(-e + \sqrt{e^2 - 4*d*f}))*(a + b*x + c*x^2)^{(3/2)})/((-4*a*f^2 - 2*b*f*(-e + \sqrt{e^2 - 4*d*f})) - c*(-e + \sqrt{e^2 - 4*d*f})^2)*(-e + \sqrt{e^2 - 4*d*f}) - 2*f*x)) + ((( -4*c*f*(b^2*f + 4*a*c*f - 2*b*c*(e - \sqrt{e^2 - 4*d*f})) - 4*c*(b*f - c*(e - \sqrt{e^2 - 4*d*f}))*(b*f + 2*c*(-e + \sqrt{e^2 - 4*d*f})) - 8*c^2*f*(b*f - c*(e - \sqrt{e^2 - 4*d*f})))*x)*\sqrt{a + b*x + c*x^2})/(8*c*f^2) - ((16*c^{(3/2)}*(b*f - c*(e - \sqrt{e^2 - 4*d*f}))*(c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})) + f*(2*a*f - b*(e - \sqrt{e^2 - 4*d*f}))))*\text{ArcTanh}[(b + 2*c*x)/(2*\sqrt{c}*\sqrt{a + b*x + c*x^2}))/f + (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\sqrt{e^2 - 4*d*f} + b*f*\sqrt{e^2 - 4*d*f}})*(32*c^2*(-e + \sqrt{e^2 - 4*d*f})*(b*f - c*(e - \sqrt{e^2 - 4*d*f}))*(c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})) + f*(2*a*f - b*(e - \sqrt{e^2 - 4*d*f})))) + 16*c*f*(b^2*f + 4*a*c*f - 2*b*c*(e - \sqrt{e^2 - 4*d*f}))*(c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*d*f})) + f*(2*a*f - b*(e - \sqrt{e^2 - 4*d*f}))))*\text{ArcTanh}[( -4*a*f - b*(-e + \sqrt{e^2 - 4*d*f})) - (2*b*f + 2*c*(-e + \sqrt{e^2 - 4*d*f}))]*x)/(2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\sqrt{e^2 - 4*d*f} + b*f*\sqrt{e^2 - 4*d*f}})*\sqrt{a + b*x + c*x^2}))/ \\
& (f*(16*a*f^2 + 8*b*f*(-e + \sqrt{e^2 - 4*d*f})) + 4*c*(-e + \sqrt{e^2 - 4*d*f})^2))/((16*c*f^2))/((-4*a*f^2 - 2*b*f*(-e + \sqrt{e^2 - 4*d*f})) - c*(-e + \sqrt{e^2 - 4*d*f})^2))/((e^2 - 4*d*f)^{(3/2)}*(a + b*x + c*x^2)^{(3/2)}) + (3*f^2*(a + x*(b + c*x))^{(3/2)})*(((4*c*f*(-4*a*f + b*(e + \sqrt{e^2 - 4*d*f})) + 2*(b*f - c*(e + \sqrt{e^2 - 4*d*f})))*(-b*f) + 2*c*(e + \sqrt{e^2 - 4*d*f})) - 4*c*f*(b*f - c*(e + \sqrt{e^2 - 4*d*f})))*x)*\sqrt{a + b*x + c*x^2})/(8*c*f^2) - (( -2*(b*f - c*(e + \sqrt{e^2 - 4*d*f})))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})) - 4*c*f*(3*a*f - b*(e + \sqrt{e^2 - 4*d*f}))))*\text{ArcTanh}[(b + 2*c*x)/(2*\sqrt{c}*\sqrt{a + b*x + c*x^2}))/(\sqrt{c}*f) - (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\sqrt{e^2 - 4*d*f} - b*f*\sqrt{e^2 - 4*d*f}})*(4*(e + \sqrt{e^2 - 4*d*f})*(b*f - c*(e + \sqrt{e^2 - 4*d*f}))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})) - 4*c*f*(3*a*f - b*(e + \sqrt{e^2 - 4*d*f})))) + 4*f*(2*c*f*(4*a*f - b*(e + \sqrt{e^2 - 4*d*f}))^2 - (e + \sqrt{e^2 - 4*d*f})*(b*f - c*(e
\end{aligned}$$

$$\begin{aligned}
& + \text{Sqrt}[e^2 - 4*d*f])*(b^2*f + 4*a*c*f - 2*b*c*(e + \text{Sqrt}[e^2 - 4*d*f])))))* \\
& \text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + \text{Sqrt}[e^2 - \\
& 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 \\
& 2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2]])]/(f*(16*a*f^2 - \\
& 8*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)))/(16*c*f^2 \\
& )))/((e^2 - 4*d*f)^(5/2)*(a + b*x + c*x^2)^(3/2)) - (9*f^2*(a + x*(b + c*x) \\
& )^(3/2)*(((4*b*c*f - 2*c*(-(b*f) + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f])) + 4*c^2*f*x) \\
& )*\text{Sqrt}[a + b*x + c*x^2])/(8*c*f^2) - ((-2*\text{Sqrt}[c]*(b^2*f^2 + 4*c^2*(e^2 - 2 \\
& *d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))) * \text{ArcT} \\
& \text{anh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/f - (2*\text{Sqrt}[2]*\text{Sqrt}[c*e \\
& ^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d \\
& *f]])*(4*c*(e + \text{Sqrt}[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 \\
& 2 - 4*d*f]) + 4*c*f*(a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))) + 4*c*f*(3*b^2*f*(e \\
& + \text{Sqrt}[e^2 - 4*d*f]) + 4*a*c*f*(e + \text{Sqrt}[e^2 - 4*d*f]) - 4*b*(2*a*f^2 + c*( \\
& e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]))) * \text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4* \\
& d*f]) - (-2*b*f + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2 \\
& *c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*\text{S} \\
& \text{qrt}[a + b*x + c*x^2]])]/(f*(16*a*f^2 - 8*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c* \\
& (e + \text{Sqrt}[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^2*(a + b*x + c*x^2 \\
& )^(3/2)) - (3*f^2*(a + x*(b + c*x))^(3/2)*(((2*b*f - 2*c*(e + \text{Sqrt}[e^2 - 4* \\
& d*f]))*(a + b*x + c*x^2)^(3/2))/((-4*a*f^2 + 2*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) \\
& - c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)*(e + \text{Sqrt}[e^2 - 4*d*f] + 2*f*x)) + (((4*c*(b \\
& *f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f])) + 4* \\
& c*f*(-(b^2*f) - 4*a*c*f + 2*b*c*(e + \text{Sqrt}[e^2 - 4*d*f])) - 8*c^2*f*(b*f - c \\
& *(e + \text{Sqrt}[e^2 - 4*d*f]))*x)*\text{Sqrt}[a + b*x + c*x^2])/(8*c*f^2) - ((16*c^(3/2) \\
& )*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) \\
& + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))) * \text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqr} \\
& \text{t}[a + b*x + c*x^2])])/f - (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 \\
& + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*(-32*c^2*(e + \text{Sqrt}[e^2 - \\
& 4*d*f])*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4* \\
& d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))) + 16*c*f*(b^2*f + 4*a*c*f - \\
& 2*b*c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*( \\
& 2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))) * \text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d \\
& *f]) - (-2*b*f + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2* \\
& c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sq} \\
& \text{rt}[a + b*x + c*x^2])]/(f*(16*a*f^2 - 8*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*( \\
& e + \text{Sqrt}[e^2 - 4*d*f])^2)))/(16*c*f^2))/(-4*a*f^2 + 2*b*f*(e + \text{Sqrt}[e^2 - 4 \\
& *d*f]) - c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x \\
& ^2)^(3/2))
\end{aligned}$$

**IntegrateAlgebraic [F]** time = 180.01, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^3,x]

[Out] \$Aborted

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d)^3,x, algorithm="fricas")

[Out] Timed out

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d)^3,x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.07, size = 178044, normalized size = 265.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d)^3,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 + ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d)^3,x, algorithm="maxima")

[Out] integrate((c\*x^2 + b\*x + a)^(3/2)/(f\*x^2 + e\*x + d)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 + ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^3,x)

[Out] int((a + b\*x + c\*x^2)^(3/2)/(d + e\*x + f\*x^2)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)/(f\*x\*\*2+e\*x+d)\*\*3,x)

[Out] Timed out



$$3.106 \quad \int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}} dx$$

Optimal. Leaf size=717

$$\frac{x\sqrt{a+bx+cx^2} (24c^2f(50a^2f^2 + 322abef + 175b^2(df + e^2)) - 252b^2cf^2(14af + 15be) - 160c^3(27af(df + e^2) + 27b^2e^2))}{3840c^5}$$

Rubi [A] time = 2.71, antiderivative size = 717, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1661, 640, 621, 206}

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)^3/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((23040\*c^5\*d^2\*e - 3465\*b^5\*f^3 + 420\*b^3\*c\*f^2\*(27\*b\*e + 34\*a\*f) - 504\*b\*c^2\*f\*(70\*a\*b\*e\*f + 22\*a^2\*f^2 + 25\*b^2\*(e^2 + d\*f)) - 640\*c^4\*(27\*b\*d\*(e^2 + d\*f) + 8\*a\*e\*(e^2 + 6\*d\*f)) + 96\*c^3\*(128\*a^2\*e\*f^2 + 275\*a\*b\*f\*(e^2 + d\*f) + 50\*b^2\*(e^3 + 6\*d\*e\*f)))\*Sqrt[a + b\*x + c\*x^2]/(7680\*c^6) + ((1155\*b^4\*f^3 - 252\*b^2\*c\*f^2\*(15\*b\*e + 14\*a\*f) + 5760\*c^4\*d\*(e^2 + d\*f) + 24\*c^2\*f\*(322\*a\*b\*e\*f + 50\*a^2\*f^2 + 175\*b^2\*(e^2 + d\*f)) - 160\*c^3\*(27\*a\*f\*(e^2 + d\*f) + 10\*b\*(e^3 + 6\*d\*e\*f)))\*x\*Sqrt[a + b\*x + c\*x^2]/(3840\*c^5) - ((231\*b^3\*f^3 - 36\*b\*c\*f^2\*(21\*b\*e + 13\*a\*f) - 320\*c^3\*(e^3 + 6\*d\*e\*f) + 24\*c^2\*f\*(32\*a\*e\*f + 35\*b\*(e^2 + d\*f)))\*x^2\*Sqrt[a + b\*x + c\*x^2]/(960\*c^4) + (f\*(99\*b^2\*f^2 - 4\*c\*f\*(81\*b\*e + 25\*a\*f) + 360\*c^2\*(e^2 + d\*f))\*x^3\*Sqrt[a + b\*x + c\*x^2]/(480\*c^3) + (f^2\*(36\*c\*e - 11\*b\*f)\*x^4\*Sqrt[a + b\*x + c\*x^2]/(60\*c^2) + (f^3\*x^5\*Sqrt[a + b\*x + c\*x^2]/(6\*c) + ((1024\*c^6\*d^3 + 231\*b^6\*f^3 - 252\*b^4\*c\*f^2\*(3\*b\*e + 5\*a\*f) - 1536\*c^5\*d\*(b\*d\*e + a\*(e^2 + d\*f)) + 840\*b^2\*c^2\*f\*(4\*a\*b\*e\*f + 2\*a^2\*f^2 + b^2\*(e^2 + d\*f)) + 384\*c^4\*(3\*b^2\*d\*(e^2 + d\*f) + 3\*a^2\*f\*(e^2 + d\*f) + 2\*a\*b\*e\*(e^2 + 6\*d\*f)) - 320\*c^3\*(9\*a^2\*b\*e\*f^2 + a^3\*f^3 + 9\*a\*b^2\*f\*(e^2 + d\*f) + b^3\*(e^3 + 6\*d\*e\*f)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])]/(1024\*c^(13/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x +

$c*x^2)^{(p+1)}/(c*(q+2*p+1)), x] + \text{Dist}[1/(c*(q+2*p+1)), \text{Int}[(a + b*x + c*x^2)^p \text{ExpandToSum}[c*(q+2*p+1)*Pq - a*e*(q-1)*x^{(q-2)} - b*e*(q+p)*x^{(q-1)} - c*e*(q+2*p+1)*x^q, x], x]] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{LeQ}[p, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}} dx &= \frac{f^3 x^5 \sqrt{a+bx+cx^2}}{6c} + \frac{\int \frac{6cd^3+18cd^2ex+18cd(e^2+df)x^2+6ce(e^2+6df)x^3-f(5af^2-18c(e^2+df))x^4+\frac{1}{2}f^2(36ce-11bf)x^5}{\sqrt{a+bx+cx^2}} dx}{6c} \\ &= \frac{f^2(36ce-11bf)x^4 \sqrt{a+bx+cx^2}}{60c^2} + \frac{f^3 x^5 \sqrt{a+bx+cx^2}}{6c} + \frac{\int \frac{30c^2d^3+90c^2d^2ex+90c^2d(e^2+df)x^2}{\sqrt{a+bx+cx^2}} dx}{60c} \\ &= \frac{f(99b^2f^2-4cf(81be+25af)+360c^2(e^2+df))x^3 \sqrt{a+bx+cx^2}}{480c^3} + \frac{f^2(36ce-11bf)x^5}{60c} \\ &= -\frac{(231b^3f^3-36bcf^2(21be+13af)-320c^3(e^3+6def)+24c^2f(32aef+35b(e^2+df)))}{960c^4} \\ &= \frac{(1155b^4f^3-252b^2cf^2(15be+14af)+5760c^4d(e^2+df)+24c^2f(322abef+50a^2f^2+25b^2e^2))}{3840c^5} \\ &= \frac{(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f(70abef+22a^2f^2+25b^2e^2))}{3840c^5} \\ &= \frac{(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f(70abef+22a^2f^2+25b^2e^2))}{3840c^5} \\ &= \frac{(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f(70abef+22a^2f^2+25b^2e^2))}{3840c^5} \end{aligned}$$

**Mathematica [A]** time = 1.35, size = 615, normalized size = 0.86

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)^3/Sqrt[a + b\*x + c\*x^2], x]

[Out] (Sqrt[a + x\*(b + c\*x)]\*(-3465\*b^5\*f^3 + 210\*b^3\*c\*f^2\*(54\*b\*e + 68\*a\*f + 11\*b\*f\*x) - 168\*b\*c^2\*f\*(66\*a^2\*f^2 + 42\*a\*b\*f\*(5\*e + f\*x) + b^2\*(75\*e^2 + 75\*d\*f + 45\*e\*f\*x + 11\*f^2\*x^2)) + 128\*c^5\*(90\*d^2\*(2\*e + f\*x) + 15\*d\*x\*(6\*e^2 + 8\*e\*f\*x + 3\*f^2\*x^2) + x^2\*(20\*e^3 + 45\*e^2\*f\*x + 36\*e\*f^2\*x^2 + 10\*f^3\*x^3)) + 48\*c^3\*(2\*a^2\*f^2\*(128\*e + 25\*f\*x) + b^2\*(100\*e^3 + 175\*e^2\*f\*x + 6\*e\*f\*(100\*d + 21\*f\*x^2) + f^2\*x\*(175\*d + 33\*f\*x^2)) + 2\*a\*b\*f\*(275\*e^2 + 161\*e\*f\*x + f\*(275\*d + 39\*f\*x^2))) - 64\*c^4\*(a\*(80\*e^3 + 135\*e^2\*f\*x + 96\*e\*f\*(5\*d + f\*x^2) + 5\*f^2\*x\*(27\*d + 5\*f\*x^2)) + b\*(270\*d^2\*f + 15\*d\*(18\*e^2 + 20\*e\*f\*x + 7\*f^2\*x^2) + x\*(50\*e^3 + 105\*e^2\*f\*x + 81\*e\*f^2\*x^2 + 22\*f^3\*x^3))))/(7680\*c^6) + ((1024\*c^6\*d^3 + 231\*b^6\*f^3 - 252\*b^4\*c\*f^2\*(3\*b\*e + 5\*a\*f) - 1536\*c^5\*d\*(b\*d\*e + a\*(e^2 + d\*f)) + 840\*b^2\*c^2\*f\*(4\*a\*b\*e\*f + 2\*a^2\*f^2 + b^2\*(e^2 + d\*f)) + 384\*c^4\*(3\*b^2\*d\*(e^2 + d\*f) + 3\*a^2\*f\*(e^2 + d\*f) + 2\*a\*b\*e\*(e^2 + 6\*d\*f)) - 320\*c^3\*(9\*a^2\*b\*e\*f^2 + a^3\*f^3 + 9\*a\*b^2\*f\*(e^2 + d\*f) + b^3\*(e^3 + 6\*d\*e\*f)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])]/(1024\*c^(13/2))

IntegrateAlgebraic [A] time = 4.22, size = 847, normalized size = 1.18

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)^3/Sqrt[a + b\*x + c\*x^2],x]

[Out] (Sqrt[a + b\*x + c\*x^2]\*(23040\*c^5\*d^2\*e - 17280\*b\*c^4\*d\*e^2 + 4800\*b^2\*c^3\*e^3 - 5120\*a\*c^4\*e^3 - 17280\*b\*c^4\*d^2\*f + 28800\*b^2\*c^3\*d\*e\*f - 30720\*a\*c^4\*d\*e\*f - 12600\*b^3\*c^2\*e^2\*f + 26400\*a\*b\*c^3\*e^2\*f - 12600\*b^3\*c^2\*d\*f^2 + 26400\*a\*b\*c^3\*d\*f^2 + 11340\*b^4\*c\*e\*f^2 - 35280\*a\*b^2\*c^2\*e\*f^2 + 12288\*a^2\*c^3\*e\*f^2 - 3465\*b^5\*f^3 + 14280\*a\*b^3\*c\*f^3 - 11088\*a^2\*b\*c^2\*f^3 + 11520\*c^5\*d\*e^2\*x - 3200\*b\*c^4\*e^3\*x + 11520\*c^5\*d^2\*f\*x - 19200\*b\*c^4\*d\*e\*f\*x + 8400\*b^2\*c^3\*e^2\*f\*x - 8640\*a\*c^4\*e^2\*f\*x + 8400\*b^2\*c^3\*d\*f^2\*x - 8640\*a\*c^4\*d\*f^2\*x - 7560\*b^3\*c^2\*e\*f^2\*x + 15456\*a\*b\*c^3\*e\*f^2\*x + 2310\*b^4\*c\*f^3\*x - 7056\*a\*b^2\*c^2\*f^3\*x + 2400\*a^2\*c^3\*f^3\*x + 2560\*c^5\*e^3\*x^2 + 15360\*c^5\*d\*e\*f\*x^2 - 6720\*b\*c^4\*e^2\*f\*x^2 - 6720\*b\*c^4\*d\*f^2\*x^2 + 6048\*b^2\*c^3\*e\*f^2\*x^2 - 6144\*a\*c^4\*e\*f^2\*x^2 - 1848\*b^3\*c^2\*f^3\*x^2 + 3744\*a\*b\*c^3\*f^3\*x^2 + 5760\*c^5\*e^2\*f\*x^3 + 5760\*c^5\*d\*f^2\*x^3 - 5184\*b\*c^4\*e\*f^2\*x^3 + 1584\*b^2\*c^3\*f^3\*x^3 - 1600\*a\*c^4\*f^3\*x^3 + 4608\*c^5\*e\*f^2\*x^4 - 1408\*b\*c^4\*f^3\*x^4 + 1280\*c^5\*f^3\*x^5))/(7680\*c^6) + ((-1024\*c^6\*d^3 + 1536\*b\*c^5\*d^2\*e - 1152\*b^2\*c^4\*d\*e^2 + 1536\*a\*c^5\*d\*e^2 + 320\*b^3\*c^3\*e^3 - 768\*a\*b\*c^4\*e^3 - 1152\*b^2\*c^4\*d^2\*f + 1536\*a\*c^5\*d^2\*f + 1920\*b^3\*c^3\*d\*e\*f - 4608\*a\*b\*c^4\*d\*e\*f - 840\*b^4\*c^2\*e^2\*f + 2880\*a\*b^2\*c^3\*e^2\*f - 1152\*a^2\*c^4\*e^2\*f - 840\*b^4\*c^2\*d\*f^2 + 2880\*a\*b^2\*c^3\*d\*f^2 - 1152\*a^2\*c^4\*d\*f^2 + 756\*b^5\*c\*e\*f^2 - 3360\*a\*b^3\*c^2\*e\*f^2 + 2880\*a^2\*b\*c^3\*e\*f^2 - 231\*b^6\*f^3 + 1260\*a\*b^4\*c\*f^3 - 1680\*a^2\*b^2\*c^2\*f^3 + 320\*a^3\*c^3\*f^3)\*Log[b + 2\*c\*x - 2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2]])/(1024\*c^(13/2))

fricas [A] time = 1.41, size = 1583, normalized size = 2.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] [-1/30720\*(15\*(1024\*c^6\*d^3 - 1536\*b\*c^5\*d^2\*e + 384\*(3\*b^2\*c^4 - 4\*a\*c^5)\*d\*e^2 - 64\*(5\*b^3\*c^3 - 12\*a\*b\*c^4)\*e^3 + (231\*b^6 - 1260\*a\*b^4\*c + 1680\*a^2\*b^2\*c^2 - 320\*a^3\*c^3)\*f^3 + 12\*(2\*(35\*b^4\*c^2 - 120\*a\*b^2\*c^3 + 48\*a^2\*c^4)\*d - (63\*b^5\*c - 280\*a\*b^3\*c^2 + 240\*a^2\*b\*c^3)\*e)\*f^2 + 24\*(16\*(3\*b^2\*c^4 - 4\*a\*c^5)\*d^2 - 16\*(5\*b^3\*c^3 - 12\*a\*b\*c^4)\*d\*e + (35\*b^4\*c^2 - 120\*a\*b^2\*c^3 + 48\*a^2\*c^4)\*e^2)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 + 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(1280\*c^6\*f^3\*x^5 + 23040\*c^6\*d^2\*e - 17280\*b\*c^5\*d\*e^2 + 128\*(36\*c^6\*e\*f^2 - 11\*b\*c^5\*f^3)\*x^4 + 320\*(15\*b^2\*c^4 - 16\*a\*c^5)\*e^3 - 21\*(165\*b^5\*c - 680\*a\*b^3\*c^2 + 528\*a^2\*b\*c^3)\*f^3 + 16\*(360\*c^6\*e^2\*f + (99\*b^2\*c^4 - 100\*a\*c^5)\*f^3 + 36\*(10\*c^6\*d - 9\*b\*c^5\*e)\*f^2)\*x^3 - 12\*(50\*(21\*b^3\*c^3 - 44\*a\*b\*c^4)\*d - (945\*b^4\*c^2 - 2940\*a\*b^2\*c^3 + 1024\*a^2\*c^4)\*e)\*f^2 + 8\*(320\*c^6\*e^3 - 3\*(77\*b^3\*c^3 - 156\*a\*b\*c^4)\*f^3 - 12\*(70\*b\*c^5\*d - (63\*b^2\*c^4 - 64\*a\*c^5)\*e)\*f^2 + 120\*(16\*c^6\*d\*e - 7\*b\*c^5\*e^2)\*f)\*x^2 - 120\*(144\*b\*c^5\*d^2 - 16\*(15\*b^2\*c^4 - 16\*a\*c^5)\*d\*e + 5\*(21\*b^3\*c^3 - 44\*a\*b\*c^4)\*e^2)\*f + 2\*(5760\*c^6\*d\*e^2 - 1600\*b\*c^5\*e^3 + 3\*(385\*b^4\*c^2 - 1176\*a\*b^2\*c^3 + 400\*a^2\*c^4)\*f^3 + 12\*(10\*(35\*b^2\*c^4 - 36\*a\*c^5)\*d - 7\*(45\*b^3\*c^3 - 92\*a\*b\*c^4)\*e)\*f^2 + 120\*(48\*c^6\*d^2 - 80\*b\*c^5\*d\*e + (35\*b^2\*c^4 - 36\*a\*c^5)\*e^2)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^7, -1/15360\*(15\*(1024\*c^6\*d^3 - 1536\*b\*c^5\*d^2\*e + 384\*(3\*b^2\*c^4 - 4\*a\*c^5)\*d\*e^2 - 64\*(5\*b^3\*c^3 - 12\*a\*b\*c^4)\*e^3 + (231\*b^6 - 1260\*a\*b^4\*c + 1680\*a^2\*b^2\*c^2 - 320\*a^3\*c^3)\*f^3 + 12\*(2\*(35\*b^4\*c^2 - 120\*a\*b^2\*c^3 + 48\*a^2\*c^4)\*d - (63\*b^5\*c - 280\*a\*b^3\*c^2 + 240\*a^2\*b\*c^3)\*e)\*f^2 + 24\*(16\*(3\*b^2\*c^4 - 4\*a\*c^5)\*d^2 - 16\*(5\*b^3\*c^3 - 12\*a\*b\*c^4)\*d\*e + (35\*b^4\*c^2 - 120\*a\*b^2\*c^3 + 48\*a^2\*c^4)\*e^2)\*f)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x +

$$a) \cdot (2cx + b) \sqrt{-c} / (c^2x^2 + bcx + ac) - 2 \cdot (1280c^6f^3x^5 + 23040c^6d^2e - 17280b^5d^2e^2 + 128 \cdot (36c^6e^2f^2 - 11b^5c^5f^3)x^4 + 320 \cdot (15b^2c^4 - 16ac^5)e^3 - 21 \cdot (165b^5c - 680ab^3c^2 + 528a^2b^3c^3)f^3 + 16 \cdot (360c^6e^2f + (99b^2c^4 - 100ac^5)f^3 + 36 \cdot (10c^6d - 9b^5c^5e)f^2)x^3 - 12 \cdot (50 \cdot (21b^3c^3 - 44ab^3c^4)d - (945b^4c^2 - 2940ab^2c^3 + 1024a^2c^4)e)f^2 + 8 \cdot (320c^6e^3 - 3 \cdot (77b^3c^3 - 156ab^3c^4)f^3 - 12 \cdot (70b^5c^5d - (63b^2c^4 - 64ac^5)e)f^2 + 120 \cdot (16c^6d^2e - 7b^5c^5e^2)f)x^2 - 120 \cdot (144b^5c^5d^2 - 16 \cdot (15b^2c^4 - 16ac^5)d^2e + 5 \cdot (21b^3c^3 - 44ab^3c^4)e^2)f + 2 \cdot (5760c^6d^2e^2 - 1600b^5c^5e^3 + 3 \cdot (385b^4c^2 - 1176ab^2c^3 + 400a^2c^4)f^3 + 12 \cdot (10 \cdot (35b^2c^4 - 36ac^5)d - 7 \cdot (45b^3c^3 - 92ab^3c^4)e)f^2 + 120 \cdot (48c^6d^2 - 80b^5c^5d^2e + (35b^2c^4 - 36ac^5)e^2)f)x) \sqrt{cx^2 + bx + a}) / c^7$$

**giac [A]** time = 0.53, size = 824, normalized size = 1.15

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{7680} \sqrt{cx^2 + bx + a} \cdot (2 \cdot (4 \cdot (2 \cdot (8 \cdot (10f^3x/c - (11b^4c^4f^3 - 36c^5f^2e)/c^6)x + (360c^5d^2f^2 + 99b^2c^3f^3 - 100ac^4f^3 - 324b^4c^4f^2e + 360c^5f^2e^2)/c^6)x - (840b^4c^4d^2f^2 + 231b^3c^2f^3 - 468ab^3c^3f^3 - 1920c^5d^2f^2e - 756b^2c^3f^2e + 768ac^4f^2e + 840b^4c^4f^2e^2 - 320c^5e^3)/c^6)x + (5760c^5d^2f + 4200b^2c^3d^2f^2 - 4320ac^4d^2f^2 + 1155b^4c^3f^3 - 3528ab^2c^2f^3 + 1200a^2c^3f^3 - 9600b^4c^4d^2f^2e - 3780b^3c^2f^2e + 7728ab^3c^3f^2e + 5760c^5d^2e^2 + 4200b^2c^3f^2e^2 - 4320ac^4f^2e^2 - 1600b^4c^4e^3)/c^6)x - (17280b^4c^4d^2f + 12600b^3c^2d^2f^2 - 26400ab^3c^3d^2f^2 + 3465b^5f^3 - 14280ab^3c^3f^3 + 11088a^2b^3c^2f^3 - 23040c^5d^2e - 28800b^2c^3d^2f^2e + 30720ac^4d^2f^2e - 11340b^4c^3f^2e + 35280ab^2c^2f^2e - 12288a^2c^3f^2e + 17280b^4c^4d^2e^2 + 12600b^3c^2f^2e^2 - 26400ab^3c^3f^2e^2 - 4800b^2c^3e^3 + 5120ac^4e^3)/c^6) - \frac{1}{1024} \cdot (1024c^6d^3 + 1152b^2c^4d^2f - 1536ac^5d^2f + 840b^4c^2d^2f^2 - 2880ab^2c^3d^2f^2 + 1152a^2c^4d^2f^2 + 231b^6f^3 - 1260ab^4c^3f^3 + 1680a^2b^2c^2f^3 - 320a^3c^3f^3 - 1536b^5c^5d^2e - 1920b^3c^3d^2f^2e + 4608ab^4c^4d^2f^2e - 756b^5c^3f^2e + 3360ab^3c^2f^2e - 2880a^2b^3c^3f^2e + 1152b^2c^4d^2e^2 - 1536ac^5d^2e^2 + 840b^4c^2f^2e^2 - 2880ab^2c^3f^2e^2 + 1152a^2c^4f^2e^2 - 320b^3c^3e^3 + 768ab^4c^4e^3) \cdot \log(\text{abs}(-2 \cdot (\sqrt{c}x - \sqrt{cx^2 + bx + a}) \sqrt{c} - b)) / c^{13/2})$

**maple [B]** time = 0.03, size = 1930, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(1/2),x)

[Out]  $3d^2e/c \cdot (cx^2 + bx + a)^{1/2} + \frac{1}{3} x^2/c \cdot (cx^2 + bx + a)^{1/2} e^3 + \frac{5}{8} b^2/c^3 \cdot (cx^2 + bx + a)^{1/2} e^3 - \frac{5}{16} b^3/c^{7/2} \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) e^3 - \frac{2}{3} a/c^2 \cdot (cx^2 + bx + a)^{1/2} e^3 - \frac{231}{512} f^3 b^5/c^6 \cdot (cx^2 + bx + a)^{1/2} + \frac{231}{1024} f^3 b^6/c^{13/2} \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) - \frac{5}{16} f^3 a^3/c^{7/2} \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) + d^3 \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) / c^{1/2} + \frac{161}{80} e^2 f^2 b/c^3 \cdot a \cdot x \cdot (cx^2 + bx + a)^{1/2} - \frac{5}{2} b/c^2 \cdot x \cdot (cx^2 + bx + a)^{1/2} \cdot d^2 e^2 f + \frac{9}{2} b/c^{5/2} \cdot a \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) \cdot d^2 e^2 f - \frac{45}{16} b^2/c^{7/2} \cdot a \cdot \ln((cx + 1/2b)/c^{1/2} + (cx^2 + bx + a)^{1/2}) \cdot e^2 f + \frac{55}{16} b/c^3 \cdot a \cdot (cx^2 + bx + a)^{1/2} \cdot d^2 f^2 + \frac{55}{16} b/c^3 \cdot a \cdot (cx^2 + bx + a)^{1/2} \cdot e^2 f - \frac{9}{8} a/c^2 \cdot x \cdot (cx^2 + bx + a)^{1/2} \cdot d^2 f^2 - \frac{147}{160} f^3 b^2/c^4 \cdot a \cdot x \cdot (cx^2 + bx + a)^{1/2} + \frac{39}{80} f^3 b/c^3$

```

a*x^2*(c*x^2+b*x+a)^(1/2)-9/8*a/c^2*x*(c*x^2+b*x+a)^(1/2)*e^2*f+2*x^2/c*(c*
x^2+b*x+a)^(1/2)*d*e*f+15/4*b^2/c^3*(c*x^2+b*x+a)^(1/2)*d*e*f-15/8*b^3/c^(7
/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*e*f-4*a/c^2*(c*x^2+b*x+a)
^(1/2)*d*e*f-27/40*e*f^2*b/c^2*x^3*(c*x^2+b*x+a)^(1/2)+63/80*e*f^2*b^2/c^3*
x^2*(c*x^2+b*x+a)^(1/2)-63/64*e*f^2*b^3/c^4*x*(c*x^2+b*x+a)^(1/2)+105/32*e*
f^2*b^3/c^(9/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-147/32*e*f^2*
b^2/c^4*a*(c*x^2+b*x+a)^(1/2)-45/16*e*f^2*b/c^(7/2)*a^2*ln((c*x+1/2*b)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))-4/5*e*f^2*a/c^2*x^2*(c*x^2+b*x+a)^(1/2)-7/8*b/c^2*
x^2*(c*x^2+b*x+a)^(1/2)*d*f^2-7/8*b/c^2*x^2*(c*x^2+b*x+a)^(1/2)*e^2*f+35/32
*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)*d*f^2+35/32*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)*e^
2*f-45/16*b^2/c^(7/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f^2+1
/6*f^3*x^5*(c*x^2+b*x+a)^(1/2)/c+9/8*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*e^2*f-5/12*b/c^2*x*(c*x^2+b*x+a)^(1/2)*e^3+3/4*b/c^(5/2)*
a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^3-105/64*b^3/c^4*(c*x^2+b*x
+a)^(1/2)*e^2*f+105/128*b^4/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*d*f^2+105/128*b^4/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
e^2*f+9/8*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f^2+3/4
*x^3/c*(c*x^2+b*x+a)^(1/2)*d*f^2+3/4*x^3/c*(c*x^2+b*x+a)^(1/2)*e^2*f-105/64
*b^3/c^4*(c*x^2+b*x+a)^(1/2)*d*f^2+3/5*e*f^2*x^4/c*(c*x^2+b*x+a)^(1/2)+189/
128*e*f^2*b^4/c^5*(c*x^2+b*x+a)^(1/2)-189/256*e*f^2*b^5/c^(11/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+8/5*e*f^2*a^2/c^3*(c*x^2+b*x+a)^(1/2)-315
/256*f^3*b^4/c^(11/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+119/64*
f^3*b^3/c^5*a*(c*x^2+b*x+a)^(1/2)+105/64*f^3*b^2/c^(9/2)*a^2*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))-231/160*f^3*b/c^4*a^2*(c*x^2+b*x+a)^(1/2)-5/2
4*f^3*a/c^2*x^3*(c*x^2+b*x+a)^(1/2)+5/16*f^3*a^2/c^3*x*(c*x^2+b*x+a)^(1/2)-
11/60*f^3*b/c^2*x^4*(c*x^2+b*x+a)^(1/2)+33/160*f^3*b^2/c^3*x^3*(c*x^2+b*x+a
)^(1/2)-77/320*f^3*b^3/c^4*x^2*(c*x^2+b*x+a)^(1/2)+77/256*f^3*b^4/c^5*x*(c*
x^2+b*x+a)^(1/2)-9/4*b/c^2*(c*x^2+b*x+a)^(1/2)*f*d^2-9/4*b/c^2*(c*x^2+b*x+a
)^(1/2)*e^2*d+9/8*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f
*d^2+9/8*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*d-3/2*
a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*d^2-3/2*a/c^(3/2)*l
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*d-3/2*d^2*e*b/c^(3/2)*ln((c*
x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/2*x/c*(c*x^2+b*x+a)^(1/2)*f*d^2+3/2
*x/c*(c*x^2+b*x+a)^(1/2)*e^2*d

```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(fx^2 + ex + d)^3}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex + fx^2)^3}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*\*3/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] Integral((d + e\*x + f\*x\*\*2)\*\*3/sqrt(a + b\*x + c\*x\*\*2), x)

$$3.107 \quad \int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=316

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2(a^2f^2+4abef+b^2(2df+e^2))-40b^2cf(3af+2be)-64c^3(a(2df+e^2)+2bdf)\right)}{128c^{9/2}}$$

**Rubi [A]** time = 0.63, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2(a^2f^2+4abef+b^2(2df+e^2))-40b^2cf(3af+2be)-64c^3(a(2df+e^2)+2bdf)\right)}{128c^{9/2}} + \frac{c\sqrt{a+bx+cx^2}(-4cf(9af+20be)+35b^2f^2+48c^2(2df+e^2))}{96c^3} + \frac{\sqrt{a+bx+cx^2}(-16c^2(6acf+9b(2df+e^2))+20bcf(11af+12be)-105b^2f^2+384c^3d)}{192c^4} + \frac{f^2\sqrt{a+bx+cx^2}(6cx-7bf)}{24c^2} + \frac{f^2\sqrt{a+bx+cx^2}}{4c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)^2/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((384\*c^3\*d\*e - 105\*b^3\*f^2 + 20\*b\*c\*f\*(12\*b\*e + 11\*a\*f) - 16\*c^2\*(16\*a\*e\*f + 9\*b\*(e^2 + 2\*d\*f)))\*Sqrt[a + b\*x + c\*x^2])/(192\*c^4) + ((35\*b^2\*f^2 - 4\*c\*f\*(20\*b\*e + 9\*a\*f) + 48\*c^2\*(e^2 + 2\*d\*f))\*x\*Sqrt[a + b\*x + c\*x^2])/(96\*c^3) + (f\*(16\*c\*e - 7\*b\*f)\*x^2\*Sqrt[a + b\*x + c\*x^2])/(24\*c^2) + (f^2\*x^3\*Sqrt[a + b\*x + c\*x^2])/(4\*c) + ((128\*c^4\*d^2 + 35\*b^4\*f^2 - 40\*b^2\*c\*f\*(2\*b\*e + 3\*a\*f) - 64\*c^3\*(2\*b\*d\*e + a\*(e^2 + 2\*d\*f)) + 48\*c^2\*(4\*a\*b\*e\*f + a^2\*f^2 + b^2\*(e^2 + 2\*d\*f)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(128\*c^(9/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{(d + ex + fx^2)^2}{\sqrt{a + bx + cx^2}} dx &= \frac{f^2 x^3 \sqrt{a + bx + cx^2}}{4c} + \frac{\int \frac{4cd^2 + 8cdex - (3af^2 - 4c(e^2 + 2df))x^2 + \frac{1}{2}f(16ce - 7bf)x^3}{\sqrt{a + bx + cx^2}} dx}{4c} \\
&= \frac{f(16ce - 7bf)x^2 \sqrt{a + bx + cx^2}}{24c^2} + \frac{f^2 x^3 \sqrt{a + bx + cx^2}}{4c} + \frac{\int \frac{12c^2 d^2 + (24c^2 de - 16acef + 7abf^2)x + \frac{1}{4}}{\sqrt{a + bx + cx^2}} dx}{192c^4} \\
&= \frac{(35b^2 f^2 - 4cf(20be + 9af) + 48c^2(e^2 + 2df))x \sqrt{a + bx + cx^2}}{96c^3} + \frac{f(16ce - 7bf)x^2 \sqrt{a + bx + cx^2}}{24c^2} \\
&= \frac{(384c^3 de - 105b^3 f^2 + 20bcf(12be + 11af) - 16c^2(16aef + 9b(e^2 + 2df))) \sqrt{a + bx + cx^2}}{192c^4} \\
&= \frac{(384c^3 de - 105b^3 f^2 + 20bcf(12be + 11af) - 16c^2(16aef + 9b(e^2 + 2df))) \sqrt{a + bx + cx^2}}{192c^4} \\
&= \frac{(384c^3 de - 105b^3 f^2 + 20bcf(12be + 11af) - 16c^2(16aef + 9b(e^2 + 2df))) \sqrt{a + bx + cx^2}}{192c^4}
\end{aligned}$$

**Mathematica [A]** time = 0.50, size = 251, normalized size = 0.79

$$\frac{\tanh^{-1}\left(\frac{bx+2a}{\sqrt{a+bx+cx^2}}\right)\left(48c^2(e^2 f^2 + 4abef + b^2(2df + e^2)) - 40f^2 c(3af + 2be) - 64c^3(a(2df + e^2) + 2bde) + 35b^3 f^2 + 128c^4 d^2\right) + \sqrt{a+bx+cx^2}\left(-8c^2(f(32e + 9fx) + b(36df + 18e^2 + 20efx + 7f^2 x^2)) + 10bcf(22af + 24be + 7bf) - 105b^2 f^2 + 16c^3(12d(2e + fx) + x(6e^2 + 8efx + 3f^2 x^2))\right)}{128c^3} + \frac{\sqrt{a+bx+cx^2}\left(-8c^2(f(32e + 9fx) + b(36df + 18e^2 + 20efx + 7f^2 x^2)) + 10bcf(22af + 24be + 7bf) - 105b^2 f^2 + 16c^3(12d(2e + fx) + x(6e^2 + 8efx + 3f^2 x^2))\right)}{192c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)^2/Sqrt[a + b\*x + c\*x^2], x]

[Out] (Sqrt[a + x\*(b + c\*x)]\*(-105\*b^3\*f^2 + 10\*b\*c\*f\*(24\*b\*e + 22\*a\*f + 7\*b\*f\*x) + 16\*c^3\*(12\*d\*(2\*e + f\*x) + x\*(6\*e^2 + 8\*e\*f\*x + 3\*f^2\*x^2)) - 8\*c^2\*(a\*f\*(32\*e + 9\*f\*x) + b\*(18\*e^2 + 36\*d\*f + 20\*e\*f\*x + 7\*f^2\*x^2)))/(192\*c^4) + ((128\*c^4\*d^2 + 35\*b^4\*f^2 - 40\*b^2\*c\*f\*(2\*b\*e + 3\*a\*f) - 64\*c^3\*(2\*b\*d\*e + a\*(e^2 + 2\*d\*f)) + 48\*c^2\*(4\*a\*b\*e\*f + a^2\*f^2 + b^2\*(e^2 + 2\*d\*f)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])/(128\*c^(9/2))

**IntegrateAlgebraic [A]** time = 1.09, size = 296, normalized size = 0.94

$$\frac{\log\left(-2\sqrt{a+bx+cx^2} + b + 2a\right)\left(-48c^2 e^2 f^2 + 120ab^2 e f^2 - 192abc^2 e f + 128ac^3 e f + 64ac^3 e^2 - 35b^4 f^2 + 80b^3 c f - 96b^2 c^2 d f - 48b^2 c^2 d^2 + 128b^2 c^2 d e - 128c^4 d^2\right) + \sqrt{a+bx+cx^2}\left(220abc^2 f^2 - 256ac^3 e f - 72ac^3 f^2 x - 105b^2 f^2 + 240b^2 c e f + 70b^2 c^2 f^2 x - 288bc^2 d f - 144bc^2 d^2 - 160b^2 c^2 e f x - 56b^2 c^2 f^2 x^2 + 384c^4 d e + 192c^2 d f x + 96c^3 d^2 x + 128c^3 e f x^2 + 48c^3 f^2 x^2\right)}{128c^3} + \frac{\sqrt{a+bx+cx^2}\left(220abc^2 f^2 - 256ac^3 e f - 72ac^3 f^2 x - 105b^2 f^2 + 240b^2 c e f + 70b^2 c^2 f^2 x - 288bc^2 d f - 144bc^2 d^2 - 160b^2 c^2 e f x - 56b^2 c^2 f^2 x^2 + 384c^4 d e + 192c^2 d f x + 96c^3 d^2 x + 128c^3 e f x^2 + 48c^3 f^2 x^2\right)}{192c^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)^2/Sqrt[a + b\*x + c\*x^2], x]

[Out] (Sqrt[a + b\*x + c\*x^2]\*(384\*c^3\*d\*e - 144\*b\*c^2\*e^2 - 288\*b\*c^2\*d\*f + 240\*b^2\*c\*e\*f - 256\*a\*c^2\*e\*f - 105\*b^3\*f^2 + 220\*a\*b\*c\*f^2 + 96\*c^3\*e^2\*x + 192\*c^3\*d\*f\*x - 160\*b\*c^2\*e\*f\*x + 70\*b^2\*c\*f^2\*x - 72\*a\*c^2\*f^2\*x + 128\*c^3\*e\*f\*x^2 - 56\*b\*c^2\*f^2\*x^2 + 48\*c^3\*f^2\*x^3))/(192\*c^4) + ((-128\*c^4\*d^2 + 128\*b\*c^3\*d\*e - 48\*b^2\*c^2\*e^2 + 64\*a\*c^3\*e^2 - 96\*b^2\*c^2\*d\*f + 128\*a\*c^3\*d\*f + 80\*b^3\*c\*e\*f - 192\*a\*b\*c^2\*e\*f - 35\*b^4\*f^2 + 120\*a\*b^2\*c\*f^2 - 48\*a^2\*c^2\*f^2)\*Log[b + 2\*c\*x - 2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2]])/(128\*c^(9/2))

**fricas [A]** time = 0.80, size = 637, normalized size = 2.02

$$\frac{\log\left(-2\sqrt{a+bx+cx^2} + b + 2a\right)\left(-48c^2 e^2 f^2 + 120ab^2 e f^2 - 192abc^2 e f + 128ac^3 e f + 64ac^3 e^2 - 35b^4 f^2 + 80b^3 c f - 96b^2 c^2 d f - 48b^2 c^2 d^2 + 128b^2 c^2 d e - 128c^4 d^2\right) + \sqrt{a+bx+cx^2}\left(220abc^2 f^2 - 256ac^3 e f - 72ac^3 f^2 x - 105b^2 f^2 + 240b^2 c e f + 70b^2 c^2 f^2 x - 288bc^2 d f - 144bc^2 d^2 - 160b^2 c^2 e f x - 56b^2 c^2 f^2 x^2 + 384c^4 d e + 192c^2 d f x + 96c^3 d^2 x + 128c^3 e f x^2 + 48c^3 f^2 x^2\right)}{128c^3} + \frac{\sqrt{a+bx+cx^2}\left(220abc^2 f^2 - 256ac^3 e f - 72ac^3 f^2 x - 105b^2 f^2 + 240b^2 c e f + 70b^2 c^2 f^2 x - 288bc^2 d f - 144bc^2 d^2 - 160b^2 c^2 e f x - 56b^2 c^2 f^2 x^2 + 384c^4 d e + 192c^2 d f x + 96c^3 d^2 x + 128c^3 e f x^2 + 48c^3 f^2 x^2\right)}{192c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(1/2), x, algorithm="fricas")



```
[Out] [1/768*(3*(128*c^4*d^2 - 128*b*c^3*d*e + 16*(3*b^2*c^2 - 4*a*c^3)*e^2 + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(2*(3*b^2*c^2 - 4*a*c^3)*d - (5*b^3*c - 12*a*b*c^2)*e)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f^2*x^3 + 384*c^4*d*e - 144*b*c^3*e^2 - 5*(21*b^3*c - 44*a*b*c^2)*f^2 + 8*(16*c^4*e*f - 7*b*c^3*f^2)*x^2 - 16*(18*b*c^3*d - (15*b^2*c^2 - 16*a*c^3)*e)*f + 2*(48*c^4*e^2 + (35*b^2*c^2 - 36*a*c^3)*f^2 + 16*(6*c^4*d - 5*b*c^3*e)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/384*(3*(128*c^4*d^2 - 128*b*c^3*d*e + 16*(3*b^2*c^2 - 4*a*c^3)*e^2 + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(2*(3*b^2*c^2 - 4*a*c^3)*d - (5*b^3*c - 12*a*b*c^2)*e)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(48*c^4*f^2*x^3 + 384*c^4*d*e - 144*b*c^3*e^2 - 5*(21*b^3*c - 44*a*b*c^2)*f^2 + 8*(16*c^4*e*f - 7*b*c^3*f^2)*x^2 - 16*(18*b*c^3*d - (15*b^2*c^2 - 16*a*c^3)*e)*f + 2*(48*c^4*e^2 + (35*b^2*c^2 - 36*a*c^3)*f^2 + 16*(6*c^4*d - 5*b*c^3*e)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

**giac [A]** time = 0.63, size = 304, normalized size = 0.96

$$\frac{1}{192} \sqrt{c^2 + bx + a} \left( \left( \frac{6f^2x}{c} - \frac{7b^2f^2 - 16c^2fe}{c^2} \right) x + \frac{96c^2df + 35b^2f^2 - 36a^2fe - 80b^2fc + 48c^2d^2}{c^2} \right) - \frac{288bc^2df + 105b^3f^2 - 220abc^2f - 384c^3d^2e - 240b^2fc + 256a^2fe + 144bc^2d^2}{c^4} - \frac{(128c^4d^2 + 96b^2c^2df - 128a^2d^2 + 35b^4f^2 - 120ab^2c^2f - 128bc^2de - 80b^3c^2fe + 48b^2c^2d^2e - 64a^2c^2e^2) \log\left(\frac{-2(\sqrt{c^2 + bx + a})\sqrt{c} - b}{c}\right)}{128c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f^2*x/c - (7*b*c^2*f^2 - 16*c^3*f*e)/c^4)*x + (96*c^3*d*f + 35*b^2*c*f^2 - 36*a*c^2*f^2 - 80*b*c^2*f*e + 48*c^3*e^2)/c^4)*x - (288*b*c^2*d*f + 105*b^3*f^2 - 220*a*b*c*f^2 - 384*c^3*d*e - 240*b^2*c*f*e + 256*a*c^2*f*e + 144*b*c^2*e^2)/c^4) - 1/128*(128*c^4*d^2 + 96*b^2*c^2*d*f - 128*a*c^3*d*f + 35*b^4*f^2 - 120*a*b^2*c*f^2 + 48*a^2*c^2*f^2 - 128*b*c^3*d*e - 80*b^3*c*f*e + 192*a*b*c^2*f*e + 48*b^2*c^2*e^2 - 64*a*c^3*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

**maple [B]** time = 0.01, size = 706, normalized size = 2.23

-----

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 2*d*e/c*(c*x^2+b*x+a)^(1/2)+1/2*x/c*(c*x^2+b*x+a)^(1/2)*e^2-3/4*b/c^2*(c*x^2+b*x+a)^(1/2)*e^2+3/8*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2-1/2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2-35/64*f^2*b^3/c^4*(c*x^2+b*x+a)^(1/2)+35/128*f^2*b^4/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/8*f^2*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-5/6*e*f*b/c^2*x*(c*x^2+b*x+a)^(1/2)+1/4*f^2*x^3*(c*x^2+b*x+a)^(1/2)/c+3/2*e*f*b/c^(5/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+d^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-5/8*e*f*b^3/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/3*e*f*a/c^2*(c*x^2+b*x+a)^(1/2)+x/c*(c*x^2+b*x+a)^(1/2)*d*f-3/2*b/c^2*(c*x^2+b*x+a)^(1/2)*d*f+3/4*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f-a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f-d*e*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/8*f^2*a/c^2*x*(c*x^2+b*x+a)^(1/2)+55/48*f^2*b/c^3*a*(c*x^2+b*x+a)^(1/2)-7/24*f^2*b/c^2*x^2*(c*x^2+b*x+a)^(1/2)+35/96*f^2*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)-15/16*f^2*b^2/c^(7/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2/3*e*f*x^2/c*(c*x^2+b*x+a)^(1/2)+5/4*e*f*b^2/c^3*(c*x^2+b*x+a)^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(fx^2 + ex + d)^2}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(1/2),x)

[Out] int((d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex + fx^2)^2}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*\*2/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((d + e\*x + f\*x\*\*2)\*\*2/sqrt(a + b\*x + c\*x\*\*2), x)

$$3.108 \quad \int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=116

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+be)+3b^2f+8c^2d)}{8c^{5/2}} + \frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c}$$

**Rubi [A]** time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+be)+3b^2f+8c^2d)}{8c^{5/2}} + \frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((4\*c\*e - 3\*b\*f)\*Sqrt[a + b\*x + c\*x^2])/(4\*c^2) + (f\*x\*Sqrt[a + b\*x + c\*x^2])/(2\*c) + ((8\*c^2\*d + 3\*b^2\*f - 4\*c\*(b\*e + a\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(8\*c^(5/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] :> Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{d + ex + fx^2}{\sqrt{a + bx + cx^2}} dx &= \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{\int \frac{2cd - af + \frac{1}{2}(4ce - 3bf)x}{\sqrt{a + bx + cx^2}} dx}{2c} \\
&= \frac{(4ce - 3bf)\sqrt{a + bx + cx^2}}{4c^2} + \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{\left(2c(2cd - af) - \frac{1}{2}b(4ce - 3bf)\right) \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{4c^2} \\
&= \frac{(4ce - 3bf)\sqrt{a + bx + cx^2}}{4c^2} + \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{\left(2c(2cd - af) - \frac{1}{2}b(4ce - 3bf)\right) \text{Subst}}{2c^2} \\
&= \frac{(4ce - 3bf)\sqrt{a + bx + cx^2}}{4c^2} + \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{(8c^2d + 3b^2f - 4c(be + af)) \tanh^{-1}\left(\frac{2\sqrt{c}x + b}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8c^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 96, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)(-4c(af+be)+3b^2f+8c^2d)}{8c^{5/2}} + \frac{\sqrt{a+x(b+cx)}(-3bf+4ce+2cfx)}{4c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((4\*c\*e - 3\*b\*f + 2\*c\*f\*x)\*Sqrt[a + x\*(b + c\*x)]/(4\*c^2) + ((8\*c^2\*d + 3\*b^2\*f - 4\*c\*(b\*e + a\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])/(8\*c^(5/2))

**IntegrateAlgebraic [A]** time = 0.48, size = 102, normalized size = 0.88

$$\frac{\log\left(-2c^{5/2}\sqrt{a + bx + cx^2} + bc^2 + 2c^3x\right)(4acf - 3b^2f + 4bce - 8c^2d)}{8c^{5/2}} + \frac{\sqrt{a + bx + cx^2}(-3bf + 4ce + 2cfx)}{4c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((4\*c\*e - 3\*b\*f + 2\*c\*f\*x)\*Sqrt[a + b\*x + c\*x^2])/(4\*c^2) + ((-8\*c^2\*d + 4\*b\*c\*e - 3\*b^2\*f + 4\*a\*c\*f)\*Log[b\*c^2 + 2\*c^3\*x - 2\*c^(5/2)\*Sqrt[a + b\*x + c\*x^2]])/(8\*c^(5/2))

**fricas [A]** time = 0.63, size = 227, normalized size = 1.96

$$\left[ \frac{(8c^2d - 4bce + (3b^2 - 4ac)f)\sqrt{c} \log\left(\frac{-8c^2x^2 - 8bcx - b^2 + 4\sqrt{cx^2 + bx + a}(2cx + b)\sqrt{c} - 4ac}{16c^3}\right) - 4(2c^2fx + 4c^2e - 3bcf)\sqrt{cx^2 + bx + a} + (8c^2d - 4bce + (3b^2 - 4ac)f)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^2 + bx + a}(2cx + b)\sqrt{c}}{2(\sqrt{c^2 + b^2 + 4ac})}\right) - 2(2c^2fx + 4c^2e - 3bcf)\sqrt{cx^2 + bx + a}}{8c^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2), x, algorithm="fricas")

[Out] [-1/16\*((8\*c^2\*d - 4\*b\*c\*e + (3\*b^2 - 4\*a\*c)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 + 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(2\*c^2\*f\*x + 4\*c^2\*e - 3\*b\*c\*f)\*sqrt(c\*x^2 + b\*x + a))/c^3, -1/8\*((8\*c^2\*d - 4\*b\*c\*e + (3\*b^2 - 4\*a\*c)\*f)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) - 2\*(2\*c^2\*f\*x + 4\*c^2\*e - 3\*b\*c\*f)\*sqrt(c\*x^2 + b\*x + a))/c^3]

**giac [A]** time = 0.50, size = 98, normalized size = 0.84

$$\frac{1}{4}\sqrt{cx^2 + bx + a}\left(\frac{2fx}{c} - \frac{3bf - 4ce}{c^2}\right) - \frac{(8c^2d + 3b^2f - 4acf - 4bce) \log\left(\left|-2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b\right|\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{4}\sqrt{c}x^2 + b x + a) * (2 * f * x / c - (3 * b * f - 4 * c * e) / c^2) - 1 / 8 * (8 * c^2 * d + 3 * b^2 * f - 4 * a * c * f - 4 * b * c * e) * \log(\text{abs}(-2 * (\sqrt{c} * x - \sqrt{c * x^2 + b * x + a})) * \sqrt{c} - b) / c^{5/2})$

**maple [A]** time = 0.01, size = 185, normalized size = 1.59

$$-\frac{af \ln\left(\frac{cx+\frac{b}{\sqrt{c}}+\sqrt{cx^2+bx+a}}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}} + \frac{3b^2 f \ln\left(\frac{cx+\frac{b}{\sqrt{c}}+\sqrt{cx^2+bx+a}}{\sqrt{c}}\right)}{8c^{\frac{5}{2}}} - \frac{be \ln\left(\frac{cx+\frac{b}{\sqrt{c}}+\sqrt{cx^2+bx+a}}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}} + \frac{d \ln\left(\frac{cx+\frac{b}{\sqrt{c}}+\sqrt{cx^2+bx+a}}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{cx^2+bx+a} fx}{2c} - \frac{3\sqrt{cx^2+bx+a} bf}{4c^2} + \frac{\sqrt{cx^2+bx+a} e}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x)

[Out]  $\frac{1}{2} * f * x * (c * x^2 + b * x + a)^{(1/2)} / c - 3 / 4 * f * b / c^2 * (c * x^2 + b * x + a)^{(1/2)} + 3 / 8 * f * b^2 / c^{5/2} * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) - 1/2 * f * a / c^{3/2} * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) + e / c * (c * x^2 + b * x + a)^{(1/2)} - 1/2 * e * b / c^{3/2} * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) + d * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) / c^{(1/2)}$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see 'assume?' for more details) Is 4\*a\*c-b^2 zero or nonzero?

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f x^2 + e x + d}{\sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(1/2),x)

[Out] int((d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(1/2), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + e x + f x^2}{\sqrt{a + b x + c x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((d + e\*x + f\*x\*\*2)/sqrt(a + b\*x + c\*x\*\*2), x)

$$3.109 \quad \int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$$

**Optimal.** Leaf size=374

$$\frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x(bf-c(\sqrt{e^2-4df}+e))-b(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

**Rubi [A]** time = 0.58, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, number of rules / integrand size = 0.111, Rules used = {983, 724, 206}

$$\frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x(bf-c(\sqrt{e^2-4df}+e))-b(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)),x]

[Out] -((Sqrt[2]\*f\*ArcTanh[(4\*a\*f - b\*(e - Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e - Sqrt[e^2 - 4\*d\*f]))\*x]/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 - (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2]))/(Sqrt[e^2 - 4\*d\*f]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 - (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]) + (Sqrt[2]\*f\*ArcTanh[(4\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e + Sqrt[e^2 - 4\*d\*f]))\*x]/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 + (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2]))/(Sqrt[e^2 - 4\*d\*f]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 + (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]])

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 724**

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

**Rule 983**

Int[1/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[1/((b - q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[(2\*c)/q, Int[1/((b + q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[c\*e - b\*f, 0] && PosQ[b^2 - 4\*a\*c]

**Rubi steps**

$$\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx = \frac{(2f) \int \frac{1}{(e-\sqrt{e^2-4df}+2fx)\sqrt{a+bx+cx^2}} dx}{\sqrt{e^2-4df}} - \frac{(2f) \int \frac{1}{(e+\sqrt{e^2-4df}+2fx)\sqrt{a+bx+cx^2}} dx}{\sqrt{e^2-4df}}$$

$$= \frac{(4f) \text{Subst} \left( \int \frac{1}{16af^2-8bf(e-\sqrt{e^2-4df})+4c(e-\sqrt{e^2-4df})^2-x^2} dx, x, \frac{4af-b(e-\sqrt{e^2-4df})}{\sqrt{e^2-4df}} \right)}{\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{2} f \tanh^{-1} \left( \frac{4af-b(e-\sqrt{e^2-4df})+2(bf-c(e-\sqrt{e^2-4df}))x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}} \right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\sqrt{2} f}{\sqrt{e^2-4df}}$$

**Mathematica [A]** time = 1.45, size = 376, normalized size = 1.01

$$\sqrt{2} f \frac{\left( \frac{\tanh^{-1} \left( \frac{4af-b(\sqrt{e^2-4df}+e-2fx)-2cx(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}} \right) - \frac{\tanh^{-1} \left( \frac{4af+b(\sqrt{e^2-4df}-c+2fx)+2cx(\sqrt{e^2-4df}-e)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2af+b(\sqrt{e^2-4df}+b(-e))+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{f(2af+b(\sqrt{e^2-4df}+b(-e))+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)),x]

[Out] (Sqrt[2]\*f\*(ArcTanh[(4\*a\*f - 2\*c\*(e + Sqrt[e^2 - 4\*d\*f]))\*x - b\*(e + Sqrt[e^2 - 4\*d\*f] - 2\*f\*x))/(2\*Sqrt[2]\*Sqrt[c\*(e^2 - 2\*d\*f + e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]))]\*Sqrt[a + x\*(b + c\*x)])/Sqrt[c\*(e^2 - 2\*d\*f + e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]))] - ArcTanh[(4\*a\*f + 2\*c\*(-e + Sqrt[e^2 - 4\*d\*f]))\*x + b\*(-e + Sqrt[e^2 - 4\*d\*f] + 2\*f\*x))/(2\*Sqrt[2]\*Sqrt[f\*(-(b\*e) + 2\*a\*f + b\*Sqrt[e^2 - 4\*d\*f]) + c\*(e^2 - 2\*d\*f - e\*Sqrt[e^2 - 4\*d\*f])]\*Sqrt[a + x\*(b + c\*x)])/Sqrt[f\*(-(b\*e) + 2\*a\*f + b\*Sqrt[e^2 - 4\*d\*f]) + c\*(e^2 - 2\*d\*f - e\*Sqrt[e^2 - 4\*d\*f])]))/Sqrt[e^2 - 4\*d\*f]

**IntegrateAlgebraic [C]** time = 0.44, size = 211, normalized size = 0.56

$$-\text{RootSum} \left[ \#1^4 f - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + \#1^2 b e + 4\#1^2 c d + 2\#1 a \sqrt{c} e - 4\#1 b \sqrt{c} d + a^2 f - a b e + b^2 d \&, \frac{b \log(-\#1 + \sqrt{a + b x + c x^2} - \sqrt{c} x) - 2\#1 \sqrt{c} \log(-\#1 + \sqrt{a + b x + c x^2} - \sqrt{c} x)}{-2\#1^3 f + 3\#1^2 \sqrt{c} e + 2\#1 a f - \#1 b e - 4\#1 c d - a \sqrt{c} e + 2b \sqrt{c} d} \& \right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)),x]

[Out] -RootSum[b^2\*d - a\*b\*e + a^2\*f - 4\*b\*Sqrt[c]\*d\*#1 + 2\*a\*Sqrt[c]\*e\*#1 + 4\*c\*d\*#1^2 + b\*e\*#1^2 - 2\*a\*f\*#1^2 - 2\*Sqrt[c]\*e\*#1^3 + f\*#1^4 &, (b\*Log[-(Sqrt[c]\*x) + Sqrt[a + b\*x + c\*x^2] - #1] - 2\*Sqrt[c]\*Log[-(Sqrt[c]\*x) + Sqrt[a + b\*x + c\*x^2] - #1]\*#1)/(2\*b\*Sqrt[c]\*d - a\*Sqrt[c]\*e - 4\*c\*d\*#1 - b\*e\*#1 + 2\*a\*f\*#1 + 3\*Sqrt[c]\*e\*#1^2 - 2\*f\*#1^3) & ]

**fricas [B]** time = 7.14, size = 11287, normalized size = 30.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d),x, algorithm="fricas")

```
[Out] 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2
*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*
a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^
2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^
2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*
a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*
d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^
3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)
*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^
3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d
^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b
*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a
^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*
c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c
)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2
*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a
*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2
- (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*
a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a
*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*
d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (
b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)
*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 -
a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d
^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 +
2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 -
4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b
*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^
3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2
- (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d
*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*
a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*
a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a
*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e +
a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*
b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c
*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3
*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^
2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*
e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^
2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c
^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)
*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*
b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*
b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*
c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*
e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3
```





$$\begin{aligned}
& ^4e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c \\
& + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)* \\
& f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a* \\
& b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^ \\
& 3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a \\
& *c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4 \\
& )*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2* \\
& e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 \\
& - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*a*b \\
& *d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + \\
& a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - \\
& a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b \\
& ^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3 \\
& *e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d \\
& ^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^ \\
& 2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e \\
& ^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2* \\
& e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^ \\
& 3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2* \\
& b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2 \\
& )*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d \\
& *e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b* \\
& c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^ \\
& 2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - \\
& (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b \\
& *d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b* \\
& e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4 \\
& *e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2 \\
& *c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^ \\
& 2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^ \\
& 2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 \\
& - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*( \\
& a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4* \\
& b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^ \\
& 2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + \\
& a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - \\
& (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e \\
& - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3 \\
& *d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c \\
& ^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^ \\
& 2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^ \\
& 2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2 \\
& *e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^ \\
& 4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^ \\
& 3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e \\
& ^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 \\
& + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b \\
& ^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2) \\
& *d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d* \\
& e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c \\
& *e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2 \\
& *c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d \\
& + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + \\
& a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - \\
& (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - \\
& 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + \\
& 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 \\
& - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c) \\
& *d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b
\end{aligned}$$

$$\begin{aligned} &^3c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 \\ &- 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3* \\ &d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2 \\ &*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e \\ &^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2 \\ &*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b \\ &*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e \\ &^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2* \\ &e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^ \\ &3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - \\ &a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a* \\ &b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 \\ &- a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b \\ &^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4* \\ &a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^ \\ &2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 \\ &- 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 \\ &- 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^ \\ &2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2 \\ &*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + ( \\ &b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 \\ &+ b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d* \\ &e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f \\ &^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2 \\ &*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^ \\ &5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e \\ &+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^ \\ &2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^ \\ &2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20 \\ &*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2 \\ &*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + \\ &6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d* \\ &e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a \\ &^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b \\ &^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - \\ &(4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*( \\ &a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a \\ &*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f \\ &+ (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + \\ &(4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b* \\ &c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^ \\ &2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a \\ &^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^ \\ &2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 \\ &- 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 \\ &- 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2 \\ &)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2* \\ &(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b \\ &^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x \end{aligned}$$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d),x, algorithm="giac")

[Out] Timed out

**maple [B]** time = 0.02, size = 761, normalized size = 2.03

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d), x)`

[Out] 
$$\frac{1}{(-4df+e^2)^{1/2}2^{1/2}} \left( \frac{(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce+2af^2-bef-2cdf+ce^2}{f^2} \right)^{1/2} \ln\left( \frac{(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce+2af^2-bef-2cdf+ce^2}{f^2+1/f(-c(-4df+e^2)^{1/2}+bf-ce)} \right) \left( x+\frac{1}{2}\frac{e+(-4df+e^2)^{1/2}}{f} \right) + \frac{1}{2}2^{1/2} \left( \frac{(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce+2af^2-bef-2cdf+ce^2}{f^2} \right)^{1/2} \left( 4\left( x+\frac{1}{2}\frac{e+(-4df+e^2)^{1/2}}{f} \right)^2c+\frac{4}{f}(-c(-4df+e^2)^{1/2}+bf-ce) \left( x+\frac{1}{2}\frac{e+(-4df+e^2)^{1/2}}{f} \right) + 2\frac{(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce+2af^2-bef-2cdf+ce^2}{f^2} \right) \left( x+\frac{1}{2}\frac{e+(-4df+e^2)^{1/2}}{f} \right) - \frac{1}{(-4df+e^2)^{1/2}2^{1/2}} \left( \frac{(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce+2af^2-bef-2cdf+ce^2}{f^2} \right)^{1/2} \ln\left( \frac{(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce+2af^2-bef-2cdf+ce^2}{f^2+(c(-4df+e^2)^{1/2}+bf-ce)/f} \left( x-\frac{1}{2}\frac{e+(-4df+e^2)^{1/2}}{f} \right) + \frac{1}{2}2^{1/2} \left( \frac{(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce+2af^2-bef-2cdf+ce^2}{f^2} \right)^{1/2} \left( 4\left( x-\frac{1}{2}\frac{e+(-4df+e^2)^{1/2}}{f} \right)^2c+\frac{4}{f}(c(-4df+e^2)^{1/2}+bf-ce) \left( x-\frac{1}{2}\frac{e+(-4df+e^2)^{1/2}}{f} \right) + 2\frac{(-4df+e^2)^{1/2}bf+(-4df+e^2)^{1/2}ce+2af^2-bef-2cdf+ce^2}{f^2} \right) \left( x-\frac{1}{2}\frac{e+(-4df+e^2)^{1/2}}{f} \right) \right)$$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d), x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*d\*f-e^2>0)', see 'assume?' for more details) Is 4\*d\*f-e^2 positive, negative or zero?

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a} (fx^2+ex+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a+b*x+c*x^2)^(1/2)*(d+e*x+f*x^2)), x)`

[Out] `int(1/((a+b*x+c*x^2)^(1/2)*(d+e*x+f*x^2)), x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d), x)`

[Out] `Integral(1/(sqrt(a+b*x+c*x**2)*(d+e*x+f*x**2)), x)`

$$3.110 \quad \int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)^2} dx$$

**Optimal.** Leaf size=789

$$\frac{(f(e - \sqrt{e^2 - 4df})(ce - bf)(2af - be + 2cd) - 2f(f(-4a^2f^2 + 3abef + b^2(e^2 - 6df)) - c(4af(e^2 - 3df) + 2\sqrt{2}(e^2 - 4df)^{3/2}((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{2af^2 - 2}))}{2\sqrt{2}(e^2 - 4df)^{3/2}((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{2af^2 - 2}}$$

**Rubi [A]** time = 8.21, antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, number of rules used = 0.148, Rules used = {974, 1032, 724, 206}

$$\frac{(f(e - \sqrt{e^2 - 4df})(ce - bf)(2af - be + 2cd) - 2f(f(-4a^2f^2 + 3abef + b^2(e^2 - 6df)) - c(4af(e^2 - 3df) + 2\sqrt{2}(e^2 - 4df)^{3/2}((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{2af^2 - 2}))}{2\sqrt{2}(e^2 - 4df)^{3/2}((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{2af^2 - 2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)^2), x]

[Out] ((f\*(b\*e^2 - 2\*b\*d\*f - a\*e\*f) - c\*(e^3 - 3\*d\*e\*f) + f\*(f\*(b\*e - 2\*a\*f) - c\*(e^2 - 2\*d\*f))\*x)\*Sqrt[a + b\*x + c\*x^2])/((e^2 - 4\*d\*f)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(d + e\*x + f\*x^2)) + ((f\*(2\*c\*d - b\*e + 2\*a\*f)\*(c\*e - b\*f)\*(e - Sqrt[e^2 - 4\*d\*f]) - 2\*f\*(3\*a\*b\*e\*f^2 - 4\*a^2\*f^3 + b^2\*f\*(e^2 - 6\*d\*f) + 2\*c^2\*d\*(e^2 - 4\*d\*f) - 4\*a\*c\*f\*(e^2 - 3\*d\*f) - b\*c\*(e^3 - 5\*d\*e\*f)))\*ArcTanh[(4\*a\*f - b\*(e - Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e - Sqrt[e^2 - 4\*d\*f]))\*x)/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 - (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2]))/(2\*Sqrt[2]\*(e^2 - 4\*d\*f)^(3/2)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 - (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]) - ((f\*(2\*c\*d - b\*e + 2\*a\*f)\*(c\*e - b\*f)\*(e + Sqrt[e^2 - 4\*d\*f]) - 2\*f\*(3\*a\*b\*e\*f^2 - 4\*a^2\*f^3 + b^2\*f\*(e^2 - 6\*d\*f) + 2\*c^2\*d\*(e^2 - 4\*d\*f) - 4\*a\*c\*f\*(e^2 - 3\*d\*f) - b\*c\*(e^3 - 5\*d\*e\*f)))\*ArcTanh[(4\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e + Sqrt[e^2 - 4\*d\*f]))\*x)/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 + (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2]))/(2\*Sqrt[2]\*(e^2 - 4\*d\*f)^(3/2)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 + (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]])

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2])), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 974

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f

```
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]
```

### Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

### Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2} (d+ex+fx^2)^2} dx = \frac{(f(be^2 - 2bdf - aef) - c(e^3 - 3def) + f(f(be - 2af) - c(e^2 - 2df))x)}{(e^2 - 4df)((cd - af)^2 - (bd - ae)(ce - bf))(d + ex + fx^2)}$$

$$= \frac{(f(be^2 - 2bdf - aef) - c(e^3 - 3def) + f(f(be - 2af) - c(e^2 - 2df))x)}{(e^2 - 4df)((cd - af)^2 - (bd - ae)(ce - bf))(d + ex + fx^2)}$$

$$= \frac{(f(be^2 - 2bdf - aef) - c(e^3 - 3def) + f(f(be - 2af) - c(e^2 - 2df))x)}{(e^2 - 4df)((cd - af)^2 - (bd - ae)(ce - bf))(d + ex + fx^2)}$$

$$= \frac{(f(be^2 - 2bdf - aef) - c(e^3 - 3def) + f(f(be - 2af) - c(e^2 - 2df))x)}{(e^2 - 4df)((cd - af)^2 - (bd - ae)(ce - bf))(d + ex + fx^2)}$$

**Mathematica [A]** time = 6.69, size = 1377, normalized size = 1.75

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
```

```
[Out] (-8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 -
4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Sqrt
[a + x*(b + c*x)]) - (8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 - 2*
b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(e + Sqrt[e^2 -
4*d*f] + 2*f*x)*Sqrt[a + x*(b + c*x)]) + (2*Sqrt[2]*f^2*Sqrt[a + b*x + c*x^
2]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*
f - b*(e - Sqrt[e^2 - 4*d*f]))]*Sqrt[a + b*x + c*x^2]))]/((e^2 - 4*d*f)^(3/
2)*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2
- 4*d*f]))]*Sqrt[a + x*(b + c*x)]) - (8*Sqrt[2]*f^2*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(2*b*f + 2
```

$$\begin{aligned}
& *c*(-e + \text{Sqrt}[e^2 - 4*d*f]))*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(-4*a*f - b*(-e \\
& + \text{Sqrt}[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2] \\
& *\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\text{Sqrt}[e^2 - 4*d*f] + b*f*\text{Sqrt}[ \\
& e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2)))]/((e^2 - 4*d*f)*(4*a*f^2 + 2*b*f*(-e \\
& + \text{Sqrt}[e^2 - 4*d*f]) + c*(-e + \text{Sqrt}[e^2 - 4*d*f])^2)*(16*a*f^2 + 8*b*f*(-e \\
& + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*(-e + \text{Sqrt}[e^2 - 4*d*f])^2)*\text{Sqrt}[a + x*(b + c*x) \\
& ]) - (2*\text{Sqrt}[2]*f^2*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 \\
& - 4*d*f]) + 2*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*(e^2 - \\
& 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))])]*\text{Sqrt} \\
& [a + b*x + c*x^2)))]/((e^2 - 4*d*f)^(3/2)*\text{Sqrt}[c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 \\
& - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))]*\text{Sqrt}[a + x*(b + c*x)]) - \\
& (8*\text{Sqrt}[2]*f^2*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d \\
& *f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*(-2*b*f + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f]))*\text{Sqrt}[a \\
& + b*x + c*x^2]*\text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) - (-2*b*f + 2*c*( \\
& e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^ \\
& 2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2)))] \\
& /((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + c*(e + \text{Sqrt}[e^2 \\
& - 4*d*f])^2)*(16*a*f^2 - 8*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*(e + \text{Sqrt}[e^2 \\
& - 4*d*f])^2)*\text{Sqrt}[a + x*(b + c*x)])
\end{aligned}$$

**IntegrateAlgebraic [C]** time = 175.85, size = 55521, normalized size = 70.37

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2)^2),x]

[Out] Result too large to show

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d)^2,x, algorithm="fricas")

[Out] Timed out

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d)^2,x, algorithm="giac")

[Out] Timed out

**maple [B]** time = 0.03, size = 3858, normalized size = 4.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d)^2,x)

[Out]  $2*f/(4*d*f - e^2)/(-4*d*f + e^2)^{(1/2)}*2^{(1/2)}/((2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 - (-4*d*f + e^2)^{(1/2)}*b*f + (-4*d*f + e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f - c*e - (-4*d*f + e^2)^{(1/2)}*c)*(x + 1/2*(e + (-4*d*f + e^2)^{(1/2)})/f)/f + (2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 - (-4*d*f + e^2)^{(1/2)}*b*f + (-4*d*f + e^2)^{(1/2)}*c*e)/f^2 + 1/2*2^{(1/2)}*((2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 - (-4*d*f + e^2)^{(1/2)}*b*f + (-4*d*f + e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}$

$$\begin{aligned}
& (1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^{2*c+4*(b*f-c*e-(-4*d*f+e^2)^{(1/2)})} \\
& *c*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d \\
& *f+e^2)^{(1/2)*b*f+(-4*d*f+e^2)^{(1/2)*c*e}/f^2)^{(1/2)}(x+1/2*(e+(-4*d*f+e^2 \\
& )^{(1/2)}))/f)+2/(4*d*f-e^2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)* \\
& b*f-(-4*d*f+e^2)^{(1/2)*c*e}*f^2/(x-1/2*f*(-4*d*f+e^2)^{(1/2)+1/2*e/f)*((x-1/ \\
& 2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^{2*c+(b*f-c*e+(-4*d*f+e^2)^{(1/2)*c)}*(x-1/2*(-e+ \\
& (-4*d*f+e^2)^{(1/2)}))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2) \\
& )*b*f-(-4*d*f+e^2)^{(1/2)*c*e}/f^2)^{(1/2)}-1/(4*d*f-e^2)*f/(2*a*f^2-b*e*f-2*c \\
& *d*f+c*e^2+(-4*d*f+e^2)^{(1/2)*b*f-(-4*d*f+e^2)^{(1/2)*c*e}*2^{(1/2)}/((2*a*f^2 \\
& -b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)*b*f-(-4*d*f+e^2)^{(1/2)*c*e}/f^2)^{(1 \\
& /2)*\ln(((b*f-c*e+(-4*d*f+e^2)^{(1/2)*c)}*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+ \\
& (2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)*b*f-(-4*d*f+e^2)^{(1/2)*c*e} \\
& /f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)*b*f-(-4*d \\
& *f+e^2)^{(1/2)*c*e}/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^{2*c+4*(b \\
& *f-c*e+(-4*d*f+e^2)^{(1/2)*c)}*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+2*(2*a*f^2 \\
& -b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)*b*f-(-4*d*f+e^2)^{(1/2)*c*e}/f^2)^{(1 \\
& /2)}(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f))*c*(-4*d*f+e^2)^{(1/2)}-1/(4*d*f-e^2)* \\
& f^2/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)*b*f-(-4*d*f+e^2)^{(1/2)* \\
& c*e}*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)*b*f-(-4*d*f+e \\
& ^2)^{(1/2)*c*e}/f^2)^{(1/2)*\ln(((b*f-c*e+(-4*d*f+e^2)^{(1/2)*c)}*(x-1/2*(-e+(-4 \\
& *d*f+e^2)^{(1/2)}))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)*b*f- \\
& (-4*d*f+e^2)^{(1/2)*c*e}/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d* \\
& f+e^2)^{(1/2)*b*f-(-4*d*f+e^2)^{(1/2)*c*e}/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e \\
& ^2)^{(1/2)}))/f)^{2*c+4*(b*f-c*e+(-4*d*f+e^2)^{(1/2)*c)}*(x-1/2*(-e+(-4*d*f+e^2) \\
& ^{(1/2)}))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)*b*f-(-4*d*f+e \\
& ^2)^{(1/2)*c*e}/f^2)^{(1/2)}(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f))*b+1/(4*d*f-e^ \\
& 2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)*b*f-(-4*d*f+e^2)^{(1/2) \\
& *c*e}*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)*b*f-(-4*d*f+ \\
& e^2)^{(1/2)*c*e}/f^2)^{(1/2)*\ln(((b*f-c*e+(-4*d*f+e^2)^{(1/2)*c)}*(x-1/2*(-e+(-4 \\
& *d*f+e^2)^{(1/2)}))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)*b*f- \\
& (-4*d*f+e^2)^{(1/2)*c*e}/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d \\
& *f+e^2)^{(1/2)*b*f-(-4*d*f+e^2)^{(1/2)*c*e}/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+ \\
& e^2)^{(1/2)}))/f)^{2*c+4*(b*f-c*e+(-4*d*f+e^2)^{(1/2)*c)}*(x-1/2*(-e+(-4*d*f+e^2) \\
& ^{(1/2)}))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)*b*f-(-4*d*f+e \\
& ^2)^{(1/2)*c*e}/f^2)^{(1/2)}(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f))*c*e+2/(4*d*f \\
& -e^2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)*b*f+(-4*d*f+e^2)^{(1/2) \\
& )*c*e}*f^2/(x+1/2/f*(-4*d*f+e^2)^{(1/2)+1/2*e/f)*((x+1/2*(e+(-4*d*f+e^2)^{(1/ \\
& 2))/f)^{2*c+(b*f-c*e-(-4*d*f+e^2)^{(1/2)*c)}*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/ \\
& f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)*b*f+(-4*d*f+e^2)^{(1/2) \\
& )*c*e}/f^2)^{(1/2)}+1/(4*d*f-e^2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2) \\
& ^{(1/2)*b*f+(-4*d*f+e^2)^{(1/2)*c*e}*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(- \\
& 4*d*f+e^2)^{(1/2)*b*f+(-4*d*f+e^2)^{(1/2)*c*e}/f^2)^{(1/2)*\ln(((b*f-c*e-(-4*d* \\
& f+e^2)^{(1/2)*c)}*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c \\
& *e^2-(-4*d*f+e^2)^{(1/2)*b*f+(-4*d*f+e^2)^{(1/2)*c*e}/f^2+1/2*2^{(1/2)}*((2*a*f \\
& ^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)*b*f+(-4*d*f+e^2)^{(1/2)*c*e}/f^2)^{(1 \\
& /2)*4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^{2*c+4*(b*f-c*e-(-4*d*f+e^2)^{(1/2) \\
& )*c)}*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d \\
& *f+e^2)^{(1/2)*b*f+(-4*d*f+e^2)^{(1/2)*c*e}/f^2)^{(1/2)}(x+1/2*(e+(-4*d*f+e^2) \\
& )^{(1/2)}))/f))*c*(-4*d*f+e^2)^{(1/2)}-1/(4*d*f-e^2)*f^2/(2*a*f^2-b*e*f-2*c*d*f+ \\
& c*e^2-(-4*d*f+e^2)^{(1/2)*b*f+(-4*d*f+e^2)^{(1/2)*c*e}*2^{(1/2)}/((2*a*f^2-b*e* \\
& f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)*b*f+(-4*d*f+e^2)^{(1/2)*c*e}/f^2)^{(1/2)*\ln \\
& (((b*f-c*e-(-4*d*f+e^2)^{(1/2)*c)}*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/f+(2*a*f \\
& ^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)*b*f+(-4*d*f+e^2)^{(1/2)*c*e}/f^2+1 \\
& /2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)*b*f+(-4*d*f+e^2) \\
& )^{(1/2)*c*e}/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^{2*c+4*(b*f-c*e- \\
& (-4*d*f+e^2)^{(1/2)*c)}*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/f+2*(2*a*f^2-b*e*f-2 \\
& *c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)*b*f+(-4*d*f+e^2)^{(1/2)*c*e}/f^2)^{(1/2)}(x+ \\
& 1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f))*b+1/(4*d*f-e^2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e \\
& ^2-(-4*d*f+e^2)^{(1/2)*b*f+(-4*d*f+e^2)^{(1/2)*c*e}*2^{(1/2)}/((2*a*f^2-b*e*f-2
\end{aligned}$$



```
*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((
(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2)))/f)/f+(2*a*f^2-
b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*
2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2)))/f)^2*c+4*(b*f-c*e-(-4
*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2)))/f)/f+2*(2*a*f^2-b*e*f-2*c*
d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2
*(e+(-4*d*f+e^2)^(1/2)))/f))*c*e-2*f/(4*d*f-e^2)/(-4*d*f+e^2)^(1/2)*2^(1/2)/
((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2)))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)))/f)
^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)))/f)/f+2
*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2)))/f))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(1/2)/(f\*x^2+e\*x+d)^2,x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(f\*x^2 + e\*x + d)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2)^2), x)

[Out] int(1/((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2)^2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x\*\*2+b\*x+a)\*\*(1/2)/(f\*x\*\*2+e\*x+d)\*\*2,x)

[Out] Timed out

$$3.111 \quad \int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=649

$$\frac{3 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \left(80c^2 f (a^2 f^2 + 6abef + 3b^2 (df + e^2)) - 280b^2 c f^2 (af + be) - 64c^3 (3af (df + e^2) + b\right)}{128c^{11/2}}$$

**Rubi [A]** time = 2.11, antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1660, 1661, 640, 621, 206}

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (2\*(3\*a\*b^4\*c\*e\*f^2 - a\*b^5\*f^3 + a\*b^3\*c\*f\*(5\*a\*f^2 - 3\*c\*(e^2 + d\*f)) - b\*c^2\*(c^3\*d^3 + 5\*a^3\*f^3 + 3\*a\*c^2\*d\*(e^2 + d\*f) - 9\*a^2\*c\*f\*(e^2 + d\*f)) - a\*b^2\*c^2\*e\*(12\*a\*f^2 - c\*(e^2 + 6\*d\*f)) + 2\*a\*c^3\*e\*(3\*c^2\*d^2 + 3\*a^2\*f^2 - a\*c\*(e^2 + 6\*d\*f)) - (2\*c^2\*d - b\*c\*e + b^2\*f - 2\*a\*c\*f)\*(c^4\*d^2 - b\*c^3\*d\*e + b^2\*c^2\*e^2 - 3\*a\*c^3\*e^2 + b^2\*c^2\*d\*f - 2\*a\*c^3\*d\*f - 2\*b^3\*c\*e\*f + 7\*a\*b\*c^2\*e\*f + b^4\*f^2 - 4\*a\*b^2\*c\*f^2 + a^2\*c^2\*f^2)\*x)/(c^5\*(b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]) - ((187\*b^3\*f^3 - 4\*b\*c\*f^2\*(114\*b\*e + 73\*a\*f) - 64\*c^3\*(e^3 + 6\*d\*e\*f) + 16\*c^2\*f\*(20\*a\*e\*f + 21\*b\*(e^2 + d\*f)))\*Sqrt[a + b\*x + c\*x^2])/(64\*c^5) + (f\*(41\*b^2\*f^2 - 4\*c\*f\*(22\*b\*e + 7\*a\*f) + 48\*c^2\*(e^2 + d\*f))\*x\*Sqrt[a + b\*x + c\*x^2])/(32\*c^4) + (f^2\*(8\*c\*e - 5\*b\*f)\*x^2\*Sqrt[a + b\*x + c\*x^2])/(8\*c^3) + (f^3\*x^3\*Sqrt[a + b\*x + c\*x^2])/(4\*c^2) + (3\*(105\*b^4\*f^3 - 280\*b^2\*c\*f^2\*(b\*e + a\*f) + 128\*c^4\*d\*(e^2 + d\*f) + 80\*c^2\*f\*(6\*a\*b\*e\*f + a^2\*f^2 + 3\*b^2\*(e^2 + d\*f)) - 64\*c^3\*(3\*a\*f\*(e^2 + d\*f) + b\*(e^3 + 6\*d\*e\*f)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(128\*c^(11/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x +



```

)) - 16*b^2*c^2*(-(a^2*f^2*(230*e + 169*f*x)) + a*c*(12*e^3 + 186*e^2*f*x +
  2*e*f*(36*d - 43*f*x^2) + f^2*x*(186*d - 13*f*x^2)) + c^2*x*(-24*d^2*f + 6
*d*(-4*e^2 + 4*e*f*x + f^2*x^2) + x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*
x^3))) + 32*c^3*(8*c^3*d^3*x - a^3*f^2*(64*e + 15*f*x) + a^2*c*(16*e^3 + 36
*e^2*f*x + f^2*x*(36*d - 5*f*x^2) - 32*e*f*(-3*d + f*x^2)) + 2*a*c^2*(-12*d
^2*(e + f*x) + 6*d*x*(-2*e^2 + 4*e*f*x + f^2*x^2) + x^2*(4*e^3 + 6*e^2*f*x
+ 4*e*f^2*x^2 + f^3*x^3))) + 16*b*c^2*(113*a^3*f^3 + 8*c^3*d^2*(d - 3*e*x)
+ a^2*c*f*(-156*e^2 - 244*e*f*x + f*(-156*d + 49*f*x^2)) + 2*a*c^2*(12*d^2*
f + 6*d*(2*e^2 + 20*e*f*x - 5*f^2*x^2) - x*(-20*e^3 + 30*e^2*f*x + 14*e*f^2
*x^2 + 3*f^3*x^3))))/(64*c^5*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + (3*(10
5*b^4*f^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*c^2*f*(6
*a*b*e*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f) + b*(e^
3 + 6*d*e*f)))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]/(128*c^(11
/2))

```

**IntegrateAlgebraic [A]** time = 5.98, size = 1100, normalized size = 1.69

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2), x]
```

```

[Out] -1/64*(-128*b*c^5*d^3 + 768*a*c^5*d^2*e - 384*a*b*c^4*d*e^2 + 192*a*b^2*c^3
*e^3 - 512*a^2*c^4*e^3 - 384*a*b*c^4*d^2*f + 1152*a*b^2*c^3*d*e*f - 3072*a^
2*c^4*d*e*f - 720*a*b^3*c^2*e^2*f + 2496*a^2*b*c^3*e^2*f - 720*a*b^3*c^2*d*
f^2 + 2496*a^2*b*c^3*d*f^2 + 840*a*b^4*c*e*f^2 - 3680*a^2*b^2*c^2*e*f^2 + 2
048*a^3*c^3*e*f^2 - 315*a*b^5*f^3 + 1680*a^2*b^3*c*f^3 - 1808*a^3*b*c^2*f^3
- 256*c^6*d^3*x + 384*b*c^5*d^2*e*x - 384*b^2*c^4*d*e^2*x + 768*a*c^5*d*e^
2*x + 192*b^3*c^3*e^3*x - 640*a*b*c^4*e^3*x - 384*b^2*c^4*d^2*f*x + 768*a*c
^5*d^2*f*x + 1152*b^3*c^3*d*e*f*x - 3840*a*b*c^4*d*e*f*x - 720*b^4*c^2*e^2*
f*x + 2976*a*b^2*c^3*e^2*f*x - 1152*a^2*c^4*e^2*f*x - 720*b^4*c^2*d*f^2*x +
  2976*a*b^2*c^3*d*f^2*x - 1152*a^2*c^4*d*f^2*x + 840*b^5*c*e*f^2*x - 4240*a
*b^3*c^2*e*f^2*x + 3904*a^2*b*c^3*e*f^2*x - 315*b^6*f^3*x + 1890*a*b^4*c*f^
3*x - 2704*a^2*b^2*c^2*f^3*x + 480*a^3*c^3*f^3*x + 64*b^2*c^4*e^3*x^2 - 256
*a*c^5*e^3*x^2 + 384*b^2*c^4*d*e*f*x^2 - 1536*a*c^5*d*e*f*x^2 - 240*b^3*c^3
*e^2*f*x^2 + 960*a*b*c^4*e^2*f*x^2 - 240*b^3*c^3*d*f^2*x^2 + 960*a*b*c^4*d*
f^2*x^2 + 280*b^4*c^2*e*f^2*x^2 - 1376*a*b^2*c^3*e*f^2*x^2 + 1024*a^2*c^4*e
*f^2*x^2 - 105*b^5*c*f^3*x^2 + 616*a*b^3*c^2*f^3*x^2 - 784*a^2*b*c^3*f^3*x^
2 + 96*b^2*c^4*e^2*f*x^3 - 384*a*c^5*e^2*f*x^3 + 96*b^2*c^4*d*f^2*x^3 - 384
*a*c^5*d*f^2*x^3 - 112*b^3*c^3*e*f^2*x^3 + 448*a*b*c^4*e*f^2*x^3 + 42*b^4*c
^2*f^3*x^3 - 208*a*b^2*c^3*f^3*x^3 + 160*a^2*c^4*f^3*x^3 + 64*b^2*c^4*e*f^2
*x^4 - 256*a*c^5*e*f^2*x^4 - 24*b^3*c^3*f^3*x^4 + 96*a*b*c^4*f^3*x^4 + 16*b
^2*c^4*f^3*x^5 - 64*a*c^5*f^3*x^5)/(c^5*(-b^2 + 4*a*c)*Sqrt[a + b*x + c*x^2
]) - (3*(128*c^4*d*e^2 - 64*b*c^3*e^3 + 128*c^4*d^2*f - 384*b*c^3*d*e*f + 2
40*b^2*c^2*e^2*f - 192*a*c^3*e^2*f + 240*b^2*c^2*d*f^2 - 192*a*c^3*d*f^2 -
  280*b^3*c*e*f^2 + 480*a*b*c^2*e*f^2 + 105*b^4*f^3 - 280*a*b^2*c*f^3 + 80*a^
2*c^2*f^3)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(128*c^(11/2))

```

**fricas [B]** time = 1.95, size = 3143, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2), x, algorithm="fricas")
```

```

[Out] [1/256*(3*(128*(a*b^2*c^4 - 4*a^2*c^5)*d*e^2 - 64*(a*b^3*c^3 - 4*a^2*b*c^4)
*e^3 + 5*(21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*f^3 + 8*
(6*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*d - 5*(7*a*b^5*c - 40*a^2*b^
3*c^2 + 48*a^3*b*c^3)*e)*f^2 + (128*(b^2*c^5 - 4*a*c^6)*d*e^2 - 64*(b^3*c^4
- 4*a*b*c^5)*e^3 + 5*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*

```

$$\begin{aligned}
& c^4) * f^3 + 8 * (6 * (5 * b^4 * c^3 - 24 * a * b^2 * c^4 + 16 * a^2 * c^5) * d - 5 * (7 * b^5 * c^2 - \\
& 40 * a * b^3 * c^3 + 48 * a^2 * b * c^4) * e) * f^2 + 16 * (8 * (b^2 * c^5 - 4 * a * c^6) * d^2 - 24 * (b \\
& ^3 * c^4 - 4 * a * b * c^5) * d * e + 3 * (5 * b^4 * c^3 - 24 * a * b^2 * c^4 + 16 * a^2 * c^5) * e^2) * f) \\
& * x^2 + 16 * (8 * (a * b^2 * c^4 - 4 * a^2 * c^5) * d^2 - 24 * (a * b^3 * c^3 - 4 * a^2 * b * c^4) * d * e \\
& + 3 * (5 * a * b^4 * c^2 - 24 * a^2 * b^2 * c^3 + 16 * a^3 * c^4) * e^2) * f + (128 * (b^3 * c^4 - 4 \\
& * a * b * c^5) * d * e^2 - 64 * (b^4 * c^3 - 4 * a * b^2 * c^4) * e^3 + 5 * (21 * b^7 - 140 * a * b^5 * c \\
& + 240 * a^2 * b^3 * c^2 - 64 * a^3 * b * c^3) * f^3 + 8 * (6 * (5 * b^5 * c^2 - 24 * a * b^3 * c^3 + 16 \\
& * a^2 * b * c^4) * d - 5 * (7 * b^6 * c - 40 * a * b^4 * c^2 + 48 * a^2 * b^2 * c^3) * e) * f^2 + 16 * (8 * \\
& (b^3 * c^4 - 4 * a * b * c^5) * d^2 - 24 * (b^4 * c^3 - 4 * a * b^2 * c^4) * d * e + 3 * (5 * b^5 * c^2 - \\
& 24 * a * b^3 * c^3 + 16 * a^2 * b * c^4) * e^2) * f) * x) * \text{sqrt}(c) * \log(-8 * c^2 * x^2 - 8 * b * c * x - \\
& b^2 - 4 * \text{sqrt}(c * x^2 + b * x + a) * (2 * c * x + b) * \text{sqrt}(c) - 4 * a * c) - 4 * (128 * b * c^6 * \\
& d^3 - 768 * a * c^6 * d^2 * e + 384 * a * b * c^5 * d * e^2 - 16 * (b^2 * c^5 - 4 * a * c^6) * f^3 * x^5 \\
& - 8 * (8 * (b^2 * c^5 - 4 * a * c^6) * e * f^2 - 3 * (b^3 * c^4 - 4 * a * b * c^5) * f^3) * x^4 - 64 * (3 \\
& * a * b^2 * c^4 - 8 * a^2 * c^5) * e^3 + (315 * a * b^5 * c - 1680 * a^2 * b^3 * c^2 + 1808 * a^3 * b * \\
& c^3) * f^3 - 2 * (48 * (b^2 * c^5 - 4 * a * c^6) * e^2 * f + (21 * b^4 * c^3 - 104 * a * b^2 * c^4 + \\
& 80 * a^2 * c^5) * f^3 + 8 * (6 * (b^2 * c^5 - 4 * a * c^6) * d - 7 * (b^3 * c^4 - 4 * a * b * c^5) * e) * f \\
& ^2) * x^3 + 8 * (6 * (15 * a * b^3 * c^3 - 52 * a^2 * b * c^4) * d - (105 * a * b^4 * c^2 - 460 * a^2 * b \\
& ^2 * c^3 + 256 * a^3 * c^4) * e) * f^2 - (64 * (b^2 * c^5 - 4 * a * c^6) * e^3 - 7 * (15 * b^5 * c^2 \\
& - 88 * a * b^3 * c^3 + 112 * a^2 * b * c^4) * f^3 - 8 * (30 * (b^3 * c^4 - 4 * a * b * c^5) * d - (35 * b \\
& ^4 * c^3 - 172 * a * b^2 * c^4 + 128 * a^2 * c^5) * e) * f^2 + 48 * (8 * (b^2 * c^5 - 4 * a * c^6) * d * \\
& e - 5 * (b^3 * c^4 - 4 * a * b * c^5) * e^2) * f) * x^2 + 48 * (8 * a * b * c^5 * d^2 - 8 * (3 * a * b^2 * c^ \\
& 4 - 8 * a^2 * c^5) * d * e + (15 * a * b^3 * c^3 - 52 * a^2 * b * c^4) * e^2) * f + (256 * c^7 * d^3 - \\
& 384 * b * c^6 * d^2 * e + 384 * (b^2 * c^5 - 2 * a * c^6) * d * e^2 - 64 * (3 * b^3 * c^4 - 10 * a * b * c^ \\
& 5) * e^3 + (315 * b^6 * c - 1890 * a * b^4 * c^2 + 2704 * a^2 * b^2 * c^3 - 480 * a^3 * c^4) * f^3 \\
& + 8 * (6 * (15 * b^4 * c^3 - 62 * a * b^2 * c^4 + 24 * a^2 * c^5) * d - (105 * b^5 * c^2 - 530 * a * b^ \\
& 3 * c^3 + 488 * a^2 * b * c^4) * e) * f^2 + 48 * (8 * (b^2 * c^5 - 2 * a * c^6) * d^2 - 8 * (3 * b^3 * c^ \\
& 4 - 10 * a * b * c^5) * d * e + (15 * b^4 * c^3 - 62 * a * b^2 * c^4 + 24 * a^2 * c^5) * e^2) * f) * x) * \text{s} \\
& \text{qrt}(c * x^2 + b * x + a) / (a * b^2 * c^6 - 4 * a^2 * c^7 + (b^2 * c^7 - 4 * a * c^8) * x^2 + (b \\
& ^3 * c^6 - 4 * a * b * c^7) * x), -1 / 128 * (3 * (128 * (a * b^2 * c^4 - 4 * a^2 * c^5) * d * e^2 - 64 * ( \\
& a * b^3 * c^3 - 4 * a^2 * b * c^4) * e^3 + 5 * (21 * a * b^6 - 140 * a^2 * b^4 * c + 240 * a^3 * b^2 * c^ \\
& 2 - 64 * a^4 * c^3) * f^3 + 8 * (6 * (5 * a * b^4 * c^2 - 24 * a^2 * b^2 * c^3 + 16 * a^3 * c^4) * d - \\
& 5 * (7 * a * b^5 * c - 40 * a^2 * b^3 * c^2 + 48 * a^3 * b * c^3) * e) * f^2 + (128 * (b^2 * c^5 - 4 * a * \\
& c^6) * d * e^2 - 64 * (b^3 * c^4 - 4 * a * b * c^5) * e^3 + 5 * (21 * b^6 * c - 140 * a * b^4 * c^2 + 2 \\
& 40 * a^2 * b^2 * c^3 - 64 * a^3 * c^4) * f^3 + 8 * (6 * (5 * b^4 * c^3 - 24 * a * b^2 * c^4 + 16 * a^2 * \\
& c^5) * d - 5 * (7 * b^5 * c^2 - 40 * a * b^3 * c^3 + 48 * a^2 * b * c^4) * e) * f^2 + 16 * (8 * (b^2 * c^ \\
& 5 - 4 * a * c^6) * d^2 - 24 * (b^3 * c^4 - 4 * a * b * c^5) * d * e + 3 * (5 * b^4 * c^3 - 24 * a * b^2 * c \\
& ^4 + 16 * a^2 * c^5) * e^2) * f) * x^2 + 16 * (8 * (a * b^2 * c^4 - 4 * a^2 * c^5) * d^2 - 24 * (a * b^ \\
& 3 * c^3 - 4 * a^2 * b * c^4) * d * e + 3 * (5 * a * b^4 * c^2 - 24 * a^2 * b^2 * c^3 + 16 * a^3 * c^4) * e^ \\
& 2) * f + (128 * (b^3 * c^4 - 4 * a * b * c^5) * d * e^2 - 64 * (b^4 * c^3 - 4 * a * b^2 * c^4) * e^3 + \\
& 5 * (21 * b^7 - 140 * a * b^5 * c + 240 * a^2 * b^3 * c^2 - 64 * a^3 * b * c^3) * f^3 + 8 * (6 * (5 * b^5 \\
& * c^2 - 24 * a * b^3 * c^3 + 16 * a^2 * b * c^4) * d - 5 * (7 * b^6 * c - 40 * a * b^4 * c^2 + 48 * a^2 * \\
& b^2 * c^3) * e) * f^2 + 16 * (8 * (b^3 * c^4 - 4 * a * b * c^5) * d^2 - 24 * (b^4 * c^3 - 4 * a * b^2 * c \\
& ^4) * d * e + 3 * (5 * b^5 * c^2 - 24 * a * b^3 * c^3 + 16 * a^2 * b * c^4) * e^2) * f) * x) * \text{sqrt}(-c) * a \\
& \text{rctan}(1 / 2 * \text{sqrt}(c * x^2 + b * x + a) * (2 * c * x + b) * \text{sqrt}(-c) / (c^2 * x^2 + b * c * x + a * c \\
& )) + 2 * (128 * b * c^6 * d^3 - 768 * a * c^6 * d^2 * e + 384 * a * b * c^5 * d * e^2 - 16 * (b^2 * c^5 - \\
& 4 * a * c^6) * f^3 * x^5 - 8 * (8 * (b^2 * c^5 - 4 * a * c^6) * e * f^2 - 3 * (b^3 * c^4 - 4 * a * b * c^5 \\
& ) * f^3) * x^4 - 64 * (3 * a * b^2 * c^4 - 8 * a^2 * c^5) * e^3 + (315 * a * b^5 * c - 1680 * a^2 * b^3 \\
& * c^2 + 1808 * a^3 * b * c^3) * f^3 - 2 * (48 * (b^2 * c^5 - 4 * a * c^6) * e^2 * f + (21 * b^4 * c^3 \\
& - 104 * a * b^2 * c^4 + 80 * a^2 * c^5) * f^3 + 8 * (6 * (b^2 * c^5 - 4 * a * c^6) * d - 7 * (b^3 * c^4 \\
& - 4 * a * b * c^5) * e) * f^2) * x^3 + 8 * (6 * (15 * a * b^3 * c^3 - 52 * a^2 * b * c^4) * d - (105 * a * b \\
& ^4 * c^2 - 460 * a^2 * b^2 * c^3 + 256 * a^3 * c^4) * e) * f^2 - (64 * (b^2 * c^5 - 4 * a * c^6) * e^ \\
& 3 - 7 * (15 * b^5 * c^2 - 88 * a * b^3 * c^3 + 112 * a^2 * b * c^4) * f^3 - 8 * (30 * (b^3 * c^4 - 4 * \\
& a * b * c^5) * d - (35 * b^4 * c^3 - 172 * a * b^2 * c^4 + 128 * a^2 * c^5) * e) * f^2 + 48 * (8 * (b^2 \\
& * c^5 - 4 * a * c^6) * d * e - 5 * (b^3 * c^4 - 4 * a * b * c^5) * e^2) * f) * x^2 + 48 * (8 * a * b * c^5 * d \\
& ^2 - 8 * (3 * a * b^2 * c^4 - 8 * a^2 * c^5) * d * e + (15 * a * b^3 * c^3 - 52 * a^2 * b * c^4) * e^2) * f \\
& + (256 * c^7 * d^3 - 384 * b * c^6 * d^2 * e + 384 * (b^2 * c^5 - 2 * a * c^6) * d * e^2 - 64 * (3 * b \\
& ^3 * c^4 - 10 * a * b * c^5) * e^3 + (315 * b^6 * c - 1890 * a * b^4 * c^2 + 2704 * a^2 * b^2 * c^3 - \\
& 480 * a^3 * c^4) * f^3 + 8 * (6 * (15 * b^4 * c^3 - 62 * a * b^2 * c^4 + 24 * a^2 * c^5) * d - (105 * \\
& b^5 * c^2 - 530 * a * b^3 * c^3 + 488 * a^2 * b * c^4) * e) * f^2 + 48 * (8 * (b^2 * c^5 - 2 * a * c^6)
\end{aligned}$$

$$*d^2 - 8*(3*b^3*c^4 - 10*a*b*c^5)*d*e + (15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*e^2)*f)*x)*\sqrt{c*x^2 + b*x + a})/(a*b^2*c^6 - 4*a^2*c^7 + (b^2*c^7 - 4*a*c^8)*x^2 + (b^3*c^6 - 4*a*b*c^7)*x)]$$

**giac [A]** time = 0.42, size = 1099, normalized size = 1.69

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(3/2),x, algorithm="giac")

[Out]  $\frac{1}{64} * ((2*(4*(2*(b^2*c^4*f^3 - 4*a*c^5*f^3)*x/(b^2*c^5 - 4*a*c^6) - (3*b^3*c^3*f^3 - 12*a*b*c^4*f^3 - 8*b^2*c^4*f^2*e + 32*a*c^5*f^2*e)/(b^2*c^5 - 4*a*c^6)))*x + (48*b^2*c^4*d*f^2 - 192*a*c^5*d*f^2 + 21*b^4*c^2*f^3 - 104*a*b^2*c^3*f^3 + 80*a^2*c^4*f^3 - 56*b^3*c^3*f^2*e + 224*a*b*c^4*f^2*e + 48*b^2*c^4*f*e^2 - 192*a*c^5*f*e^2)/(b^2*c^5 - 4*a*c^6))*x - (240*b^3*c^3*d*f^2 - 960*a*b*c^4*d*f^2 + 105*b^5*c*f^3 - 616*a*b^3*c^2*f^3 + 784*a^2*b*c^3*f^3 - 384*b^2*c^4*d*f*e + 1536*a*c^5*d*f*e - 280*b^4*c^2*f^2*e + 1376*a*b^2*c^3*f^2*e - 1024*a^2*c^4*f^2*e + 240*b^3*c^3*f*e^2 - 960*a*b*c^4*f*e^2 - 64*b^2*c^4*e^3 + 256*a*c^5*e^3)/(b^2*c^5 - 4*a*c^6))*x - (256*c^6*d^3 + 384*b^2*c^4*d^2*f - 768*a*c^5*d^2*f + 720*b^4*c^2*d*f^2 - 2976*a*b^2*c^3*d*f^2 + 1152*a^2*c^4*d*f^2 + 315*b^6*f^3 - 1890*a*b^4*c*f^3 + 2704*a^2*b^2*c^2*f^3 - 480*a^3*c^3*f^3 - 384*b*c^5*d^2*e - 1152*b^3*c^3*d*f*e + 3840*a*b*c^4*d*f*e - 840*b^5*c*f^2*e + 4240*a*b^3*c^2*f^2*e - 3904*a^2*b*c^3*f^2*e + 384*b^2*c^4*d*e^2 - 768*a*c^5*d*e^2 + 720*b^4*c^2*f*e^2 - 2976*a*b^2*c^3*f*e^2 + 1152*a^2*c^4*f*e^2 - 192*b^3*c^3*e^3 + 640*a*b*c^4*e^3)/(b^2*c^5 - 4*a*c^6))*x - (128*b*c^5*d^3 + 384*a*b*c^4*d^2*f + 720*a*b^3*c^2*d*f^2 - 2496*a^2*b*c^3*d*f^2 + 315*a*b^5*f^3 - 1680*a^2*b^3*c*f^3 + 1808*a^3*b*c^2*f^3 - 768*a*c^5*d^2*e - 1152*a*b^2*c^3*d*f*e + 3072*a^2*c^4*d*f*e - 840*a*b^4*c*f^2*e + 3680*a^2*b^2*c^2*f^2*e - 2048*a^3*c^3*f^2*e + 384*a*b*c^4*d*e^2 + 720*a*b^3*c^2*f*e^2 - 2496*a^2*b*c^3*f*e^2 - 192*a*b^2*c^3*e^3 + 512*a^2*c^4*e^3)/(b^2*c^5 - 4*a*c^6))/sqrt(c*x^2 + b*x + a) - 3/128*(128*c^4*d^2*f + 240*b^2*c^2*d*f^2 - 192*a*c^3*d*f^2 + 105*b^4*f^3 - 280*a*b^2*c*f^3 + 80*a^2*c^2*f^3 - 384*b*c^3*d*f*e - 280*b^3*c*f^2*e + 480*a*b*c^2*f^2*e + 128*c^4*d*e^2 + 240*b^2*c^2*f*e^2 - 192*a*c^3*f*e^2 - 64*b*c^3*e^3)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)$

**maple [B]** time = 0.03, size = 2827, normalized size = 4.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(3/2),x)

[Out]  $2*a/c^2/(c*x^2+b*x+a)^(1/2)*e^3+2*d^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+315/128*f^3*b^4/c^(11/2)*\ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+15/8*f^3*a^2/c^(7/2)*\ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+315/256*f^3*b^5/c^6/(c*x^2+b*x+a)^(1/2)+1/4*f^3*x^5/c/(c*x^2+b*x+a)^(1/2)+3/c^(3/2)*\ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*d^2+3/c^(3/2)*\ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*d-3*d^2*e/c/(c*x^2+b*x+a)^(1/2)+x^2/c/(c*x^2+b*x+a)^(1/2)*e^3-3/4*b^2/c^3/(c*x^2+b*x+a)^(1/2)*e^3-3/2*b/c^(5/2)*\ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^3+115/4*e*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-9*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*e*f+12*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*e*f-39/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*f^2-39/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^2*f-16*e*f^2*a^2/c^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+21/32*f^3*b^2/c^3*x^3/(c*x^2+b*x+a)^(1/2)-105/64*f^3*b^3/c^4*x^2/(c*x^2+b*x+a)^(1/2)-315/128*f^3*b^4/c^5*x/(c*x^2+b*x+a)^(1/2)+315/256*f^3*b^7/c^6/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-105/16*f^3*b^2/c^(9/2)*a*\ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))$

$$\begin{aligned} &)^{(1/2)} - 5/8 f^3 a / c^2 x^3 / (c x^2 + b x + a)^{(1/2)} - 15/8 f^3 a^2 / c^3 x / (c x^2 + b x + a)^{(1/2)} - 3 x / c / (c x^2 + b x + a)^{(1/2)} * f^d - 3 x / c / (c x^2 + b x + a)^{(1/2)} * e^{2d} + \\ &3/2 x^3 / c / (c x^2 + b x + a)^{(1/2)} * d f^2 + 3/2 x^3 / c / (c x^2 + b x + a)^{(1/2)} * e^{2f} + 45/16 b^3 / c^4 / (c x^2 + b x + a)^{(1/2)} * d f^2 + 45/16 b^3 / c^4 / (c x^2 + b x + a)^{(1/2)} * e^{2f} + \\ &45/8 b^2 / c^{(7/2)} * \ln((c x + 1/2 b) / c^{(1/2)} + (c x^2 + b x + a)^{(1/2)}) * d f^2 + 45/8 b^2 / c^{(7/2)} * \ln((c x + 1/2 b) / c^{(1/2)} + (c x^2 + b x + a)^{(1/2)}) * e^{2f} - 9/2 a / c^{(5/2)} * \\ &\ln((c x + 1/2 b) / c^{(1/2)} + (c x^2 + b x + a)^{(1/2)}) * d f^2 - 9/2 a / c^{(5/2)} * \ln((c x + 1/2 b) / c^{(1/2)} + (c x^2 + b x + a)^{(1/2)}) * e^{2f} + 3/2 b / c^2 x / (c x^2 + b x + a)^{(1/2)} * e^{3-} \\ &3/4 b^4 / c^3 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * e^3 + e f^2 x^4 / c / (c x^2 + b x + a)^{(1/2)} - 105/32 e f^2 b^4 / c^5 / (c x^2 + b x + a)^{(1/2)} - 105/16 e f^2 b^3 / c^{(9/2)} * \ln((c x + 1/2 b) / c^{(1/2)} + (c x^2 + b x + a)^{(1/2)}) - 8 e f^2 a^2 / c^3 / (c x^2 + b x + a)^{(1/2)} \\ &+ 3/2 b / c^2 / (c x^2 + b x + a)^{(1/2)} * f^d + 3/2 b / c^2 / (c x^2 + b x + a)^{(1/2)} * e^{2d} - 105/16 f^3 b^3 / c^5 a / (c x^2 + b x + a)^{(1/2)} + 113/16 f^3 b / c^4 a^2 / (c x^2 + b x + a)^{(1/2)} + 24 a / c b / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * x d e f + 9 b / c^2 x / (c x^2 + b x + a)^{(1/2)} * d e f - 9/2 b^4 / c^3 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * d e f + 4 a / c b / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * x e^3 + 3 b^2 / c / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * x f^d + 3 b^2 / c / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * x e^{2d} - 8 e f^2 a^2 / c^3 b^2 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} + 45/8 b^4 / c^3 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * x d f^2 + 45/8 b^4 / c^3 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * x e^{2f} - 39/4 b^3 / c^3 a / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * d f^2 - 39/4 b^3 / c^3 a / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * e^{2f} - 105/16 e f^2 b^5 / c^4 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * x + 115/8 e f^2 b^4 / c^4 a / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} - 45/4 e f^2 b / c^3 a x / (c x^2 + b x + a)^{(1/2)} + 113/8 f^3 b^2 / c^3 a^2 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * x - 105/8 f^3 b^4 / c^4 a / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * x + 105/16 e f^2 b^3 / c^4 x / (c x^2 + b x + a)^{(1/2)} - 105/32 e f^2 b^6 / c^5 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} + 115/8 e f^2 b^2 / c^4 a / (c x^2 + b x + a)^{(1/2)} + 45/4 e f^2 b / c^{(7/2)} a * \ln((c x + 1/2 b) / c^{(1/2)} + (c x^2 + b x + a)^{(1/2)}) - 4 e f^2 a / c^2 x^2 / (c x^2 + b x + a)^{(1/2)} + 35/128 f^3 b^6 / c^5 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * x - 105/16 f^3 b^5 / c^5 a / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} + 105/16 f^3 b^2 / c^4 a x / (c x^2 + b x + a)^{(1/2)} + 49/16 f^3 b / c^3 a x^2 / (c x^2 + b x + a)^{(1/2)} + 113/16 f^3 b^3 / c^4 a^2 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} + 6 x^2 / c / (c x^2 + b x + a)^{(1/2)} * d e f - 9/2 b^2 / c^3 / (c x^2 + b x + a)^{(1/2)} * d e f - 3/2 b^3 / c^2 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * x e^3 - 9 b / c^{(5/2)} * \ln((c x + 1/2 b) / c^{(1/2)} + (c x^2 + b x + a)^{(1/2)}) * d e f + 12 a / c^2 / (c x^2 + b x + a)^{(1/2)} * d e f + 2 a / c^2 b^2 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * e^3 + 3/2 b^3 / c^2 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * f^d + 3/2 b^3 / c^2 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * e^{2d} - 6 d^2 e b / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * x - 3 d^2 e b^2 / c / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} - 15/4 b / c^2 x^2 / (c x^2 + b x + a)^{(1/2)} * d f^2 - 15/4 b / c^2 x^2 / (c x^2 + b x + a)^{(1/2)} * e^{2f} - 45/8 b^2 / c^3 x / (c x^2 + b x + a)^{(1/2)} * e^{2f} - 7/4 e f^2 b / c^2 x^3 / (c x^2 + b x + a)^{(1/2)} + 35/8 e f^2 b^2 / c^3 x^2 / (c x^2 + b x + a)^{(1/2)} + 45/16 b^5 / c^4 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * e^{2f} - 39/4 b / c^3 a / (c x^2 + b x + a)^{(1/2)} * d f^2 - 39/4 b / c^3 a / (c x^2 + b x + a)^{(1/2)} * e^{2f} + 9/2 a / c^2 x / (c x^2 + b x + a)^{(1/2)} * d f^2 + 9/2 a / c^2 x / (c x^2 + b x + a)^{(1/2)} * e^{2f} - 3/8 f^3 b / c^2 x^4 / (c x^2 + b x + a)^{(1/2)} + 45/16 b^5 / c^4 / (4 a c - b^2) / (c x^2 + b x + a)^{(1/2)} * d f^2 \end{aligned}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(fx^2 + ex + d)^3}{(cx^2 + bx + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(3/2), x)

[Out] int((d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex + fx^2)^3}{(a + bx + cx^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*\*3/(c\*x\*\*2+b\*x+a)\*\*(3/2), x)

[Out] Integral((d + e\*x + f\*x\*\*2)\*\*3/(a + b\*x + c\*x\*\*2)\*\*(3/2), x)



$$3.112 \quad \int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=309

$$\frac{2(-x(c^2(2a^2f^2 + 6abef + b^2(2df + e^2)) - 2b^2cf(2af + be) - 2c^3(a(2df + e^2) + bde) + b^4f^2 + 2c^4d^2) - bc}{c^3(b^2 - 4ac)\sqrt{a + bx + cx^2}}$$

**Rubi [A]** time = 0.45, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, number of rules / integrand size = 0.185, Rules used = {1660, 1661, 640, 621, 206}

$$\frac{2(-x(c^2(2a^2f^2 + 6abef + b^2(2df + e^2)) - 2b^2cf(2af + be) - 2c^3(a(2df + e^2) + bde) + b^4f^2 + 2c^4d^2) - bc(-3a^2f^2 + ac(2df + e^2) + c^2d^2) + 2ab^2cef - ab^3f^2 + 4ac^2e(ad - ef))}{c^3(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{\tanh^{-1}\left(\frac{bx}{2c\sqrt{a+bx+cx^2}}\right)(-12cf(af + 2be) + 15b^2f^2 + 8c^2(2df + e^2))}{8c^{7/2}} + \frac{f\sqrt{a + bx + cx^2}(8ce - 7bf)}{4c^3} + \frac{f^2x\sqrt{a + bx + cx^2}}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (2\*(2\*a\*b^2\*c\*e\*f - a\*b^3\*f^2 + 4\*a\*c^2\*e\*(c\*d - a\*f) - b\*c\*(c^2\*d^2 - 3\*a^2\*f^2 + a\*c\*(e^2 + 2\*d\*f)) - (2\*c^4\*d^2 + b^4\*f^2 - 2\*b^2\*c\*f\*(b\*e + 2\*a\*f) - 2\*c^3\*(b\*d\*e + a\*(e^2 + 2\*d\*f)) + c^2\*(6\*a\*b\*e\*f + 2\*a^2\*f^2 + b^2\*(e^2 + 2\*d\*f)))\*x)/(c^3\*(b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]) + (f\*(8\*c\*e - 7\*b\*f)\*Sqrt[a + b\*x + c\*x^2])/(4\*c^3) + (f^2\*x\*Sqrt[a + b\*x + c\*x^2])/(2\*c^2) + ((15\*b^2\*f^2 - 12\*c\*f\*(2\*b\*e + a\*f) + 8\*c^2\*(e^2 + 2\*d\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(8\*c^(7/2))

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1660

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

#### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d + ex + fx^2)^2}{(a + bx + cx^2)^{3/2}} dx = \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4))}{c^3(b^2 - 4ac)}$$

$$= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4))}{c^3(b^2 - 4ac)}$$

$$= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4))}{c^3(b^2 - 4ac)}$$

$$= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4))}{c^3(b^2 - 4ac)}$$

$$= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4))}{c^3(b^2 - 4ac)}$$

**Mathematica [A]** time = 0.73, size = 288, normalized size = 0.93

$$\frac{4bc(-13a^2f^2 + ac(4df + 2a^2 + 20bf^2 - 5f^2x^2) + 2^2d(d - 2ax)) + 8c^2(f^2(8c + 3fx) + ac(x(-2a^2 + 4efx + f^2x^2) - 4d(e + fx) + 2c^2d^2a) + b^2f(15af + cx(5fx - 24a)) - 2b^2c(af(12a + 3fx) + cx(-8df - 4c^2 + 4efx + f^2x^2)) + 15b^2f^2x + \log(2\sqrt{a+bx+cx^2} + b + 2cx) - 12c^2f(e^2 + 2df) + 8c^2(2df + f^2))}{4c^3(4ac - b^2)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2), x]
```

```
[Out] (15*b^4*f^2*x + b^3*f*(15*a*f + c*x*(-24*e + 5*f*x)) + 4*b*c*(-13*a^2*f^2 + 2*c^2*d*(d - 2*e*x) + a*c*(2*e^2 + 4*d*f + 20*e*f*x - 5*f^2*x^2)) - 2*b^2*c*(a*f*(12*e + 31*f*x) + c*x*(-4*e^2 - 8*d*f + 4*e*f*x + f^2*x^2)) + 8*c^2*(2*c^2*d^2*x + a^2*f*(8*e + 3*f*x) + a*c*(-4*d*(e + f*x) + x*(-2*e^2 + 4*e*f*x + f^2*x^2)))/(4*c^3*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + ((15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])/(8*c^(7/2))
```

**IntegrateAlgebraic [A]** time = 2.16, size = 375, normalized size = 1.21

$$\frac{\log(-2\sqrt{a+bx+cx^2} + b + 2cx) (12ac^2f^2 - 15b^2f^2 + 24bc^2f - 16c^2df - 8c^2d^2) + 52b^2bc^2f - 64a^2c^2f - 24a^2f^2fx - 15ab^2f^2 + 24a^2cf + 62ab^2f^2x - 16ab^2df - 8ab^2c^2 - 80abc^2fx + 20ab^2f^2x^2 + 32ac^2b^2 + 32ac^2dfx + 16ac^2d^2x - 8ac^2f^2x - 15b^2fx + 24b^2fx - 5b^2fx^2 - 16b^2dfx - 8b^2d^2x + 8b^2dfx^2 + 2b^2d^2fx^2 - 8b^2d^2x - 16b^2d^2x - 16c^2d^2}{4c^3(4ac - b^2)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2), x]
```

```
[Out] -1/4*(-8*b*c^3*d^2 + 32*a*c^3*d*e - 8*a*b*c^2*e^2 - 16*a*b*c^2*d*f + 24*a*b^2*c*e*f - 64*a^2*c^2*e*f - 15*a*b^3*f^2 + 52*a^2*b*c*f^2 - 16*c^4*d^2*x + 16*b*c^3*d*e*x - 8*b^2*c^2*e^2*x + 16*a*c^3*e^2*x - 16*b^2*c^2*d*f*x + 32*a*c^3*d*f*x + 24*b^3*c*e*f*x - 80*a*b*c^2*e*f*x - 15*b^4*f^2*x + 62*a*b^2*c*f^2*x - 24*a^2*c^2*f^2*x + 8*b^2*c^2*e*f*x^2 - 32*a*c^3*e*f*x^2 - 5*b^3*c*f
```

$$\frac{\begin{aligned} &^2*x^2 + 20*a*b*c^2*f^2*x^2 + 2*b^2*c^2*f^2*x^3 - 8*a*c^3*f^2*x^3)/(c^3*(-b \\ &^2 + 4*a*c)*\text{Sqrt}[a + b*x + c*x^2]) + ((-8*c^2*e^2 - 16*c^2*d*f + 24*b*c*e*f \\ &- 15*b^2*f^2 + 12*a*c*f^2)*\text{Log}[b*c^3 + 2*c^4*x - 2*c^{(7/2)}*\text{Sqrt}[a + b*x + \\ &c*x^2]])/(8*c^{(7/2)}) \end{aligned}}$$

**fricas** [B] time = 1.24, size = 1305, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(3/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} &[-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*e^2 + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3 \\ &*c^2)*f^2 + (8*(b^2*c^3 - 4*a*c^4)*e^2 + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2 \\ &*c^3)*f^2 + 8*(2*(b^2*c^3 - 4*a*c^4)*d - 3*(b^3*c^2 - 4*a*b*c^3)*e)*f)*x^2 \\ &+ 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*d - 3*(a*b^3*c - 4*a^2*b*c^2)*e)*f + (8*(b^3 \\ &*c^2 - 4*a*b*c^3)*e^2 + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f^2 + 8*(2*(b \\ &^3*c^2 - 4*a*b*c^3)*d - 3*(b^4*c - 4*a*b^2*c^2)*e)*f)*x)*\text{sqrt}(c)*\text{log}(-8*c^2 \\ &*x^2 - 8*b*c*x - b^2 + 4*\text{sqrt}(c*x^2 + b*x + a)*(2*c*x + b)*\text{sqrt}(c) - 4*a*c) \\ &+ 4*(8*b*c^4*d^2 - 32*a*c^4*d*e + 8*a*b*c^3*e^2 - 2*(b^2*c^3 - 4*a*c^4)*f^2 \\ &2*x^3 + (15*a*b^3*c - 52*a^2*b*c^2)*f^2 - (8*(b^2*c^3 - 4*a*c^4)*e*f - 5*(b \\ &^3*c^2 - 4*a*b*c^3)*f^2)*x^2 + 8*(2*a*b*c^3*d - (3*a*b^2*c^2 - 8*a^2*c^3)*e \\ &)*f + (16*c^5*d^2 - 16*b*c^4*d*e + 8*(b^2*c^3 - 2*a*c^4)*e^2 + (15*b^4*c - \\ &62*a*b^2*c^2 + 24*a^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 2*a*c^4)*d - (3*b^3*c^2 - \\ &10*a*b*c^3)*e)*f)*x)*\text{sqrt}(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c \\ &^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*((8*(a*b^2*c^2 - 4*a^2*c \\ &^3)*e^2 + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f^2 + (8*(b^2*c^3 - 4*a*c \\ &^4)*e^2 + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 4*a \\ &*c^4)*d - 3*(b^3*c^2 - 4*a*b*c^3)*e)*f)*x^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)* \\ &d - 3*(a*b^3*c - 4*a^2*b*c^2)*e)*f + (8*(b^3*c^2 - 4*a*b*c^3)*e^2 + 3*(5*b^5 \\ &- 24*a*b^3*c + 16*a^2*b*c^2)*f^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*d - 3*(b^4*c \\ &- 4*a*b^2*c^2)*e)*f)*x)*\text{sqrt}(-c)*\text{arctan}(1/2*\text{sqrt}(c*x^2 + b*x + a)*(2*c*x \\ &+ b)*\text{sqrt}(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(8*b*c^4*d^2 - 32*a*c^4*d*e + 8* \\ &a*b*c^3*e^2 - 2*(b^2*c^3 - 4*a*c^4)*f^2*x^3 + (15*a*b^3*c - 52*a^2*b*c^2)*f \\ &^2 - (8*(b^2*c^3 - 4*a*c^4)*e*f - 5*(b^3*c^2 - 4*a*b*c^3)*f^2)*x^2 + 8*(2*a \\ &*b*c^3*d - (3*a*b^2*c^2 - 8*a^2*c^3)*e)*f + (16*c^5*d^2 - 16*b*c^4*d*e + 8* \\ &(b^2*c^3 - 2*a*c^4)*e^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*f^2 + 8*(2 \\ &*(b^2*c^3 - 2*a*c^4)*d - (3*b^3*c^2 - 10*a*b*c^3)*e)*f)*x)*\text{sqrt}(c*x^2 + b*x \\ &+ a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b \\ &c^5)*x] \end{aligned}$$

**giac** [A] time = 0.35, size = 407, normalized size = 1.32

$$\frac{\left(\frac{2(b^2c^2 - 4ac^3)f^2}{b^2c^2 - 4ac^3} - \frac{5b^4c^2 - 20ab^2c^2 - 8b^2c^2f^2 + 32a^2c^2}{b^2c^2 - 4ac^3}\right)x - \frac{16c^4d^2 + 16b^2c^2df - 32a^2c^2d^2 + 12a^2c^2f^2 - 16b^2c^2d^2 + 24b^2c^2f^2 + 80ab^2c^2d^2 - 16ac^2d^2}{4\sqrt{cx^2 + bx + a}} - \frac{8b^3c^2 - 16ab^2c^2f^2 + 15a^2c^2f^2 - 32a^2c^2d^2 - 24ab^2c^2d^2 + 64a^2c^2f^2}{b^2c^2 - 4ac^3} \cdot \frac{(16c^2df + 15b^2f^2 - 12ac^2 - 24b^2c^2)\log\left(-2\left(\sqrt{cx^2 + bx + a}\right)\sqrt{c - b}\right)}{8c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(3/2),x, algorithm="giac")

[Out] 
$$\begin{aligned} &1/4*(((2*(b^2*c^2*f^2 - 4*a*c^3*f^2)*x/(b^2*c^3 - 4*a*c^4) - (5*b^3*c*f^2 - \\ &20*a*b*c^2*f^2 - 8*b^2*c^2*f*e + 32*a*c^3*f*e)/(b^2*c^3 - 4*a*c^4))*x - (1 \\ &6*c^4*d^2 + 16*b^2*c^2*d*f - 32*a*c^3*d*f + 15*b^4*f^2 - 62*a*b^2*c*f^2 + 2 \\ &4*a^2*c^2*f^2 - 16*b*c^3*d*e - 24*b^3*c*f*e + 80*a*b*c^2*d*f + 8*b^2*c^2*e^2 \\ &- 16*a*c^3*e^2)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d^2 + 16*a*b*c^2*d*f + \\ &15*a*b^3*f^2 - 52*a^2*b*c*f^2 - 32*a*c^3*d*e - 24*a*b^2*c*f*e + 64*a^2*c^2* \\ &f*e + 8*a*b*c^2*e^2)/(b^2*c^3 - 4*a*c^4))/\text{sqrt}(c*x^2 + b*x + a) - 1/8*(16*c \\ &^2*d*f + 15*b^2*f^2 - 12*a*c*f^2 - 24*b*c*f*e + 8*c^2*e^2)*\text{log}(\text{abs}(-2*(\text{sqrt} \\ &(c)*x - \text{sqrt}(c*x^2 + b*x + a))*\text{sqrt}(c) - b))/c^{(7/2)} \end{aligned}$$

**maple** [B] time = 0.02, size = 1011, normalized size = 3.27

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x)`

[Out] 
$$\begin{aligned} & -2*d*e/c/(c*x^2+b*x+a)^{(1/2)}-x/c/(c*x^2+b*x+a)^{(1/2)}*e^2+1/2*b/c^2/(c*x^2+b*x+a)^{(1/2)}*e^2+2/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*f+1/2*f^2*x^3/c/(c*x^2+b*x+a)^{(1/2)}+15/16*f^2*b^3/c^4/(c*x^2+b*x+a)^{(1/2)}+15/8*f^2*b^2/c^{(7/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-3/2*f^2*a/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+2*d^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}+4*e*f*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}+2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*d*f-13/2*f^2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x-3*e*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x+8*e*f*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x-13/4*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}-4*d*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x+15/8*f^2*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x-2*d*e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}+3*e*f*b/c^2*x/(c*x^2+b*x+a)^{(1/2)}-3/2*e*f*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}+b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*e^2+b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*d*f+1/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2+1/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*e^2+2*e*f*x^2/c/(c*x^2+b*x+a)^{(1/2)}-3/2*e*f*b^2/c^3/(c*x^2+b*x+a)^{(1/2)}-3*e*f*b/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+4*e*f*a/c^2/(c*x^2+b*x+a)^{(1/2)}-5/4*f^2*b/c^2*x^2/(c*x^2+b*x+a)^{(1/2)}-15/8*f^2*b^2/c^3*x/(c*x^2+b*x+a)^{(1/2)}+15/16*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}-13/4*f^2*b/c^3*a/(c*x^2+b*x+a)^{(1/2)}+3/2*f^2*a/c^2*x/(c*x^2+b*x+a)^{(1/2)}-2*x/c/(c*x^2+b*x+a)^{(1/2)}*d*f+b/c^2/(c*x^2+b*x+a)^{(1/2)}*d*f \end{aligned}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f x^2 + e x + d)^2}{(c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x)`

[Out] `int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + e x + f x^2)^2}{(a + b x + c x^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(3/2),x)`

[Out] `Integral((d + e*x + f*x**2)**2/(a + b*x + c*x**2)**(3/2), x)`

$$3.113 \quad \int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}} dx$$

Optimal. Leaf size=111

$$\frac{2\left(c\left(2ae - b\left(\frac{af}{c} + d\right)\right) - x(-2acf + b^2f - bce + 2c^2d)\right)}{c(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{f \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1660, 12, 621, 206}

$$\frac{2\left(c\left(2ae - b\left(\frac{af}{c} + d\right)\right) - x(-2acf + b^2f - bce + 2c^2d)\right)}{c(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{f \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (2\*(c\*(2\*a\*e - b\*(d + (a\*f)/c)) - (2\*c^2\*d - b\*c\*e + b^2\*f - 2\*a\*c\*f)\*x))/(c\*(b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]) + (f\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/c^(3/2)

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{d + ex + fx^2}{(a + bx + cx^2)^{3/2}} dx &= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} - \frac{2 \int -\frac{(b^2 - 4ac)f}{2c \sqrt{a + bx + cx^2}} dx}{b^2 - 4ac} \\
&= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} + \frac{f \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{c} \\
&= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} + \frac{(2f) \text{Subst} \left( \int \frac{1}{4c - x^2} dx, x, \frac{b}{\sqrt{a + bx + cx^2}} \right)}{c} \\
&= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} + \frac{f \tanh^{-1} \left( \frac{b + 2cx}{2\sqrt{c} \sqrt{a + bx + cx^2}} \right)}{c^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.31, size = 113, normalized size = 1.02

$$\frac{\frac{2\sqrt{c}(abf - 2ac(e + fx) + b^2fx + bc(d - ex) + 2c^2dx)}{\sqrt{a + x(b + cx)}} - f(b^2 - 4ac) \log(2\sqrt{c} \sqrt{a + x(b + cx)} + b + 2cx)}{c^{3/2}(4ac - b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(3/2), x]

[Out] ((2\*sqrt[c]\*(a\*b\*f + 2\*c^2\*d\*x + b^2\*f\*x + b\*c\*(d - e\*x) - 2\*a\*c\*(e + f\*x))/sqrt[a + x\*(b + c\*x)] - (b^2 - 4\*a\*c)\*f\*Log[b + 2\*c\*x + 2\*sqrt[c]\*sqrt[a + x\*(b + c\*x)]])/(c^(3/2)\*(-b^2 + 4\*a\*c))

**IntegrateAlgebraic [A]** time = 0.55, size = 111, normalized size = 1.00

$$\frac{2(abf - 2ace - 2acfx + b^2fx + bcd - bcex + 2c^2dx)}{c(4ac - b^2) \sqrt{a + bx + cx^2}} - \frac{f \log(-2c^{3/2} \sqrt{a + bx + cx^2} + bc + 2c^2x)}{c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (2\*(b\*c\*d - 2\*a\*c\*e + a\*b\*f + 2\*c^2\*d\*x - b\*c\*e\*x + b^2\*f\*x - 2\*a\*c\*f\*x))/(c\*(-b^2 + 4\*a\*c)\*sqrt[a + b\*x + c\*x^2]) - (f\*Log[b\*c + 2\*c^2\*x - 2\*c^(3/2)\*sqrt[a + b\*x + c\*x^2]])/c^(3/2)

**fricas [B]** time = 0.78, size = 429, normalized size = 3.86

$$\frac{\left( \frac{(b^2 - 4ac)^2 f^2 + (b^2 - 4ac) f x + (ab^2 - 4a^2 c) f \sqrt{c} \log\left(-8c^2 x^2 - 8b^2 c x - b^2 - 4\sqrt{c^2 + bx + a}(2cx + b)\sqrt{c - 4ac}\right) - 4((b^2 d - 2ac^2 e + abcf + (2c^2 d - bc^2 e + (b^2 - 2ac^2) f) x) \sqrt{c^2 + bx + a}}{2(ab^2 - 4a^2 c) + (b^2 - 4ac)^2} + (b^2 - 4ac)^2 \right) \sqrt{c} \arctan\left(\frac{\sqrt{c^2 + bx + a}}{2\sqrt{c} \sqrt{a + bx + cx^2}}\right) + 2(b^2 d - 2ac^2 e + abcf + (2c^2 d - bc^2 e + (b^2 - 2ac^2) f) x) \sqrt{c^2 + bx + a}}{ab^2 - 4a^2 c + (b^2 - 4ac)^2} + (b^2 - 4ac)^2 \right) / (c^2 x^2 + b^2 c x + a^2 c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2), x, algorithm="fricas")

[Out] [1/2\*((b^2\*c - 4\*a\*c^2)\*f\*x^2 + (b^3 - 4\*a\*b\*c)\*f\*x + (a\*b^2 - 4\*a^2\*c)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(b\*c^2\*d - 2\*a\*c^2\*e + a\*b\*c\*f + (2\*c^3\*d - b\*c^2\*e + (b^2\*c - 2\*a\*c^2)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a\*b^2\*c^2 - 4\*a^2\*c^3 + (b^2\*c^3 - 4\*a\*c^4)\*x^2 + (b^3\*c^2 - 4\*a\*b\*c^3)\*x), -(((b^2\*c - 4\*a\*c^2)\*f\*x^2 + (b^3 - 4\*a\*b\*c)\*f\*x + (a\*b^2 - 4\*a^2\*c)\*f)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) + 2\*(b\*c^2\*d

$- 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*x)*\sqrt{c*x^2 + b*x + a})/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x]$

**giac** [A] time = 0.31, size = 122, normalized size = 1.10

$$\frac{2 \left( \frac{(2c^2d + b^2f - 2acf - bce)x}{b^2c - 4ac^2} + \frac{bcd + abf - 2ace}{b^2c - 4ac^2} \right) f \log \left( \left| -2 \left( \sqrt{c}x - \sqrt{cx^2 + bx + a} \right) \sqrt{c} - b \right| \right)}{\sqrt{cx^2 + bx + a} \cdot \frac{3}{c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="giac")

[Out]  $-2*((2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*x/(b^2*c - 4*a*c^2) + (b*c*d + a*b*f - 2*a*c*e)/(b^2*c - 4*a*c^2))/\sqrt{c*x^2 + b*x + a} - f*\log(\text{abs}(-2*(\sqrt{c}(c*x - \sqrt{c*x^2 + b*x + a}))*\sqrt{c} - b))/c^{3/2}$

**maple** [B] time = 0.01, size = 249, normalized size = 2.24

$$\frac{\frac{b^2fx}{(4ac-b^2)\sqrt{cx^2+bx+a}c} - \frac{2bex}{(4ac-b^2)\sqrt{cx^2+bx+a}} + \frac{b^2f}{2(4ac-b^2)\sqrt{cx^2+bx+a}c^2} - \frac{b^2e}{(4ac-b^2)\sqrt{cx^2+bx+a}c} - \frac{fx}{\sqrt{cx^2+bx+a}c} + \frac{2(2cx+b)d}{(4ac-b^2)\sqrt{cx^2+bx+a}} + \frac{f \ln \left( \frac{cx+\frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2+bx+a} \right)}{c^{\frac{3}{2}}} + \frac{bf}{2\sqrt{cx^2+bx+a}c^2} - \frac{e}{\sqrt{cx^2+bx+a}c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x)

[Out]  $-f*x/c/(c*x^2+b*x+a)^{1/2} + 1/2*f*b/c^2/(c*x^2+b*x+a)^{1/2} + f*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{1/2} * x + 1/2*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{1/2} + f/c^{3/2}*\ln((c*x+1/2*b)/c^{1/2}+(c*x^2+b*x+a)^{1/2}) - e/c/(c*x^2+b*x+a)^{1/2} - 2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{1/2} * x - e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{1/2} + 2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^{1/2}$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see 'assume?' for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [B] time = 3.73, size = 143, normalized size = 1.29

$$\frac{f \ln \left( \frac{b+cx}{\sqrt{c}} + \sqrt{cx^2 + bx + a} \right)}{c^{3/2}} - \frac{e(4a + 2bx)}{(4ac - b^2)\sqrt{cx^2 + bx + a}} + \frac{d \left( \frac{b}{2} + cx \right)}{\left( ac - \frac{b^2}{4} \right) \sqrt{cx^2 + bx + a}} + \frac{f \left( \frac{ab}{2} - x \left( ac - \frac{b^2}{2} \right) \right)}{c \left( ac - \frac{b^2}{4} \right) \sqrt{cx^2 + bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(3/2),x)

[Out]  $(f*\log((b/2 + c*x)/c^{1/2} + (a + b*x + c*x^2)^{1/2}))/c^{3/2} - (e*(4*a + 2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^{1/2}) + (d*(b/2 + c*x))/((a*c - b^2/4)*(a + b*x + c*x^2)^{1/2}) + (f*((a*b)/2 - x*(a*c - b^2/2)))/(c*(a*c - b^2/4)*(a + b*x + c*x^2)^{1/2})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{(a + bx + cx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(3/2), x)

[Out] Integral((d + e\*x + f\*x\*\*2)/(a + b\*x + c\*x\*\*2)\*\*(3/2), x)



$$3.114 \quad \int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$$

**Optimal.** Leaf size=666

$$\frac{2(-cx(-2acf + b^2f - bce + 2c^2d) - bc(cd - 3af) - 2ac^2e + b^3(-f) + b^2ce)}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} \frac{f(f(2af - b(\sqrt{e^2 - 4df} + e)))}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 -$$

**Rubi [A]** time = 1.83, antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, number of rules / integrand size = 0.148, Rules used = {974, 1032, 724, 206}

$$\frac{2(-cx(-2acf + b^2f - bce + 2c^2d) - bc(cd - 3af) - 2ac^2e + b^3(-f) + b^2ce)}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} \frac{f(f(2af - b(\sqrt{e^2 - 4df} + e)))}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))} + \frac{f(f(2af - b(\sqrt{e^2 - 4df} + e)))}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))} \frac{f(f(2af - b(\sqrt{e^2 - 4df} + e)))}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2)), x]

[Out] (2\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f) - c\*(2\*c^2\*d - b\*c\*e + b^2\*f - 2\*a\*c\*f)\*x))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*Sqrt[a + b\*x + c\*x^2]) - (f\*(c\*(e^2 - 2\*d\*f + e\*Sqrt[e^2 - 4\*d\*f])) + f\*(2\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f])))\*ArcTanh[(4\*a\*f - b\*(e - Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e - Sqrt[e^2 - 4\*d\*f]))\*x)/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 - (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2])]/(Sqrt[2]\*Sqrt[e^2 - 4\*d\*f]\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*Sqrt[c\*(e^2 - 2\*d\*f - e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e - Sqrt[e^2 - 4\*d\*f])))] + (f\*(c\*(e^2 - 2\*d\*f - e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e - Sqrt[e^2 - 4\*d\*f])))\*ArcTanh[(4\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f]) + 2\*(b\*f - c\*(e + Sqrt[e^2 - 4\*d\*f]))\*x)/(2\*Sqrt[2]\*Sqrt[c\*e^2 - 2\*c\*d\*f - b\*e\*f + 2\*a\*f^2 + (c\*e - b\*f)\*Sqrt[e^2 - 4\*d\*f]]\*Sqrt[a + b\*x + c\*x^2])]/(Sqrt[2]\*Sqrt[e^2 - 4\*d\*f]\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*Sqrt[c\*(e^2 - 2\*d\*f + e\*Sqrt[e^2 - 4\*d\*f]) + f\*(2\*a\*f - b\*(e + Sqrt[e^2 - 4\*d\*f])))]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 974

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[2\*c\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*(b^2\*c\*e - 2\*a\*c^2\*e - b^3\*f - b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) + (2\*f\*(2\*a\*c^2\*e - b^2\*c\*e + b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f))\*(b\*f\*

```
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a + bx + cx^2)^{3/2} (d + ex + fx^2)} dx = \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} + \dots$$

**Mathematica [A]** time = 5.34, size = 700, normalized size = 1.05

$$2f \left[ \frac{-2(-2(2af+e(\sqrt{c^2-4df}+a))+2b^2f-b(\sqrt{c^2-4df}+e-2fz))}{(f^2-4ae)\sqrt{c^2+4b^2+4c^2}} \frac{4af-\sqrt{c^2-4df}+2z(\sqrt{c^2-4df}+e)}{2\sqrt{c^2+4b^2+4c^2}\sqrt{(2af+e(\sqrt{c^2-4df}+a))-\sqrt{c^2-4df}-2af+e^2}} + \frac{2(c(\sqrt{c^2-4df}-e)-2af+2b^2f+bc(\sqrt{c^2-4df}+e+2fz))}{(b^2-4ac)\sqrt{c^2+4b^2+4c^2}} \frac{\sqrt{2}f^2 \tanh^{-1}\left(\frac{4af-\sqrt{c^2-4df}+2z(\sqrt{c^2-4df}+e)}{2\sqrt{c^2+4b^2+4c^2}\sqrt{(2af+e(\sqrt{c^2-4df}+a))-\sqrt{c^2-4df}-2af+e^2}}\right)}{(f(2af-\sqrt{c^2-4df}+e))-\sqrt{c^2-4df}-2af+e^2}} + \frac{\sqrt{2}f^2\sqrt{(2af+e(\sqrt{c^2-4df}-e))-\sqrt{c^2-4df}-2af+e^2}}{(f(b(-\sqrt{c^2-4df}-2af)-\sqrt{c^2-4df}-2af+e^2))} \tanh^{-1}\left(\frac{4af-\sqrt{c^2-4df}+2z(\sqrt{c^2-4df}+e)}{2\sqrt{c^2+4b^2+4c^2}\sqrt{(2af+e(\sqrt{c^2-4df}+a))-\sqrt{c^2-4df}-2af+e^2}}\right)}{(f(b(-\sqrt{c^2-4df}-2af)-\sqrt{c^2-4df}-2af+e^2))} \right] \sqrt{a+bx+cx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
[Out] (2*f*((2*b^2*f + b*c*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*c*(-2*a*f + c*(-e
+ Sqrt[e^2 - 4*d*f])*x))/((b^2 - 4*a*c)*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f])))*Sqrt[a + x*(b + c*x)]) - (2*
(2*b^2*f - b*c*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + Sqrt[e
^2 - 4*d*f])*x))/((b^2 - 4*a*c)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) +
c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*f^2*ArcTanh
[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x)
)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e +
Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])))/(c*(e^2 - 2*d*f + e*Sqrt[e^2
- 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))^(3/2) - (Sqrt[2]*f^2*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*
d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2
- 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
```

$$\frac{f*(2*a*f + b*(-e + \text{Sqrt}[e^2 - 4*d*f]))*\text{Sqrt}[a + x*(b + c*x)]}{(c*(-e^2 + 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(-2*a*f + b*(e - \text{Sqrt}[e^2 - 4*d*f])))^2} / \text{Sqrt}[e^2 - 4*d*f]$$

**IntegrateAlgebraic [C]** time = 1.61, size = 730, normalized size = 1.10

RootSum[...]

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2)),x]

[Out] 
$$\frac{(-2*(b*c^2*d - b^2*c*e + 2*a*c^2*e + b^3*f - 3*a*b*c*f + 2*c^3*d*x - b*c^2*e*x + b^2*c*f*x - 2*a*c^2*f*x)) / ((b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)*\text{Sqrt}[a + b*x + c*x^2]) + \text{RootSum}[b^2*d - a*b*e + a^2*f - 4*b*\text{Sqrt}[c]*d*#1 + 2*a*\text{Sqrt}[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*\text{Sqrt}[c]*e*#1^3 + f*#1^4 \& , (- (b*c*e^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - #1)] + b*c*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - #1] + b^2*e*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - #1] + a*c*e*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - #1] - 2*a*b*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - #1] + 2*c^(3/2)*e^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - #1]*#1 - 2*c^(3/2)*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - #1]*#1 - 2*b*\text{Sqrt}[c]*e*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - #1]*#1 + 2*a*\text{Sqrt}[c]*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - #1]*#1 - c*e*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - #1]*#1^2 + b*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - #1]*#1^2) / (2*b*\text{Sqrt}[c]*d - a*\text{Sqrt}[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*\text{Sqrt}[c]*e*#1^2 - 2*f*#1^3) \& ] / (c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)$$

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

*sage2*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d),x, algorithm="giac")

[Out] sage2

**maple [B]** time = 0.03, size = 4099, normalized size = 6.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x^2+b\*x+a)^(3/2)/(f\*x^2+e\*x+d),x)

[Out] 
$$\frac{-2/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)*f^2/((x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}-4*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d$$



```
*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-
(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b^2+2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*
f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2
/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f
+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2)*b*c*e-2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)*f^2*2^(1/2)/((2*a*f^2-b*e*f-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((
b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-
b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*
2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-
4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f))
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

$$3.115 \quad \int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}} dx$$

**Optimal.** Leaf size=891

$$\frac{x\sqrt{cx^2+bx+a}f^3}{2c^3} + \frac{(12ce-11bf)\sqrt{cx^2+bx+a}f^2}{4c^4} + \frac{(24(e^2+df)c^2-20f(3be+af)c+35b^2f^2)\tanh^{-1}\left(\frac{b-2\sqrt{c}\sqrt{cx^2+bx+a}}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{9/2}}$$

**Rubi [A]** time = 1.77, antiderivative size = 891, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1660, 1661, 640, 621, 206}

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(5/2), x]

[Out] (2\*(3\*a\*b^4\*c\*e\*f^2 - a\*b^5\*f^3 + a\*b^3\*c\*f\*(5\*a\*f^2 - 3\*c\*(e^2 + d\*f)) - b\*c^2\*(c^3\*d^3 + 5\*a^3\*f^3 + 3\*a\*c^2\*d\*(e^2 + d\*f) - 9\*a^2\*c\*f\*(e^2 + d\*f)) - a\*b^2\*c^2\*e\*(12\*a\*f^2 - c\*(e^2 + 6\*d\*f)) + 2\*a\*c^3\*e\*(3\*c^2\*d^2 + 3\*a^2\*f^2 - a\*c\*(e^2 + 6\*d\*f)) - (2\*c^2\*d - b\*c\*e + b^2\*f - 2\*a\*c\*f)\*(c^4\*d^2 - b\*c^3\*d\*e + b^2\*c^2\*e^2 - 3\*a\*c^3\*e^2 + b^2\*c^2\*d\*f - 2\*a\*c^3\*d\*f - 2\*b^3\*c\*e\*f + 7\*a\*b\*c^2\*e\*f + b^4\*f^2 - 4\*a\*b^2\*c\*f^2 + a^2\*c^2\*f^2)\*x)/(3\*c^5\*(b^2 - 4\*a\*c)\*(a + b\*x + c\*x^2)^(3/2)) - (2\*(3\*b^6\*c\*e\*f^2 - b^7\*f^3 + 3\*b^5\*c\*f\*(6\*a\*f^2 - c\*(e^2 + d\*f)) - 3\*b^3\*c^2\*(29\*a^2\*f^3 + c^2\*d\*(e^2 + d\*f) - 10\*a\*c\*f\*(e^2 + d\*f)) - 4\*b\*c^3\*(2\*c^3\*d^3 - 29\*a^3\*f^3 + 3\*a\*c^2\*d\*(e^2 + d\*f) + 24\*a^2\*c\*f\*(e^2 + d\*f)) - 24\*a^2\*c^4\*e\*(6\*a\*f^2 - c\*(e^2 + 6\*d\*f)) - b^4\*c^2\*e\*(42\*a\*f^2 - c\*(e^2 + 6\*d\*f)) + 6\*b^2\*c^3\*e\*(2\*c^2\*d^2 + 28\*a^2\*f^2 - a\*c\*(e^2 + 6\*d\*f)) - c\*(16\*c^6\*d^3 - 10\*b^6\*f^3 + 3\*b^4\*c\*f^2\*(7\*b\*e + 26\*a\*f) - 24\*c^5\*d\*(b\*d\*e - a\*(e^2 + d\*f)) - 6\*b^2\*c^2\*f\*(25\*a\*b\*e\*f + 27\*a^2\*f^2 + 2\*b^2\*(e^2 + d\*f)) + 6\*c^4\*(b^2\*d\*(e^2 + d\*f) - 16\*a^2\*f\*(e^2 + d\*f) - 2\*a\*b\*e\*(e^2 + 6\*d\*f)) + c^3\*(240\*a^2\*b\*e\*f^2 + 56\*a^3\*f^3 + 84\*a\*b^2\*f\*(e^2 + d\*f) + b^3\*(e^3 + 6\*d\*e\*f)))\*x)/(3\*c^5\*(b^2 - 4\*a\*c)^2\*sqrt[a + b\*x + c\*x^2]) + (f^2\*(12\*c\*e - 11\*b\*f)\*sqrt[a + b\*x + c\*x^2])/(4\*c^4) + (f^3\*x\*sqrt[a + b\*x + c\*x^2])/(2\*c^3) + (f\*(35\*b^2\*f^2 - 20\*c\*f\*(3\*b\*e + a\*f) + 24\*c^2\*(e^2 + d\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*sqrt[c]\*sqrt[a + b\*x + c\*x^2])])/(8\*c^(9/2))

**Rule 206**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 621**

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 640**

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]



$$\begin{aligned}
& + c^2 x^3 (-16e^2 - 16df + 6efx + f^2 x^2) - 3b^5 f (35a^2 f^2 - 10acfx (12e + 23fx) + c^2 x^2 (24e^2 + 24df - 80efx + 7f^2 x^2)) - 48b^2 c^2 (27a^4 f^3 - 4c^4 d^2 x^2 (d - ex) + a^2 c^2 (-4d^2 f + 4e^3 x - 64ef^2 x^3 + 7f^3 x^4 - 4d^2 e (e - 6fx)) - 2a^3 c^3 (d^3 - e^3 x^3 + 3d^2 e x^2 (e - 2fx) + 3d^2 x^2 (-e + fx)) - 2a^3 c^3 f (5e^2 + 39efx + f(5d - 14fx^2))) - 8b^3 c^2 (-95a^3 f^3 + c^3 (d^3 - e^3 x^3 + 9d^2 x^2 (e - fx) - 3d^2 e x^2 (3e + 2fx)) - 3a^2 c^2 f x^2 (18e^2 - 74efx + f(18d + 7fx^2)) + 3a^2 c^2 f (3e^2 + 105efx + f(3d + 29fx^2))) + 32c^3 (4c^4 d^3 x^3 + 3a^4 f^2 (16e + 5fx) + 6a^3 c^3 d x^2 (d^2 + e^2 x^2 + d f x^2) - 2a^3 c^3 (2e^3 + 9e^2 f x + f^2 x^2 (9d - 10fx^2) + 12ef(d - 3fx^2)) - 3a^2 c^2 (2d^2 e + 4d f x^2 (3e + 2fx) + x^2 (2e^3 + 8e^2 f x - 6ef^2 x^2 - f^3 x^3))) - 48b^2 c^2 (a^3 f^2 (25e + 63fx) - c^3 d x^2 (d^2 + e^2 x^2 + d x^2 (-6e + fx)) + a^2 c^2 f x^2 (-21e^2 - 12efx + 7f(-3d + 7fx^2)) + a^2 c^2 (d^2 (e - 6fx) - 2d x^2 (3e^2 - 3efx + 7f^2 x^2) + x^2 (e^3 - 14e^2 f x + 6ef^2 x^2 + f^3 x^3))) / (12c^4 (b^2 - 4ac)^2 (a + x(b + cx))^(3/2)) + (f(35b^2 f^2 - 20c^2 f(3be + af) + 24c^2 (e^2 + df)) * Log[b + 2cx + 2*sqrt[c]*sqrt[a + x(b + cx)]])/(8c^(9/2))
\end{aligned}$$

**IntegrateAlgebraic [A]** time = 37.60, size = 1411, normalized size = 1.58

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + ex + fx^2)^3/(a + bx + cx^2)^(5/2), x]

[Out]  $(-8b^3 c^4 d^3 + 96a b c^5 d^3 - 48a^2 b^2 c^4 d^2 e - 192a^2 c^5 d^2 e + 192a^2 b c^4 d e^2 - 128a^3 c^4 e^3 + 192a^2 b c^4 d^2 f - 768a^3 c^4 d e f - 72a^2 b^3 c^2 e^2 f + 480a^3 b c^3 e^2 f - 72a^2 b^3 c^2 d f^2 + 480a^3 b c^3 d f^2 + 180a^2 b^4 c e f^2 - 1200a^3 b^2 c^2 e f^2 + 1536a^4 c^3 e f^2 - 105a^2 b^5 f^3 + 760a^3 b^3 c f^3 - 1296a^4 b c^2 f^3 + 48b^2 c^5 d^3 x + 192a^2 c^6 d^3 x - 72b^3 c^4 d^2 e x - 288a b c^5 d^2 e x + 288a^2 b^2 c^4 d e^2 x - 192a^2 b c^4 e^3 x + 288a^2 b^2 c^4 d^2 f x - 1152a^2 b c^4 d e f x - 144a^2 b^4 c^2 e^2 f x + 1008a^2 b^2 c^3 e^2 f x - 576a^3 c^4 e^2 f x - 144a^2 b^4 c^2 d f^2 x + 1008a^2 b^2 c^3 d f^2 x - 576a^3 c^4 d f^2 x + 360a^2 b^5 c e f^2 x - 2520a^2 b^3 c^2 e f^2 x + 3744a^3 b c^3 e f^2 x - 210a^2 b^6 f^3 x + 1590a^2 b^4 c f^3 x - 3024a^3 b^2 c^2 f^3 x + 480a^4 c^3 f^3 x + 192b^2 c^6 d^3 x^2 - 288b^2 c^5 d^2 e x^2 + 72b^3 c^4 d e^2 x^2 + 288a b c^5 d e^2 x^2 - 48a^2 b^2 c^4 e^3 x^2 - 192a^2 c^5 e^3 x^2 + 72b^3 c^4 d^2 f x^2 + 288a b c^5 d^2 f x^2 - 288a^2 b^2 c^4 d e f x^2 - 1152a^2 c^5 d e f x^2 - 72b^5 c^2 e^2 f x^2 + 432a^2 b^3 c^3 e^2 f x^2 - 72b^5 c^2 d f^2 x^2 + 432a^2 b^3 c^3 d f^2 x^2 + 180b^6 c e f^2 x^2 - 1080a^2 b^4 c^2 e f^2 x^2 + 576a^2 b^2 c^3 e f^2 x^2 + 2304a^3 c^4 e f^2 x^2 - 105b^7 f^3 x^2 + 690a^2 b^5 c f^3 x^2 - 696a^2 b^3 c^2 f^3 x^2 - 1344a^3 b c^3 f^3 x^2 + 128c^7 d^3 x^3 - 192b^2 c^6 d^2 e x^3 + 48b^2 c^5 d e^2 x^3 + 192a^2 c^6 d e^2 x^3 + 8b^3 c^4 e^3 x^3 - 96a^2 b c^5 e^3 x^3 + 48b^2 c^5 d^2 f x^3 + 192a^2 c^6 d^2 f x^3 + 48b^3 c^4 d e f x^3 - 576a^2 b c^5 d e f x^3 - 96b^4 c^3 e^2 f x^3 + 672a^2 b^2 c^4 e^2 f x^3 - 768a^2 c^5 e^2 f x^3 - 96b^4 c^3 d f^2 x^3 + 672a^2 b^2 c^4 d f^2 x^3 - 768a^2 c^5 d f^2 x^3 + 240b^5 c^2 e f^2 x^3 - 1776a^2 b^3 c^3 e f^2 x^3 + 3072a^2 b c^4 e f^2 x^3 - 140b^6 c f^3 x^3 + 1116a^2 b^4 c^2 f^3 x^3 - 2352a^2 b^2 c^3 f^3 x^3 + 640a^3 c^4 f^3 x^3 + 36b^4 c^3 e f^2 x^4 - 288a^2 b^2 c^4 e f^2 x^4 + 576a^2 c^5 e f^2 x^4 - 21b^5 c^2 f^3 x^4 + 168a^2 b^3 c^3 f^3 x^4 - 336a^2 b c^4 f^3 x^4 + 6b^4 c^3 f^3 x^5 - 48a^2 b^2 c^4 f^3 x^5 + 96a^2 c^5 f^3 x^5) / (12c^4 (-b^2 + 4ac)^2 (a + bx + cx^2)^(3/2)) + ((-24c^2 e^2 f - 24c^2 d f^2 + 60b c e f^2 - 35b^2 f^3 + 20a c f^3) * Log[b^2 c^4 + 2c^5 x - 2c^(9/2) * sqrt[a + bx + cx^2]]) / (8c^(9/2))$

**fricas [B]** time = 5.38, size = 3995, normalized size = 4.48

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(5/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/48*(3*((24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*e^2*f + 5*(7*b^6*c^2 - \\ & 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f^3 + 12*(2*(b^4*c^4 - 8*a*b^2 \\ & *c^5 + 16*a^2*c^6)*d - 5*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e)*f^2)*x^4 \\ & + 24*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^2*f + 5*(7*a^2*b^6 - 60* \\ & a^3*b^4*c + 144*a^4*b^2*c^2 - 64*a^5*c^3)*f^3 + 2*(24*(b^5*c^3 - 8*a*b^3*c^4 \\ & + 16*a^2*b*c^5)*e^2*f + 5*(7*b^7*c - 60*a*b^5*c^2 + 144*a^2*b^3*c^3 - 64* \\ & a^3*b*c^4)*f^3 + 12*(2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d - 5*(b^6*c^2 \\ & - 8*a*b^4*c^3 + 16*a^2*b^2*c^4)*e)*f^2)*x^3 + 12*(2*(a^2*b^4*c^2 - 8*a^3* \\ & b^2*c^3 + 16*a^4*c^4)*d - 5*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*e)*f \\ & ^2 + (24*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*e^2*f + 5*(7*b^8 - 46*a*b^6*c \\ & + 24*a^2*b^4*c^2 + 224*a^3*b^2*c^3 - 128*a^4*c^4)*f^3 + 12*(2*(b^6*c^2 - 6 \\ & *a*b^4*c^3 + 32*a^3*c^5)*d - 5*(b^7*c - 6*a*b^5*c^2 + 32*a^3*b*c^4)*e)*f^2) \\ & *x^2 + 2*(24*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e^2*f + 5*(7*a*b^7 \\ & - 60*a^2*b^5*c + 144*a^3*b^3*c^2 - 64*a^4*b*c^3)*f^3 + 12*(2*(a*b^5*c^2 - 8 \\ & *a^2*b^3*c^3 + 16*a^3*b*c^4)*d - 5*(a*b^6*c - 8*a^2*b^4*c^2 + 16*a^3*b^2*c^ \\ & 3)*e)*f^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + \\ & a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(192*a^2*b*c^5*d*e^2 - 128*a^3*c^5*e^3 \\ & + 6*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*f^3*x^5 + 3*(12*(b^4*c^4 - 8*a*b^ \\ & 2*c^5 + 16*a^2*c^6)*e*f^2 - 7*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*f^3)*x \\ & ^4 - 8*(b^3*c^5 - 12*a*b*c^6)*d^3 - 48*(a*b^2*c^5 + 4*a^2*c^6)*d^2*e - (105 \\ & *a^2*b^5*c - 760*a^3*b^3*c^2 + 1296*a^4*b*c^3)*f^3 + 4*(32*c^8*d^3 - 48*b*c \\ & ^7*d^2*e + 12*(b^2*c^6 + 4*a*c^7)*d*e^2 + 2*(b^3*c^5 - 12*a*b*c^6)*e^3 - (3 \\ & 5*b^6*c^2 - 279*a*b^4*c^3 + 588*a^2*b^2*c^4 - 160*a^3*c^5)*f^3 - 12*(2*(b^4 \\ & *c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*d - (5*b^5*c^3 - 37*a*b^3*c^4 + 64*a^2*b*c^ \\ & 5)*e)*f^2 + 12*((b^2*c^6 + 4*a*c^7)*d^2 + (b^3*c^5 - 12*a*b*c^6)*d*e - 2*(b \\ & ^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*e^2)*f)*x^3 - 12*(2*(3*a^2*b^3*c^3 - 20*a \\ & ^3*b*c^4)*d - (15*a^2*b^4*c^2 - 100*a^3*b^2*c^3 + 128*a^4*c^4)*e)*f^2 + 3*( \\ & 64*b*c^7*d^3 - 96*b^2*c^6*d^2*e + 24*(b^3*c^5 + 4*a*b*c^6)*d*e^2 - 16*(a*b^ \\ & 2*c^5 + 4*a^2*c^6)*e^3 - (35*b^7*c - 230*a*b^5*c^2 + 232*a^2*b^3*c^3 + 448* \\ & a^3*b*c^4)*f^3 - 12*(2*(b^5*c^3 - 6*a*b^3*c^4)*d - (5*b^6*c^2 - 30*a*b^4*c^ \\ & 3 + 16*a^2*b^2*c^4 + 64*a^3*c^5)*e)*f^2 + 24*((b^3*c^5 + 4*a*b*c^6)*d^2 - 4 \\ & *(a*b^2*c^5 + 4*a^2*c^6)*d*e - (b^5*c^3 - 6*a*b^3*c^4)*e^2)*f)*x^2 + 24*(8* \\ & a^2*b*c^5*d^2 - 32*a^3*c^5*d*e - (3*a^2*b^3*c^3 - 20*a^3*b*c^4)*e^2)*f + 6* \\ & (48*a*b^2*c^5*d*e^2 - 32*a^2*b*c^5*e^3 + 8*(b^2*c^6 + 4*a*c^7)*d^3 - 12*(b^ \\ & 3*c^5 + 4*a*b*c^6)*d^2*e - (35*a*b^6*c - 265*a^2*b^4*c^2 + 504*a^3*b^2*c^3 \\ & - 80*a^4*c^4)*f^3 - 12*(2*(a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*d - (5*a* \\ & b^5*c^2 - 35*a^2*b^3*c^3 + 52*a^3*b*c^4)*e)*f^2 + 24*(2*a*b^2*c^5*d^2 - 8*a \\ & ^2*b*c^5*d*e - (a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*e^2)*f)*x)*sqrt(c*x^ \\ & 2 + b*x + a))/(a^2*b^4*c^5 - 8*a^3*b^2*c^6 + 16*a^4*c^7 + (b^4*c^7 - 8*a*b^ \\ & 2*c^8 + 16*a^2*c^9)*x^4 + 2*(b^5*c^6 - 8*a*b^3*c^7 + 16*a^2*b*c^8)*x^3 + (b \\ & ^6*c^5 - 6*a*b^4*c^6 + 32*a^3*c^8)*x^2 + 2*(a*b^5*c^5 - 8*a^2*b^3*c^6 + 16* \\ & a^3*b*c^7)*x), -1/24*(3*((24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*e^2*f + 5 \\ & *(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f^3 + 12*(2*(b^4 \\ & *c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d - 5*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^ \\ & 5)*e)*f^2)*x^4 + 24*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^2*f + 5*(7 \\ & *a^2*b^6 - 60*a^3*b^4*c + 144*a^4*b^2*c^2 - 64*a^5*c^3)*f^3 + 2*(24*(b^5*c^ \\ & 3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e^2*f + 5*(7*b^7*c - 60*a*b^5*c^2 + 144*a^2 \\ & *b^3*c^3 - 64*a^3*b*c^4)*f^3 + 12*(2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5) \\ & *d - 5*(b^6*c^2 - 8*a*b^4*c^3 + 16*a^2*b^2*c^4)*e)*f^2)*x^3 + 12*(2*(a^2*b^ \\ & 4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d - 5*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a \\ & ^4*b*c^3)*e)*f^2 + (24*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*e^2*f + 5*(7*b^ \\ & 8 - 46*a*b^6*c + 24*a^2*b^4*c^2 + 224*a^3*b^2*c^3 - 128*a^4*c^4)*f^3 + 12*( \\ & 2*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*d - 5*(b^7*c - 6*a*b^5*c^2 + 32*a^3* \\ & b*c^4)*e)*f^2)*x^2 + 2*(24*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e^2*f \\ & + 5*(7*a*b^7 - 60*a^2*b^5*c + 144*a^3*b^3*c^2 - 64*a^4*b*c^3)*f^3 + 12*(2* \\ & (a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*d - 5*(a*b^6*c - 8*a^2*b^4*c^2 + \end{aligned}$$

$$16a^3b^2c^3)e)f^2)x)\sqrt{-c})\arctan(1/2\sqrt{cx^2 + bx + a})(2cx + b)\sqrt{-c)/(c^2x^2 + bcx + ac)) - 2*(192a^2b^5c^5d^2e^2 - 128a^3c^5e^3 + 6*(b^4c^4 - 8a^2b^2c^5 + 16a^2c^6)*f^3x^5 + 3*(12*(b^4c^4 - 8a^2b^2c^5 + 16a^2c^6)*e)f^2 - 7*(b^5c^3 - 8a^2b^3c^4 + 16a^2b^2c^5)*f^3)*x^4 - 8*(b^3c^5 - 12a^2b^2c^6)*d^3 - 48*(a^2b^2c^5 + 4a^2c^6)*d^2e - (105a^2b^5c - 760a^3b^3c^2 + 1296a^4b^2c^3)*f^3 + 4*(32c^8d^3 - 48b^2c^7d^2e + 12*(b^2c^6 + 4a^2c^7)*d^2e^2 + 2*(b^3c^5 - 12a^2b^2c^6)*e^3 - (35b^6c^2 - 279a^2b^4c^3 + 588a^2b^2c^4 - 160a^3c^5)*f^3 - 12*(2*(b^4c^4 - 7a^2b^2c^5 + 8a^2c^6)*d - (5b^5c^3 - 37a^2b^3c^4 + 64a^2b^2c^5)*e)*f^2 + 12*((b^2c^6 + 4a^2c^7)*d^2 + (b^3c^5 - 12a^2b^2c^6)*d*e - 2*(b^4c^4 - 7a^2b^2c^5 + 8a^2c^6)*e^2)*f)*x^3 - 12*(2*(3a^2b^3c^3 - 20a^3b^2c^4)*d - (15a^2b^4c^2 - 100a^3b^2c^3 + 128a^4c^4)*e)*f^2 + 3*(64b^2c^7d^3 - 96b^2c^6d^2e + 24*(b^3c^5 + 4a^2b^2c^6)*d^2e^2 - 16*(a^2b^2c^5 + 4a^2c^6)*e^3 - (35b^7c - 230a^2b^5c^2 + 232a^2b^3c^3 + 448a^3b^2c^4)*f^3 - 12*(2*(b^5c^3 - 6a^2b^3c^4)*d - (5b^6c^2 - 30a^2b^4c^3 + 16a^2b^2c^4 + 64a^3c^5)*e)*f^2 + 24*((b^3c^5 + 4a^2b^2c^6)*d^2 - 4*(a^2b^2c^5 + 4a^2c^6)*d*e - (b^5c^3 - 6a^2b^3c^4)*e^2)*f)*x^2 + 24*(8a^2b^2c^5d^2 - 32a^3c^5d^2e - (3a^2b^3c^3 - 20a^3b^2c^4)*e^2)*f + 6*(48a^2b^2c^5d^2e^2 - 32a^2b^2c^5e^3 + 8*(b^2c^6 + 4a^2c^7)*d^3 - 12*(b^3c^5 + 4a^2b^2c^6)*d^2e - (35a^2b^6c - 265a^2b^4c^2 + 504a^3b^2c^3 - 80a^4c^4)*f^3 - 12*(2*(a^2b^4c^3 - 7a^2b^2c^4 + 4a^3c^5)*d - (5a^2b^5c^2 - 35a^2b^3c^3 + 52a^3b^2c^4)*e)*f^2 + 24*(2a^2b^2c^5d^2 - 8a^2b^2c^5d^2e - (a^2b^4c^3 - 7a^2b^2c^4 + 4a^3c^5)*e^2)*f)*x)*\sqrt{cx^2 + bx + a})/(a^2b^4c^5 - 8a^3b^2c^6 + 16a^4c^7 + (b^4c^7 - 8a^2b^2c^8 + 16a^2c^9)*x^4 + 2*(b^5c^6 - 8a^2b^3c^7 + 16a^2b^2c^8)*x^3 + (b^6c^5 - 6a^2b^4c^6 + 32a^3c^8)*x^2 + 2*(a^2b^5c^5 - 8a^2b^3c^6 + 16a^3b^2c^7)*x)]$$

**giac [A]** time = 0.45, size = 1401, normalized size = 1.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+bx+a)^(5/2),x, algorithm="giac")

[Out]  $1/12*(((3*(2*(b^4c^3f^3 - 8a^2b^2c^4f^3 + 16a^2c^5f^3)*x/(b^4c^4 - 8a^2b^2c^5 + 16a^2c^6) - (7b^5c^2f^3 - 56a^2b^3c^3f^3 + 112a^2b^2c^4f^3 - 12b^4c^3f^2e + 96a^2b^2c^4f^2e - 192a^2c^5f^2e)/(b^4c^4 - 8a^2b^2c^5 + 16a^2c^6))*x + 4*(32c^7d^3 + 12b^2c^5d^2f + 48a^2c^6d^2f - 24b^4c^3d^2f^2 + 168a^2b^2c^4d^2f^2 - 192a^2c^5d^2f^2 - 35b^6c^2f^3 + 279a^2b^4c^2f^3 - 588a^2b^2c^3f^3 + 160a^3c^4f^3 - 48b^2c^6d^2e + 12b^3c^4d^2f^2e - 144a^2b^2c^5d^2f^2e + 60b^5c^2f^2e - 44a^2b^3c^3f^2e + 768a^2b^2c^4f^2e + 12b^2c^5d^2e^2 + 48a^2c^6d^2e^2 - 24b^4c^3f^2e^2 + 168a^2b^2c^4f^2e^2 - 192a^2c^5f^2e^2 + 2b^3c^4e^3 - 24a^2b^2c^5e^3)/(b^4c^4 - 8a^2b^2c^5 + 16a^2c^6))*x + 3*(64b^2c^6d^3 + 24b^3c^4d^2f + 96a^2b^2c^5d^2f - 24b^5c^2d^2f^2 + 144a^2b^3c^3d^2f^2 - 35b^7f^3 + 230a^2b^5c^2f^3 - 232a^2b^3c^2f^3 - 448a^3b^2c^3f^3 - 96b^2c^5d^2e - 96a^2b^2c^4d^2f^2e - 384a^2c^5d^2f^2e + 60b^6c^2f^2e - 360a^2b^4c^2f^2e + 192a^2b^2c^3f^2e + 768a^3c^4f^2e + 24b^3c^4d^2e^2 + 96a^2b^2c^5d^2e^2 - 24b^5c^2f^2e^2 + 144a^2b^3c^3f^2e^2 - 16a^2b^2c^4e^3 - 64a^2c^5e^3)/(b^4c^4 - 8a^2b^2c^5 + 16a^2c^6))*x + 6*(8b^2c^5d^3 + 32a^2c^6d^3 + 48a^2b^2c^4d^2f - 24a^2b^4c^2d^2f^2 + 168a^2b^2c^3d^2f^2 - 96a^3c^4d^2f^2 - 35a^2b^6f^3 + 265a^2b^4c^2f^3 - 504a^3b^2c^2f^3 + 80a^4c^3f^3 - 12b^3c^4d^2e - 48a^2b^2c^5d^2e - 192a^2b^2c^4d^2f^2e + 60a^2b^5c^2f^2e - 420a^2b^3c^2f^2e + 624a^3b^2c^3f^2e + 48a^2b^2c^4d^2e^2 - 24a^2b^4c^2f^2e^2 + 168a^2b^2c^3f^2e^2 - 96a^3c^4f^2e^2 - 32a^2b^2c^4e^3)/(b^4c^4 - 8a^2b^2c^5 + 16a^2c^6))*x - (8b^3c^4d^3 - 96a^2b^2c^5d^3 - 192a^2b^2c^4d^2f + 72a^2b^3c^2d^2f^2 - 480a^3b^2c^3d^2f^2 + 105a^2b^5f^3 - 760a^3b^3c^2f^3 + 1296a^4b^2c^2f^3 + 48a^2b^2c^4d^2e + 192a^2c^5d^2e + 768$

$$\begin{aligned} & *a^3*c^4*d*f*e - 180*a^2*b^4*c*f^2*e + 1200*a^3*b^2*c^2*f^2*e - 1536*a^4*c^3*f^2*e - 192*a^2*b*c^4*d*e^2 + 72*a^2*b^3*c^2*f*e^2 - 480*a^3*b*c^3*f*e^2 \\ & + 128*a^3*c^4*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))/(c*x^2 + b*x + a)^{3/2} - 1/8*(24*c^2*d*f^2 + 35*b^2*f^3 - 20*a*c*f^3 - 60*b*c*f^2*e + 24*c^2*f*e^2) \\ & *log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^{9/2} \end{aligned}$$

**maple [B]** time = 0.03, size = 4635, normalized size = 5.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(5/2),x)

[Out] 
$$\begin{aligned} & 5/6*f^3*a/c^2*x^3/(c*x^2+b*x+a)^{3/2}+5/2*f^3*a/c^3*x/(c*x^2+b*x+a)^{1/2}-6 \\ & *b/c*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*x*d*e*f-5/2*f^3*a/c^3*b^2/(4*a*c-b^2 \\ & )/(c*x^2+b*x+a)^{1/2}*x-33/4*f^3*b^2/c^3*a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2} \\ & )*x-66*f^3*b^2/c^2*a^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*x-1/8*b^4/c^3/(4*a \\ & *c-b^2)/(c*x^2+b*x+a)^{3/2}*x*e^2*f+4*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2} \\ & )*x*d*e*f-3*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*d*e*f-48*b*a/(4*a*c- \\ & b^2)^2/(c*x^2+b*x+a)^{1/2}*x*d*e*f-24*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2} \\ & )*d*e*f+12*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*x*d*f^2+12*b^2/c*a/ \\ & (4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*x*e^2*f+3/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+ \\ & b*x+a)^{3/2}*x*d*f^2+3/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*x*e^2*f+ \\ & 12*e*f^2*a^2/c^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*x+96*e*f^2*a^2/c*b/(4*a* \\ & c-b^2)^2/(c*x^2+b*x+a)^{1/2}*x-19/4*e*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+ \\ & a)^{3/2}*x-38*e*f^2*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*x+1/2*b^3/c \\ & ^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*x*d*e*f+2/3*d^3/(4*a*c-b^2)/(c*x^2+b*x+a \\ & )^{3/2}*b-x^2/c/(c*x^2+b*x+a)^{3/2}*e^3+1/24*b^2/c^3/(c*x^2+b*x+a)^{3/2}*e^ \\ & 3-2/3*a/c^2/(c*x^2+b*x+a)^{3/2}*e^3+3/c^{5/2}*ln((c*x+1/2*b)/c^{1/2}+(c*x^2 \\ & +b*x+a)^{1/2})*d*f^2+3/c^{5/2}*ln((c*x+1/2*b)/c^{1/2}+(c*x^2+b*x+a)^{1/2}))* \\ & e^2*f+35/16*f^3*b^3/c^5/(c*x^2+b*x+a)^{1/2}-5/2*f^3*a/c^{7/2}*ln((c*x+1/2*b) \\ & )/c^{1/2}+(c*x^2+b*x+a)^{1/2}))+35/8*f^3*b^2/c^{9/2}*ln((c*x+1/2*b)/c^{1/2}+ \\ & (c*x^2+b*x+a)^{1/2}))+1/2*f^3*x^5/c/(c*x^2+b*x+a)^{3/2}-35/384*f^3*b^5/c^6/( \\ & c*x^2+b*x+a)^{3/2}-d^2*e/c/(c*x^2+b*x+a)^{3/2}-33/16*f^3*b^2/c^4*a*x/(c*x^2 \\ & +b*x+a)^{3/2}-33/8*f^3*b^3/c^4*a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}+23/16*f^ \\ & 3*b^5/c^5*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}+23/2*f^3*b^5/c^4*a/(4*a*c-b^2)^ \\ & 2/(c*x^2+b*x+a)^{1/2}-5/4*f^3*a/c^4*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{1/2}-35/ \\ & 192*f^3*b^6/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*x+35/8*f^3*b^4/c^4/(4*a*c-b \\ & ^2)/(c*x^2+b*x+a)^{1/2}*x+1/4*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*f*d^2 \\ & +1/4*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*e^2*d+4*b^2/(4*a*c-b^2)^2/(c*x \\ & ^2+b*x+a)^{1/2}*x*f*d^2+4*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*x*e^2*d+2*b \\ & ^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*f*d^2+2*b^3/c/(4*a*c-b^2)^2/(c*x^2+b \\ & *x+a)^{1/2}*e^2*d+2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*x*f*d^2-15/4*e*f^2*b^ \\ & 4/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^{1/2}+3/2*b/c^2*x^2/(c*x^2+b*x+a)^{3/2}*d*f \\ & ^2+3/2*b/c^2*x^2/(c*x^2+b*x+a)^{3/2}*e^2*f+3/8*b^2/c^3*x/(c*x^2+b*x+a)^{3/2} \\ & )*d*f^2+3/8*b^2/c^3*x/(c*x^2+b*x+a)^{3/2}*e^2*f-1/16*b^5/c^4/(4*a*c-b^2)/(c \\ & *x^2+b*x+a)^{3/2}*d*f^2-1/16*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*e^2*f- \\ & 1/2*b^5/c^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*d*f^2-1/2*b^5/c^3/(4*a*c-b^2) \\ & ^2/(c*x^2+b*x+a)^{1/2}*e^2*f+b/c^3*a/(c*x^2+b*x+a)^{3/2}*d*f^2+b/c^3*a/(c*x \\ & ^2+b*x+a)^{3/2}*e^2*f+3/2/c^3*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{1/2}*d*f^2+3/2 \\ & /c^3*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{1/2}*e^2*f+12*e*f^2*a/c^2*x^2/(c*x^2+b* \\ & x+a)^{3/2}+5/2*e*f^2*b/c^2*x^3/(c*x^2+b*x+a)^{3/2}-15/4*e*f^2*b^2/c^3*x^2/( \\ & c*x^2+b*x+a)^{3/2}-15/16*e*f^2*b^3/c^4*x/(c*x^2+b*x+a)^{3/2}+5/32*e*f^2*b^6 \\ & /c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}+5/4*e*f^2*b^6/c^4/(4*a*c-b^2)^2/(c*x^2 \\ & +b*x+a)^{1/2}-3*e*f^2*b^2/c^4*a/(c*x^2+b*x+a)^{3/2}+15/2*e*f^2*b/c^3*x/(c*x \\ & ^2+b*x+a)^{1/2}-35/24*f^3*b^6/c^4/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*x-33/4* \\ & f^3*b/c^3*a*x^2/(c*x^2+b*x+a)^{3/2}-33*f^3*b^3/c^3*a^2/(4*a*c-b^2)^2/(c*x^2 \\ & +b*x+a)^{1/2}+2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{3/2}*x*e^2*d+8*a/(4*a*c-b^2)^2 \\ & /c*x^2+b*x+a)^{1/2}*b*f*d^2+8*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{1/2}*b*e^2*d- \end{aligned}$$

$$\begin{aligned}
& 2*d^2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x-d^2*e*b^2/c/(4*a*c-b^2)/(c*x^2+ \\
& b*x+a)^{(3/2)}-6*x^2/c/(c*x^2+b*x+a)^{(3/2)}*d*e*f+1/4*b^2/c^3/(c*x^2+b*x+a)^{(3/2)} \\
& *d*e*f+1/12*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x*e^3+2/3*b^3/c/(4*a \\
& *c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x*e^3-1/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a) \\
& ^{(3/2)}*e^3-8*b*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x*e^3-4*b^2/c*a/(4*a*c-b \\
& ^2)^2/(c*x^2+b*x+a)^{(1/2)}*e^3-4*a/c^2/(c*x^2+b*x+a)^{(3/2)}*d*e*f-16*d^2*e*b* \\
& c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x-b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{( \\
& 1/2)}*x*e^2*f+3/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*d*f^2+3/4*b^3/c^ \\
& 3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*e^2*f+6*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+ \\
& b*x+a)^{(1/2)}*d*f^2+6*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*e^2*f+3/c^ \\
& 2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*d*f^2-b^4/c^2/(4*a*c-b^2)^2/(c*x^2+ \\
& b*x+a)^{(1/2)}*x*d*f^2-3/2*b/c^2*x/(c*x^2+b*x+a)^{(3/2)}*d*e*f+1/4*b^4/c^3/(4*a \\
& *c-b^2)/(c*x^2+b*x+a)^{(3/2)}*d*e*f+2*b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/ \\
& 2)}*d*e*f-b/c*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x*e^3+5/2*e*f^2*b^5/c^3/(4*a \\
& *c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x+48*e*f^2*a^2/c^2*b^2/(4*a*c-b^2)^2/(c*x^2+b \\
& *x+a)^{(1/2)}+3*e*f^2*a/c^3*b*x/(c*x^2+b*x+a)^{(3/2)}+6*e*f^2*a^2/c^3*b^2/(4*a* \\
& c-b^2)/(c*x^2+b*x+a)^{(3/2)}-19/8*e*f^2*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{( \\
& 3/2)}-19*e*f^2*b^4/c^3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}-15/2*e*f^2*b^3/c^ \\
& 3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x+5/16*e*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+b \\
& *x+a)^{(3/2)}*x+23/8*f^3*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x+23*f^3*b \\
& ^4/c^3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x+1/2*b^2/c/(4*a*c-b^2)/(c*x^2+b \\
& *x+a)^{(3/2)}*x*e^2*d+a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*b*f*d^2+a/c/(4*a*c- \\
& b^2)/(c*x^2+b*x+a)^{(3/2)}*b*e^2*d+16*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x \\
& *f*d^2+16*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x*e^2*d+3/c^2*b^2/(4*a*c-b^ \\
& 2)/(c*x^2+b*x+a)^{(1/2)}*x*e^2*f-1/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}* \\
& x*d*f^2+1/2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x*f*d^2-5/4*f^3*a/c^4*b/( \\
& c*x^2+b*x+a)^{(1/2)}+173/96*f^3*b^3/c^5*a/(c*x^2+b*x+a)^{(3/2)}-35/8*f^3*b^2/c^ \\
& 4*x/(c*x^2+b*x+a)^{(1/2)}+35/16*f^3*b^5/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}-1 \\
& 1/2*f^3*b/c^4*a^2/(c*x^2+b*x+a)^{(3/2)}-7/4*f^3*b/c^2*x^4/(c*x^2+b*x+a)^{(3/2)} \\
& -35/24*f^3*b^2/c^3*x^3/(c*x^2+b*x+a)^{(3/2)}+35/16*f^3*b^3/c^4*x^2/(c*x^2+b*x \\
& +a)^{(3/2)}+35/64*f^3*b^4/c^5*x/(c*x^2+b*x+a)^{(3/2)}-35/384*f^3*b^7/c^6/(4*a*c \\
& -b^2)/(c*x^2+b*x+a)^{(3/2)}-35/48*f^3*b^7/c^5/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/ \\
& 2)}+8*e*f^2*a^2/c^3/(c*x^2+b*x+a)^{(3/2)}-15/2*e*f^2*b/c^(7/2)*ln((c*x+1/2*b)/ \\
& c^(1/2)+(c*x^2+b*x+a)^(1/2))+3*e*f^2*x^4/c/(c*x^2+b*x+a)^{(3/2)}+5/32*e*f^2*b \\
& ^4/c^5/(c*x^2+b*x+a)^{(3/2)}-15/4*e*f^2*b^2/c^4/(c*x^2+b*x+a)^{(1/2)}-x^3/c/(c* \\
& x^2+b*x+a)^{(3/2)}*d*f^2-x^3/c/(c*x^2+b*x+a)^{(3/2)}*e^2*f-1/16*b^3/c^4/(c*x^2+ \\
& b*x+a)^{(3/2)}*d*f^2-1/16*b^3/c^4/(c*x^2+b*x+a)^{(3/2)}*e^2*f-3/c^2*x/(c*x^2+b* \\
& x+a)^{(1/2)}*d*f^2-3/c^2*x/(c*x^2+b*x+a)^{(1/2)}*e^2*f+3/2/c^3*b/(c*x^2+b*x+a)^ \\
& (1/2)*d*f^2+3/2/c^3*b/(c*x^2+b*x+a)^{(1/2)}*e^2*f-1/4*b/c^2*x/(c*x^2+b*x+a)^{( \\
& 3/2)}*e^3+1/24*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*e^3+1/3*b^4/c^2/(4*a* \\
& c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*e^3-3/2*x/c/(c*x^2+b*x+a)^{(3/2)}*f*d^2-3/2*x/c/ \\
& (c*x^2+b*x+a)^{(3/2)}*e^2*d+1/4*b/c^2/(c*x^2+b*x+a)^{(3/2)}*f*d^2+1/4*b/c^2/(c* \\
& x^2+b*x+a)^{(3/2)}*e^2*d-8*d^2*e*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}+4/3*d^ \\
& 3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x*c+32/3*d^3*c^2/(4*a*c-b^2)^2/(c*x^2+b*x \\
& +a)^{(1/2)}*x+16/3*d^3*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*b
\end{aligned}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^3/(c\*x^2+b\*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f x^2 + e x + d)^3}{(c x^2 + b x + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(5/2), x)

[Out] int((d + e\*x + f\*x^2)^3/(a + b\*x + c\*x^2)^(5/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*\*3/(c\*x\*\*2+b\*x+a)\*\*(5/2), x)

[Out] Timed out

$$3.116 \quad \int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}} dx$$

**Optimal.** Leaf size=444

$$\frac{2(-x(c^2(2a^2f^2 + 6abef + b^2(2df + e^2)) - 2b^2cf(2af + be) - 2c^3(a(2df + e^2) + bde) + b^4f^2 + 2c^4d^2) - bc(-3c^3(b^2 - 4ac)(a + bx + cx^2)^{3/2})}{3c^3(b^2 - 4ac)(a + bx + cx^2)^{3/2}}$$

**Rubi [A]** time = 0.45, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1660, 12, 621, 206}

$$\frac{2(-3c^3(c^2(2a^2f^2 + 6abef + b^2(2df + e^2)) - 2b^2cf(2af + be) - 2c^3(a(2df + e^2) + bde) + b^4f^2 + 2c^4d^2) - bc(-3c^3(b^2 - 4ac)(a + bx + cx^2)^{3/2})}{3c^3(b^2 - 4ac)(a + bx + cx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(5/2), x]

[Out] (2\*(2\*a\*b^2\*c\*e\*f - a\*b^3\*f^2 + 4\*a\*c^2\*e\*(c\*d - a\*f) - b\*c\*(c^2\*d^2 - 3\*a^2\*f^2 + a\*c\*(e^2 + 2\*d\*f)) - (2\*c^4\*d^2 + b^4\*f^2 - 2\*b^2\*c\*f\*(b\*e + 2\*a\*f) - 2\*c^3\*(b\*d\*e + a\*(e^2 + 2\*d\*f)) + c^2\*(6\*a\*b\*e\*f + 2\*a^2\*f^2 + b^2\*(e^2 + 2\*d\*f)))\*x)/(3\*c^3\*(b^2 - 4\*a\*c)\*(a + b\*x + c\*x^2)^(3/2)) - (2\*(2\*b^4\*c\*e\*f + 48\*a^2\*c^3\*e\*f - b^5\*f^2 + 4\*b^2\*c^2\*e\*(2\*c\*d - 3\*a\*f) + b^3\*c\*(10\*a\*f^2 - c\*(e^2 + 2\*d\*f)) - 4\*b\*c^2\*(2\*c^2\*d^2 + 8\*a^2\*f^2 + a\*c\*(e^2 + 2\*d\*f)) - 2\*c\*(8\*c^4\*d^2 - 2\*b^4\*f^2 + b^2\*c\*f\*(b\*e + 14\*a\*f) - c^3\*(8\*b\*d\*e - 4\*a\*(e^2 + 2\*d\*f)) - c^2\*(12\*a\*b\*e\*f + 16\*a^2\*f^2 - b^2\*(e^2 + 2\*d\*f)))\*x)/(3\*c^3\*(b^2 - 4\*a\*c)^2\*sqrt[a + b\*x + c\*x^2]) + (f^2\*ArcTanh[(b + 2\*c\*x)/(2\*sqrt[c]\*sqrt[a + b\*x + c\*x^2])])/c^(5/2)

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\int \frac{(d + ex + fx^2)^2}{(a + bx + cx^2)^{5/2}} dx = \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + 3c^3(b^2 - 4a^2)))}{3c^3(b^2 - 4a^2)}$$

$$= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + 3c^3(b^2 - 4a^2)))}{3c^3(b^2 - 4a^2)}$$

$$= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + 3c^3(b^2 - 4a^2)))}{3c^3(b^2 - 4a^2)}$$

$$= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + 3c^3(b^2 - 4a^2)))}{3c^3(b^2 - 4a^2)}$$

**Mathematica [A]** time = 1.26, size = 387, normalized size = 0.87

$$\frac{2 \left( (2ab^2cef + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + 3c^3(b^2 - 4a^2))) \right)}{3c^3(b^2 - 4a^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(5/2),x]

[Out] (2\*(-3\*b^5\*f^2\*x^2 - 2\*b^4\*f^2\*x\*(3\*a + 2\*c\*x^2) + 4\*b\*c\*(5\*a^3\*f^2 + 2\*c^3\*d\*x^2\*(3\*d - 2\*e\*x) + 2\*a^2\*c\*(e^2 + 2\*d\*f - 6\*e\*f\*x) + 3\*a\*c^2\*(d - e\*x)\*(d + x\*(-e + 2\*f\*x))) + b^3\*(-3\*a^2\*f^2 + 18\*a\*c\*f^2\*x^2 + c^2\*(-d^2 + 6\*d\*x\*(-e + f\*x) + e\*x^2\*(3\*e + 2\*f\*x))) + 8\*c^2\*(2\*c^3\*d^2\*x^3 - a^3\*f\*(4\*e + 3\*f\*x) + a\*c^2\*x\*(3\*d^2 + e^2\*x^2 + 2\*d\*f\*x^2) - 2\*a^2\*c\*(d\*e + f\*x^2\*(3\*e + 2\*f\*x))) + 2\*b^2\*c\*(21\*a^2\*f^2\*x + c^2\*x\*(3\*d^2 + e^2\*x^2 + 2\*d\*x\*(-6\*e + f\*x)) - 2\*a\*c\*(d\*(e - 6\*f\*x) + x\*(-3\*e^2 + 3\*e\*f\*x - 7\*f^2\*x^2))))/(3\*c^2\*(b^2 - 4\*a\*c)^2\*(a + x\*(b + c\*x))^(3/2)) + (f^2\*Log[b + 2\*c\*x + 2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)]])/c^(5/2)

**IntegrateAlgebraic [A]** time = 3.16, size = 555, normalized size = 1.25

$$\frac{2 \left( (2ab^2cef + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + 3c^3(b^2 - 4a^2))) \right)}{3c^3(b^2 - 4a^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(5/2),x]

[Out] (-2\*(b^3\*c^2\*d^2 - 12\*a\*b\*c^3\*d^2 + 4\*a\*b^2\*c^2\*d\*e + 16\*a^2\*c^3\*d\*e - 8\*a^2\*b\*c^2\*e^2 - 16\*a^2\*b\*c^2\*d\*f + 32\*a^3\*c^2\*e\*f + 3\*a^2\*b^3\*f^2 - 20\*a^3\*b\*c\*f^2 - 6\*b^2\*c^3\*d^2\*x - 24\*a\*c^4\*d^2\*x + 6\*b^3\*c^2\*d\*e\*x + 24\*a\*b\*c^3\*d\*e\*x - 12\*a\*b^2\*c^2\*e^2\*x - 24\*a\*b^2\*c^2\*d\*f\*x + 48\*a^2\*b\*c^2\*e\*f\*x + 6\*a\*b^4\*f^2\*x - 42\*a^2\*b^2\*c\*f^2\*x + 24\*a^3\*c^2\*f^2\*x - 24\*b\*c^4\*d^2\*x^2 + 24\*b^2\*c^3\*d\*e\*x^2 - 3\*b^3\*c^2\*e^2\*x^2 - 12\*a\*b\*c^3\*e^2\*x^2 - 6\*b^3\*c^2\*d\*f\*x^2 - 24\*a\*b\*c^3\*d\*f\*x^2 + 12\*a\*b^2\*c^2\*e\*f\*x^2 + 48\*a^2\*c^3\*e\*f\*x^2 + 3\*b^5\*f^2\*x^2 - 18\*a\*b^3\*c\*f^2\*x^2 - 16\*c^5\*d^2\*x^3 + 16\*b\*c^4\*d\*e\*x^3 - 2\*b^2\*c^3\*e^2\*x^3 - 8\*a\*c^4\*e^2\*x^3 - 4\*b^2\*c^3\*d\*f\*x^3 - 16\*a\*c^4\*d\*f\*x^3 - 2\*b^3\*c^2\*e\*f\*x^3 + 24\*a\*b\*c^3\*e\*f\*x^3 + 4\*b^4\*c\*f^2\*x^3 - 28\*a\*b^2\*c^2\*f^2\*x^3 + 32\*

$$a^2*c^3*f^2*x^3)/(3*c^2*(-b^2 + 4*a*c)^2*(a + b*x + c*x^2)^(3/2)) - (f^2*\log[b*c^2 + 2*c^3*x - 2*c^(5/2)*\sqrt{a + b*x + c*x^2}])/c^(5/2)$$

**fricas** [A] time = 3.50, size = 1581, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(5/2),x, algorithm="fricas")

[Out] [1/6\*(3\*((b^4\*c^2 - 8\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*f^2\*x^4 + 2\*(b^5\*c - 8\*a\*b^3\*c^2 + 16\*a^2\*b\*c^3)\*f^2\*x^3 + (b^6 - 6\*a\*b^4\*c + 32\*a^3\*c^3)\*f^2\*x^2 + 2\*(a\*b^5 - 8\*a^2\*b^3\*c + 16\*a^3\*b\*c^2)\*f^2\*x + (a^2\*b^4 - 8\*a^3\*b^2\*c + 16\*a^4\*c^2)\*f^2)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) + 4\*(8\*a^2\*b\*c^3\*e^2 + 2\*(8\*c^6\*d^2 - 8\*b\*c^5\*d\*e + (b^2\*c^4 + 4\*a\*c^5)\*e^2 - 2\*(b^4\*c^2 - 7\*a\*b^2\*c^3 + 8\*a^2\*c^4)\*f^2 + (2\*(b^2\*c^4 + 4\*a\*c^5)\*d + (b^3\*c^3 - 12\*a\*b\*c^4)\*e)\*f)\*x^3 - (b^3\*c^3 - 12\*a\*b\*c^4)\*d^2 - 4\*(a\*b^2\*c^3 + 4\*a^2\*c^4)\*d\*e - (3\*a^2\*b^3\*c - 20\*a^3\*b\*c^2)\*f^2 + 3\*(8\*b\*c^5\*d^2 - 8\*b^2\*c^4\*d\*e + (b^3\*c^3 + 4\*a\*b\*c^4)\*e^2 - (b^5\*c - 6\*a\*b^3\*c^2)\*f^2 + 2\*((b^3\*c^3 + 4\*a\*b\*c^4)\*d - 2\*(a\*b^2\*c^3 + 4\*a^2\*c^4)\*e)\*f)\*x^2 + 16\*(a^2\*b\*c^3\*d - 2\*a^3\*c^3\*e)\*f + 6\*(2\*a\*b^2\*c^3\*e^2 + (b^2\*c^4 + 4\*a\*c^5)\*d^2 - (b^3\*c^3 + 4\*a\*b\*c^4)\*d\*e - (a\*b^4\*c - 7\*a^2\*b^2\*c^2 + 4\*a^3\*c^3)\*f^2 + 4\*(a\*b^2\*c^3\*d - 2\*a^2\*b\*c^3\*e)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a^2\*b^4\*c^3 - 8\*a^3\*b^2\*c^4 + 16\*a^4\*c^5 + (b^4\*c^5 - 8\*a\*b^2\*c^6 + 16\*a^2\*c^7)\*x^4 + 2\*(b^5\*c^4 - 8\*a\*b^3\*c^5 + 16\*a^2\*b\*c^6)\*x^3 + (b^6\*c^3 - 6\*a\*b^4\*c^4 + 32\*a^3\*c^6)\*x^2 + 2\*(a\*b^5\*c^3 - 8\*a^2\*b^3\*c^4 + 16\*a^3\*b\*c^5)\*x), -1/3\*(3\*((b^4\*c^2 - 8\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*f^2\*x^4 + 2\*(b^5\*c - 8\*a\*b^3\*c^2 + 16\*a^2\*b\*c^3)\*f^2\*x^3 + (b^6 - 6\*a\*b^4\*c + 32\*a^3\*c^3)\*f^2\*x^2 + 2\*(a\*b^5 - 8\*a^2\*b^3\*c + 16\*a^3\*b\*c^2)\*f^2\*x + (a^2\*b^4 - 8\*a^3\*b^2\*c + 16\*a^4\*c^2)\*f^2)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) - 2\*(8\*a^2\*b\*c^3\*e^2 + 2\*(8\*c^6\*d^2 - 8\*b\*c^5\*d\*e + (b^2\*c^4 + 4\*a\*c^5)\*e^2 - 2\*(b^4\*c^2 - 7\*a\*b^2\*c^3 + 8\*a^2\*c^4)\*f^2 + (2\*(b^2\*c^4 + 4\*a\*c^5)\*d + (b^3\*c^3 - 12\*a\*b\*c^4)\*e)\*f)\*x^3 - (b^3\*c^3 - 12\*a\*b\*c^4)\*d^2 - 4\*(a\*b^2\*c^3 + 4\*a^2\*c^4)\*d\*e - (3\*a^2\*b^3\*c - 20\*a^3\*b\*c^2)\*f^2 + 3\*(8\*b\*c^5\*d^2 - 8\*b^2\*c^4\*d\*e + (b^3\*c^3 + 4\*a\*b\*c^4)\*e^2 - (b^5\*c - 6\*a\*b^3\*c^2)\*f^2 + 2\*((b^3\*c^3 + 4\*a\*b\*c^4)\*d - 2\*(a\*b^2\*c^3 + 4\*a^2\*c^4)\*e)\*f)\*x^2 + 16\*(a^2\*b\*c^3\*d - 2\*a^3\*c^3\*e)\*f + 6\*(2\*a\*b^2\*c^3\*e^2 + (b^2\*c^4 + 4\*a\*c^5)\*d^2 - (b^3\*c^3 + 4\*a\*b\*c^4)\*d\*e - (a\*b^4\*c - 7\*a^2\*b^2\*c^2 + 4\*a^3\*c^3)\*f^2 + 4\*(a\*b^2\*c^3\*d - 2\*a^2\*b\*c^3\*e)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a^2\*b^4\*c^3 - 8\*a^3\*b^2\*c^4 + 16\*a^4\*c^5 + (b^4\*c^5 - 8\*a\*b^2\*c^6 + 16\*a^2\*c^7)\*x^4 + 2\*(b^5\*c^4 - 8\*a\*b^3\*c^5 + 16\*a^2\*b\*c^6)\*x^3 + (b^6\*c^3 - 6\*a\*b^4\*c^4 + 32\*a^3\*c^6)\*x^2 + 2\*(a\*b^5\*c^3 - 8\*a^2\*b^3\*c^4 + 16\*a^3\*b\*c^5)\*x)]

**giac** [A] time = 0.34, size = 587, normalized size = 1.32

$$\frac{f^2 \log\left(\frac{-2(\sqrt{c}x - \sqrt{c^2x^2 + bx + a})\sqrt{c} - b}{c}\right)}{c^{5/2}} + \frac{2}{3} \left( \frac{(2(8c^5d^2 + 2b^2c^3d^2f + 8a^2c^4d^2f - 2b^4c^3f^2 + 14ab^2c^2f^2 - 16a^2c^3f^2 - 8b^2c^4d^2e + b^3c^2f^2e - 12ab^2c^3f^2e + b^2c^3e^2 + 4a^2c^4e^2)*x + (b^4c^2 - 8ab^2c^3 + 16a^2c^4) + 3(8b^2c^4d^2 + 2b^3c^2d^2f + 8ab^2c^3d^2f - b^5f^2 + 6ab^3c^2f^2 - 8b^2c^3d^2e - 4ab^2c^2f^2e - 16a^2c^3f^2e + b^3c^2e^2 + 4ab^2c^3e^2))/(b^4c^2 - 8ab^2c^3 + 16a^2c^4)*x + 6(b^2c^3d^2 + 4a^2c^4d^2 + 4ab^2c^2d^2f - ab^4f^2 + 7a^2b^2c^2f^2 - 4a^3c^2f^2 - b^3c^2d^2e - 4ab^2c^3d^2e - 8a^2b^2c^2f^2e + 2ab^2c^2e^2))/(b^4c^2 - 8ab^2c^3 + 16a^2c^4)}{3(c^2 + bx + a)^{5/2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(5/2),x, algorithm="giac")

[Out] -f^2\*log(abs(-2\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*sqrt(c) - b))/c^(5/2) + 2/3\*(((2\*(8\*c^5\*d^2 + 2\*b^2\*c^3\*d^2\*f + 8\*a^2\*c^4\*d^2\*f - 2\*b^4\*c^3\*f^2 + 14\*a\*b^2\*c^2\*f^2 - 16\*a^2\*c^3\*f^2 - 8\*b^2\*c^4\*d^2\*e + b^3\*c^2\*f^2\*e - 12\*a\*b^2\*c^3\*f^2\*e + b^2\*c^3\*e^2 + 4\*a^2\*c^4\*e^2)\*x/(b^4\*c^2 - 8\*a\*b^2\*c^3 + 16\*a^2\*c^4) + 3\*(8\*b^2\*c^4\*d^2 + 2\*b^3\*c^2\*d^2\*f + 8\*a\*b^2\*c^3\*d^2\*f - b^5\*f^2 + 6\*a\*b^3\*c^2\*f^2 - 8\*b^2\*c^3\*d^2\*e - 4\*a\*b^2\*c^2\*f^2\*e - 16\*a^2\*c^3\*f^2\*e + b^3\*c^2\*e^2 + 4\*a\*b^2\*c^3\*e^2)/(b^4\*c^2 - 8\*a\*b^2\*c^3 + 16\*a^2\*c^4))\*x + 6\*(b^2\*c^3\*d^2 + 4\*a^2\*c^4\*d^2 + 4\*a\*b^2\*c^2\*d^2\*f - a\*b^4\*f^2 + 7\*a^2\*b^2\*c^2\*f^2 - 4\*a^3\*c^2\*f^2 - b^3\*c^2\*d^2\*e - 4\*a\*b^2\*c^3\*d^2\*e - 8\*a^2\*b^2\*c^2\*f^2\*e + 2\*a\*b^2\*c^2\*e^2))/(b^4\*c^2 - 8\*a\*b^2\*c^3 + 16\*a^2\*c^4)



$$*a^2*c^4)*x - (b^3*c^2*d^2 - 12*a*b*c^3*d^2 - 16*a^2*b*c^2*d*f + 3*a^2*b^3*f^2 - 20*a^3*b*c*f^2 + 4*a*b^2*c^2*d*e + 16*a^2*c^3*d*e + 32*a^3*c^2*f*e - 8*a^2*b*c^2*e^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)/(c*x^2 + b*x + a)^{(3/2)}$$

**maple [B]** time = 0.01, size = 1786, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(5/2),x)

[Out] 
$$\begin{aligned} & -2/3*d*e/c/(c*x^2+b*x+a)^{(3/2)}+2/3*d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*b-1/ \\ & 2*x/c/(c*x^2+b*x+a)^{(3/2)}*e^2+1/12*b/c^2/(c*x^2+b*x+a)^{(3/2)}*e^2-1/3*f^2*x^ \\ & 3/c/(c*x^2+b*x+a)^{(3/2)}-1/48*f^2*b^3/c^4/(c*x^2+b*x+a)^{(3/2)}-f^2/c^2*x/(c*x \\ & ^2+b*x+a)^{(1/2)}+1/2*f^2/c^3*b/(c*x^2+b*x+a)^{(1/2)}-2*e*f*b/c*a/(4*a*c-b^2)/( \\ & c*x^2+b*x+a)^{(3/2)}*x+16/3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*b*d*f-1/24*f^ \\ & 2*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x-1/3*f^2*b^4/c^2/(4*a*c-b^2)^2/( \\ & c*x^2+b*x+a)^{(1/2)}*x+1/4*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}+2*f^ \\ & 2*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}+f^2/c^2*b^2/(4*a*c-b^2)/(c*x^ \\ & 2+b*x+a)^{(1/2)}*x-4/3*d*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x-2/3*d*e*b^2/c/ \\ & (4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}-1/2*e*f*b/c^2*x/(c*x^2+b*x+a)^{(3/2)}+1/12*e* \\ & f*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}+2/3*e*f*b^4/c^2/(4*a*c-b^2)^2/(c* \\ & x^2+b*x+a)^{(1/2)}+1/6*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x*e^2+1/6*b^3/c^ \\ & 2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*d*f+8/3*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{( \\ & 1/2)}*x*d*f+4/3*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*d*f+4/3*a/(4*a*c-b^2 \\ & )/(c*x^2+b*x+a)^{(3/2)}*x*d*f+1/3*a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}*b*e^2+1 \\ & 6/3*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x*e^2+f^2/c^(5/2)*ln((c*x+1/2*b)/ \\ & c^(1/2)+(c*x^2+b*x+a)^(1/2))-2*e*f*x^2/c/(c*x^2+b*x+a)^{(3/2)}+1/12*e*f*b^2/c \\ & ^3/(c*x^2+b*x+a)^{(3/2)}-4/3*e*f*a/c^2/(c*x^2+b*x+a)^{(3/2)}+1/2*f^2*b/c^2*x^2/ \\ & (c*x^2+b*x+a)^{(3/2)}+1/8*f^2*b^2/c^3*x/(c*x^2+b*x+a)^{(3/2)}-1/48*f^2*b^5/c^4/ \\ & (4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)}-1/6*f^2*b^5/c^3/(4*a*c-b^2)^2/(c*x^2+b*x+a) \\ & ^{(1/2)}+1/3*f^2*b/c^3*a/(c*x^2+b*x+a)^{(3/2)}+1/2*f^2/c^3*b^3/(4*a*c-b^2)/(c*x \\ & ^2+b*x+a)^{(1/2)}+2/3*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*e^2+2/3*a/(4*a* \\ & c-b^2)/(c*x^2+b*x+a)^{(3/2)}*x*e^2+8/3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*b* \\ & e^2-16/3*d*e*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}+4/3*d^2/(4*a*c-b^2)/(c*x \\ & ^2+b*x+a)^{(3/2)}*x*c+32/3*d^2*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x+16/3*d \\ & ^2*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*b-x/c/(c*x^2+b*x+a)^{(3/2)}*d*f+1/6*b/ \\ & c^2/(c*x^2+b*x+a)^{(3/2)}*d*f+32/3*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x*d* \\ & f-32/3*d*e*b*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)}*x-16*e*f*b*a/(4*a*c-b^2)^2 \\ & /((c*x^2+b*x+a)^(1/2)*x-8*e*f*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2))+1/6* \\ & e*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+1/2*f^2*b^2/c^2*a/(4*a*c-b^2) \\ & /((c*x^2+b*x+a)^(3/2))*x+4*f^2*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/ \\ & 3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*d*f+2/3*a/c/(4*a*c-b^2)/(c*x^2+b* \\ & x+a)^(3/2)*b*d*f+1/12*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^2+4/3*b^2/( \\ & 4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^2+4/3*e*f*b^3/c/(4*a*c-b^2)^2/(c*x^2+b \\ & *x+a)^(1/2)*x-e*f*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2) \end{aligned}$$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^2/(c\*x^2+b\*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f x^2 + e x + d)^2}{(c x^2 + b x + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(5/2), x)

[Out] int((d + e\*x + f\*x^2)^2/(a + b\*x + c\*x^2)^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*\*2/(c\*x\*\*2+b\*x+a)\*\*(5/2), x)

[Out] Timed out

$$3.117 \quad \int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}} dx$$

**Optimal.** Leaf size=131

$$\frac{2\left(c\left(2ae - b\left(\frac{af}{c} + d\right)\right) - x(-2acf + b^2f - bce + 2c^2d)\right)}{3c(b^2 - 4ac)(a + bx + cx^2)^{3/2}} + \frac{2(b + 2cx)\left(4af + \frac{b^2f}{c} - 4be + 8cd\right)}{3(b^2 - 4ac)^2 \sqrt{a + bx + cx^2}}$$

**Rubi [A]** time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {1660, 12, 613}

$$\frac{2\left(c\left(2ae - b\left(\frac{af}{c} + d\right)\right) - x(-2acf + b^2f - bce + 2c^2d)\right)}{3c(b^2 - 4ac)(a + bx + cx^2)^{3/2}} + \frac{2(b + 2cx)\left(4af + \frac{b^2f}{c} - 4be + 8cd\right)}{3(b^2 - 4ac)^2 \sqrt{a + bx + cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(5/2), x]

[Out] (2\*(c\*(2\*a\*e - b\*(d + (a\*f)/c)) - (2\*c^2\*d - b\*c\*e + b^2\*f - 2\*a\*c\*f)\*x))/(3\*c\*(b^2 - 4\*a\*c)\*(a + b\*x + c\*x^2)^(3/2)) + (2\*(8\*c\*d - 4\*b\*e + 4\*a\*f + (b^2\*f)/c)\*(b + 2\*c\*x))/(3\*(b^2 - 4\*a\*c)^2\*Sqrt[a + b\*x + c\*x^2])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 613

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-3/2), x\_Symbol] := Simp[(-2\*(b + 2\*c\*x))/((b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

#### Rubi steps

$$\int \frac{d + ex + fx^2}{(a + bx + cx^2)^{5/2}} dx = \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{3c (b^2 - 4ac) (a + bx + cx^2)^{3/2}} - \frac{2 \int \frac{8cd - 4be + 4af + \frac{b^2f}{c}}{2(a + bx + cx^2)^{3/2}} dx}{3 (b^2 - 4ac)}$$

$$= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{3c (b^2 - 4ac) (a + bx + cx^2)^{3/2}} - \frac{\left( 8cd - 4be + 4af + \frac{b^2f}{c} \right) \int \frac{1}{(a + bx + cx^2)^{3/2}} dx}{3 (b^2 - 4ac)}$$

$$= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{3c (b^2 - 4ac) (a + bx + cx^2)^{3/2}} + \frac{2 \left( 8cd - 4be + 4af + \frac{b^2f}{c} \right) (b + cx)}{3 (b^2 - 4ac)^2 \sqrt{a + bx + cx^2}}$$

**Mathematica [A]** time = 0.38, size = 147, normalized size = 1.12

$$\frac{8b(2a^2f + 3ac(d - ex + fx^2) - 2c^2x^2(ex - 3d)) + 16c(-a^2e + acx(3d + fx^2) + 2c^2dx^3) - 4b^2(ae - 6fx) - cx(3d - 6ex + fx^2) - 2b^3(d + 3x(e - fx))}{3(b^2 - 4ac)^2(a + x(b + cx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(5/2), x]

[Out] (-2\*b^3\*(d + 3\*x\*(e - f\*x)) + 16\*c\*(-(a^2\*e) + 2\*c^2\*d\*x^3 + a\*c\*x\*(3\*d + f\*x^2)) - 4\*b^2\*(a\*(e - 6\*f\*x) - c\*x\*(3\*d - 6\*e\*x + f\*x^2)) + 8\*b\*(2\*a^2\*f - 2\*c^2\*x^2\*(-3\*d + e\*x) + 3\*a\*c\*(d - e\*x + f\*x^2)))/(3\*(b^2 - 4\*a\*c)^2\*(a + x\*(b + c\*x))^(3/2))

**IntegrateAlgebraic [A]** time = 1.17, size = 176, normalized size = 1.34

$$\frac{2(-8a^2bf + 8a^2ce + 2ab^2e - 12ab^2fx - 12abcd + 12abcex - 12abcfx^2 - 24ac^2dx - 8ac^2fx^3 + b^3d + 3b^3ex - 3b^3fx^2 - 6b^2cdx + 12b^2cex^2 - 2b^2cfx^3 - 24b^2dx^2 + 8bc^2ex^3 - 16c^3dx^3)}{3(b^2 - 4ac)^2(a + bx + cx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(5/2), x]

[Out] (-2\*(b^3\*d - 12\*a\*b\*c\*d + 2\*a\*b^2\*e + 8\*a^2\*c\*e - 8\*a^2\*b\*f - 6\*b^2\*c\*d\*x - 24\*a\*c^2\*d\*x + 3\*b^3\*e\*x + 12\*a\*b\*c\*e\*x - 12\*a\*b^2\*f\*x - 24\*b\*c^2\*d\*x^2 + 12\*b^2\*c\*e\*x^2 - 3\*b^3\*f\*x^2 - 12\*a\*b\*c\*f\*x^2 - 16\*c^3\*d\*x^3 + 8\*b\*c^2\*e\*x^3 - 2\*b^2\*c\*f\*x^3 - 8\*a\*c^2\*f\*x^3))/(3\*(b^2 - 4\*a\*c)^2\*(a + b\*x + c\*x^2)^(3/2))

**fricas [B]** time = 2.60, size = 286, normalized size = 2.18

$$\frac{2(8a^2bf + 2(8c^3d - 4bc^2e + (b^2c + 4ac^2)f)x^3 + 3(8bc^2d - 4b^2ce + (b^3 + 4abc)f)x^2 - (b^3 - 12abc)d - 2(ab^2 + 4a^2c)e + 3(4ab^2f + 2(b^2c + 4ac^2)d - (b^3 + 4abc)e)x)\sqrt{cx^2 + bx + a}}{3(a^2b^4 - 8a^3b^2c + 16a^4c^2 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^4 + 2(b^5c - 8ab^3c^2 + 16a^2bc^3)x^3 + (b^6 - 6ab^4c + 32a^3c^3)x^2 + 2(ab^5 - 8a^2b^3c + 16a^3bc^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(5/2), x, algorithm="fricas")

[Out] 2/3\*(8\*a^2\*b\*f + 2\*(8\*c^3\*d - 4\*b\*c^2\*e + (b^2\*c + 4\*a\*c^2)\*f)\*x^3 + 3\*(8\*b\*c^2\*d - 4\*b^2\*c\*e + (b^3 + 4\*a\*b\*c)\*f)\*x^2 - (b^3 - 12\*a\*b\*c)\*d - 2\*(a\*b^2 + 4\*a^2\*c)\*e + 3\*(4\*a\*b^2\*f + 2\*(b^2\*c + 4\*a\*c^2)\*d - (b^3 + 4\*a\*b\*c)\*e)\*x)\*sqrt(c\*x^2 + b\*x + a)/(a^2\*b^4 - 8\*a^3\*b^2\*c + 16\*a^4\*c^2 + (b^4\*c^2 - 8\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*x^4 + 2\*(b^5\*c - 8\*a\*b^3\*c^2 + 16\*a^2\*b\*c^3)\*x^3 + (b^6 - 6\*a\*b^4\*c + 32\*a^3\*c^3)\*x^2 + 2\*(a\*b^5 - 8\*a^2\*b^3\*c + 16\*a^3\*b\*c^2)\*x)

**giac [A]** time = 0.32, size = 240, normalized size = 1.83

$$\frac{2 \left( \left( \frac{2(8c^3d + b^2cf + 4ac^2f - 4bc^2e)x}{b^4 - 8ab^2c + 16a^2c^2} + \frac{3(8bc^2d + b^3f + 4abcf - 4b^2ce)}{b^4 - 8ab^2c + 16a^2c^2} \right) x + \frac{3(2b^2cd + 8ac^2d + 4ab^2f - b^3e - 4abce)}{b^4 - 8ab^2c + 16a^2c^2} \right) x - \frac{b^3d - 12abcd - 8a^2bf + 2ab^2e + 8a^2ce}{b^4 - 8ab^2c + 16a^2c^2}}{3(cx^2 + bx + a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(5/2),x, algorithm="giac")

[Out]  $\frac{2}{3} * \left( \left( \frac{2 * (8 * c^3 * d + b^2 * c * f + 4 * a * c^2 * f - 4 * b * c^2 * e) * x}{b^4 - 8 * a * b^2 * c + 16 * a^2 * c^2} + 3 * (8 * b * c^2 * d + b^3 * f + 4 * a * b * c * f - 4 * b^2 * c * e) \right) / (b^4 - 8 * a * b^2 * c + 16 * a^2 * c^2) \right) * x + 3 * \left( \frac{2 * b^2 * c * d + 8 * a * c^2 * d + 4 * a * b^2 * f - b^3 * e - 4 * a * b * c * e}{b^4 - 8 * a * b^2 * c + 16 * a^2 * c^2} \right) * x - \frac{(b^3 * d - 12 * a * b * c * d - 8 * a^2 * b * f + 2 * a * b^2 * e + 8 * a^2 * c * e)}{(b^4 - 8 * a * b^2 * c + 16 * a^2 * c^2)} / (c * x^2 + b * x + a)^{\frac{3}{2}}$

**maple [A]** time = 0.01, size = 185, normalized size = 1.41

$$\frac{\frac{16}{3}ac^2fx^3 + \frac{4}{3}b^2cfx^3 - \frac{16}{3}bc^2ex^3 + \frac{32}{3}c^3dx^3 + 8abcfx^2 + 2b^3fx^2 - 8b^2cex^2 + 16bc^2dx^2 + 8ab^2fx - 8abce + 16a^2cx - 2b^3ex + 4b^2cdx + \frac{16}{3}a^2bf - \frac{16}{3}a^2ce - \frac{4}{3}ab^2e + 8abcd - \frac{2}{3}b^3d}{(cx^2 + bx + a)^{\frac{3}{2}}(16a^2c^2 - 8ab^2c + b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(5/2),x)

[Out]  $\frac{2}{3} / (c * x^2 + b * x + a)^{\frac{3}{2}} * \left( \frac{8 * a * c^2 * f * x^3 + 2 * b^2 * c * f * x^3 - 8 * b * c^2 * e * x^3 + 16 * c^3 * d * x^3 + 12 * a * b * c * f * x^2 + 3 * b^3 * f * x^2 - 12 * b^2 * c * e * x^2 + 24 * b * c^2 * d * x^2 + 12 * a * b^2 * f * x - 12 * a * b * c * e * x + 24 * a * c^2 * d * x - 3 * b^3 * e * x + 6 * b^2 * c * d * x + 8 * a^2 * b * f - 8 * a^2 * c * e - 2 * a * b^2 * e + 12 * a * b * c * d - b^3 * d}{16 * a^2 * c^2 - 8 * a * b^2 * c + b^4} \right)$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad [B]** time = 3.71, size = 175, normalized size = 1.34

$$\frac{2(8fa^2b - 8ea^2c + 12fab^2x - 2ea^2b + 12fabcx^2 - 12eabcx + 12dabc + 8fa^2c^2x^3 + 24da^2cx + 3fb^3x^2 - 3eb^3x - db^3 + 2fb^2cx^3 - 12eb^2cx^2 + 6db^2cx - 8eb^2c^2x^3 + 24db^2c^2x^2 + 16dc^3x^3)}{3(4ac - b^2)^2(cx^2 + bx + a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(5/2),x)

[Out]  $\frac{(2 * (16 * c^3 * d * x^3 - b^3 * d + 3 * b^3 * f * x^2 - 2 * a * b^2 * e + 8 * a^2 * b * f - 8 * a^2 * c * e - 3 * b^3 * e * x + 24 * a * c^2 * d * x + 12 * a * b^2 * f * x + 6 * b^2 * c * d * x + 24 * b * c^2 * d * x^2 - 12 * b^2 * c * e * x^2 + 8 * a * c^2 * f * x^3 - 8 * b * c^2 * e * x^3 + 2 * b^2 * c * f * x^3 + 12 * a * b * c * d - 12 * a * b * c * e * x + 12 * a * b * c * f * x^2)) / (3 * (4 * a * c - b^2)^2 * (a + b * x + c * x^2)^{\frac{3}{2}})}$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(5/2),x)

[Out] Timed out

$$3.118 \quad \int \frac{1}{\sqrt{-7+2x+5x^2}(8+12x+5x^2)} dx$$

**Optimal.** Leaf size=51

$$\frac{1}{10} \tan^{-1} \left( \frac{5(x+2)}{2\sqrt{5x^2+2x-7}} \right) + \frac{1}{5} \tanh^{-1} \left( \frac{5(x+1)}{\sqrt{5x^2+2x-7}} \right)$$

**Rubi [A]** time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {986, 1029, 203, 207}

$$\frac{1}{10} \tan^{-1} \left( \frac{5(x+2)}{2\sqrt{5x^2+2x-7}} \right) + \frac{1}{5} \tanh^{-1} \left( \frac{5(x+1)}{\sqrt{5x^2+2x-7}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-7 + 2\*x + 5\*x^2]\*(8 + 12\*x + 5\*x^2)),x]

[Out] ArcTan[(5\*(2 + x))/(2\*Sqrt[-7 + 2\*x + 5\*x^2])]/10 + ArcTanh[(5\*(1 + x))/Sqrt[-7 + 2\*x + 5\*x^2]]/5

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

#### Rule 986

Int[1/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] :> With[{q = Rt[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f)], Dist[1/(2\*q), Int[(c\*d - a\*f + q + (c\*e - b\*f)\*x)/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[1/(2\*q), Int[(c\*d - a\*f - q + (c\*e - b\*f)\*x)/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[c\*e - b\*f, 0] && NegQ[b^2 - 4\*a\*c]

#### Rule 1029

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] :> Dist[-2\*g\*(g\*b - 2\*a\*h), Subst[Int[1/Simp[g\*(g\*b - 2\*a\*h)\*(b^2 - 4\*a\*c) - (b\*d - a\*e)\*x^2, x], x], x, Simp[g\*b - 2\*a\*h - (b\*h - 2\*g\*c)\*x, x]/Sqrt[d + e\*x + f\*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && NeQ[b\*d - a\*e, 0] && EqQ[h^2\*(b\*d - a\*e) - 2\*g\*h\*(c\*d - a\*f) + g^2\*(c\*e - b\*f), 0]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-7+2x+5x^2} (8+12x+5x^2)} dx &= -\left( \frac{1}{50} \int \frac{-100-50x}{\sqrt{-7+2x+5x^2} (8+12x+5x^2)} dx \right) + \frac{1}{50} \int \frac{1}{\sqrt{-7+2x+5x^2}} dx \\ &= 400 \operatorname{Subst} \left( \int \frac{1}{160000+100x^2} dx, x, \frac{200+100x}{\sqrt{-7+2x+5x^2}} \right) + 1600 \operatorname{Subst} \left( \int \frac{1}{160000+100x^2} dx, x, \frac{200+100x}{\sqrt{-7+2x+5x^2}} \right) \\ &= \frac{1}{10} \tan^{-1} \left( \frac{5(2+x)}{2\sqrt{-7+2x+5x^2}} \right) + \frac{1}{5} \tanh^{-1} \left( \frac{5(1+x)}{\sqrt{-7+2x+5x^2}} \right) \end{aligned}$$

**Mathematica [C]** time = 0.04, size = 81, normalized size = 1.59

$$\left( \frac{1}{10} - \frac{i}{20} \right) \tanh^{-1} \left( \frac{\left( \frac{1}{100} + \frac{i}{50} \right) ((100-40i)x + (164-8i))}{\sqrt{5x^2+2x-7}} \right) - \left( \frac{1}{20} - \frac{i}{10} \right) \tanh^{-1} \left( \frac{\left( \frac{1}{50} + \frac{i}{100} \right) ((-100-40i)x - (164+8i))}{\sqrt{5x^2+2x-7}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-7 + 2\*x + 5\*x^2]\*(8 + 12\*x + 5\*x^2)),x]

[Out] (-1/20 + I/10)\*ArcTan[((1/50 + I/100)\*((-164 - 8\*I) - (100 + 40\*I)\*x))/Sqrt[-7 + 2\*x + 5\*x^2]] + (1/10 - I/20)\*ArcTanh[((1/100 + I/50)\*((164 - 8\*I) + (100 - 40\*I)\*x))/Sqrt[-7 + 2\*x + 5\*x^2]]

**IntegrateAlgebraic [A]** time = 0.34, size = 53, normalized size = 1.04

$$\frac{1}{10} \tan^{-1} \left( \frac{\frac{5x}{2} + 5}{\sqrt{5x^2+2x-7}} \right) + \frac{1}{5} \tanh^{-1} \left( \frac{5x+5}{\sqrt{5x^2+2x-7}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[-7 + 2\*x + 5\*x^2]\*(8 + 12\*x + 5\*x^2)),x]

[Out] ArcTan[(5 + (5\*x)/2)/Sqrt[-7 + 2\*x + 5\*x^2]]/10 + ArcTanh[(5 + 5\*x)/Sqrt[-7 + 2\*x + 5\*x^2]]/5

**fricas [B]** time = 0.45, size = 154, normalized size = 3.02

$$\frac{1}{20} \arctan \left( \frac{27x^2 + 20\sqrt{5x^2+2x-7}(x+2) + 36x}{31x^2 + 16x - 56} \right) + \frac{1}{20} \arctan \left( \frac{27x^2 - 20\sqrt{5x^2+2x-7}(x+2) + 36x}{31x^2 + 16x - 56} \right) + \frac{1}{20} \log \left( \frac{15x^2 + 5\sqrt{5x^2+2x-7}(x+1) + 26x + 9}{x^2} \right) - \frac{1}{20} \log \left( \frac{15x^2 - 5\sqrt{5x^2+2x-7}(x+1) + 26x + 9}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+12\*x+8)/(5\*x^2+2\*x-7)^(1/2),x, algorithm="fricas")

[Out] 1/20\*arctan((27\*x^2 + 20\*sqrt(5\*x^2 + 2\*x - 7)\*(x + 2) + 36\*x)/(31\*x^2 + 16\*x - 56)) + 1/20\*arctan(-(27\*x^2 - 20\*sqrt(5\*x^2 + 2\*x - 7)\*(x + 2) + 36\*x)/(31\*x^2 + 16\*x - 56)) + 1/20\*log((15\*x^2 + 5\*sqrt(5\*x^2 + 2\*x - 7)\*(x + 1) + 26\*x + 9)/x^2) - 1/20\*log((15\*x^2 - 5\*sqrt(5\*x^2 + 2\*x - 7)\*(x + 1) + 26\*x + 9)/x^2)

**giac [B]** time = 0.28, size = 205, normalized size = 4.02

$$-\frac{1}{10} \arctan \left( \frac{5\sqrt{5}x + 6\sqrt{5} - 5\sqrt{5x^2+2x-7} + 5}{2(\sqrt{5}+5)} \right) - \frac{1}{10} \arctan \left( \frac{5\sqrt{5}x + 6\sqrt{5} - 5\sqrt{5x^2+2x-7} - 5}{2(\sqrt{5}-5)} \right) + \frac{1}{10} \log \left( 5(\sqrt{5x^2+2x-7})^2 + 2(\sqrt{5x^2+2x-7})(6\sqrt{5}+5) + 20\sqrt{5} + 65 \right) - \frac{1}{10} \log \left( 5(\sqrt{5x^2+2x-7})^2 + 2(\sqrt{5x^2+2x-7})(6\sqrt{5}-5) - 20\sqrt{5} + 65 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5\*x^2+12\*x+8)/(5\*x^2+2\*x-7)^(1/2),x, algorithm="giac")

[Out] -1/10\*arctan(-1/2\*(5\*sqrt(5)\*x + 6\*sqrt(5) - 5\*sqrt(5\*x^2 + 2\*x - 7) + 5)/(sqrt(5) + 5)) - 1/10\*arctan(1/2\*(5\*sqrt(5)\*x + 6\*sqrt(5) - 5\*sqrt(5\*x^2 + 2\*x - 7) + 5)/(sqrt(5) - 5)) + 1/10\*log(5\*(sqrt(5\*x^2 + 2\*x - 7))^2 + 2\*(sqrt(5\*x^2 + 2\*x - 7))\*(6\*sqrt(5) + 5) + 20\*sqrt(5) + 65) - 1/10\*log(5\*(sqrt(5\*x^2 + 2\*x - 7))^2 + 2\*(sqrt(5\*x^2 + 2\*x - 7))\*(6\*sqrt(5) - 5) - 20\*sqrt(5) + 65)

$\frac{(x-7) - 5}{(\sqrt{5} - 5)} + \frac{1}{10} \log(5(\sqrt{5}x - \sqrt{5x^2 + 2x - 7})^2 + 2(\sqrt{5}x - \sqrt{5x^2 + 2x - 7}))(6\sqrt{5} + 5) + 20\sqrt{5} + 65) - \frac{1}{10} \log(5(\sqrt{5}x - \sqrt{5x^2 + 2x - 7})^2 + 2(\sqrt{5}x - \sqrt{5x^2 + 2x - 7}))(6\sqrt{5} - 5) - 20\sqrt{5} + 65)$

**maple** [B] time = 0.02, size = 144, normalized size = 2.82

$$\frac{\sqrt{-\frac{4(x+2)^2}{(-x-1)^2} + 9} \left( 2 \operatorname{arctanh} \left( \frac{\sqrt{-\frac{4(x+2)^2}{(-x-1)^2} + 9}}{5} \right) + \operatorname{arctan} \left( \frac{5 \sqrt{-\frac{4(x+2)^2}{(-x-1)^2} + 9} (x+2)}{2 \left( \frac{4(x+2)^2}{(-x-1)^2} - 9 \right) (-x-1)} \right) \right)}{10 \sqrt{-\frac{4(x+2)^2}{(-x-1)^2} - 9} \left( 1 + \frac{x+2}{-x-1} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x)`

[Out]  $-1/10 * (-4*(x+2)^2/(-1-x)^2+9)^(1/2) * (2*\operatorname{arctanh}(1/5*(-4*(x+2)^2/(-1-x)^2+9)^(1/2)) + \operatorname{arctan}(5/2*(-4*(x+2)^2/(-1-x)^2+9)^(1/2)/(4*(x+2)^2/(-1-x)^2-9)*(x+2)/(-1-x)))/(-4*(x+2)^2/(-1-x)^2-9)/(1+(x+2)/(-1-x))^2)^(1/2)/(1+(x+2)/(-1-x))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x^2 + 12x + 8)\sqrt{5x^2 + 2x - 7}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((5*x^2 + 12*x + 8)*sqrt(5*x^2 + 2*x - 7)), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{5x^2 + 2x - 7} (5x^2 + 12x + 8)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2*x + 5*x^2 - 7)^(1/2)*(12*x + 5*x^2 + 8)),x)`

[Out] `int(1/((2*x + 5*x^2 - 7)^(1/2)*(12*x + 5*x^2 + 8)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)(5x+7)} (5x^2 + 12x + 8)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5*x**2+12*x+8)/(5*x**2+2*x-7)**(1/2),x)`

[Out] `Integral(1/(sqrt((x - 1)*(5*x + 7))*(5*x**2 + 12*x + 8)), x)`



# Chapter 4

## Appendix

### Local contents

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## 4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

**Mathematica format** Mathematica\_syntax\_CAS\_integration\_elementary\_version.zip

**Maple and Mupad format** Maple\_syntax\_CAS\_integration\_elementary\_version.zip

**Sympy format** SYMPY\_syntax\_CAS\_integration\_elementary\_version.zip

**Sage math format** SAGE\_syntax\_CAS\_integration\_elementary\_version.zip

## 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```

(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType, expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]], 2]],
            Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
            If[ElementaryFunctionQ[Head[expn]],
              Max[3, ExpnType[expn[[1]]]],
              If[SpecialFunctionQ[Head[expn]],
                Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
                If[HypergeometricFunctionQ[Head[expn]],
                  Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
                  If[AppellFunctionQ[Head[expn]],
                    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
                    If[Head[expn]===RootSum,
                      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
                      If[Head[expn]===Integrate || Head[expn]===Int,
                        Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
                        9]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

```

```

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

```

```

HypergeometricFunctionQ[func_] :=

```

```
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## 4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#                    if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#                    see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
end if;
```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

### 4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                 asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                 asinh,acosh,atanh,acoth,asech,acsch
                 ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
                 fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                 gamma,loggamma,digamma,zeta,polylog,LambertW,
                 elliptic_f,elliptic_e,elliptic_pi,exp_polar
                 ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

def is_atom(expn):

```

```

try:
    if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
    else:
        return False

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+`') or
    type(expn,'*`)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function

```



```

def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

#### 4.2.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False

```

```

else:
    return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
        hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):

```

```

        return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
            return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
        return False

    except AttributeError as error:
        return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```